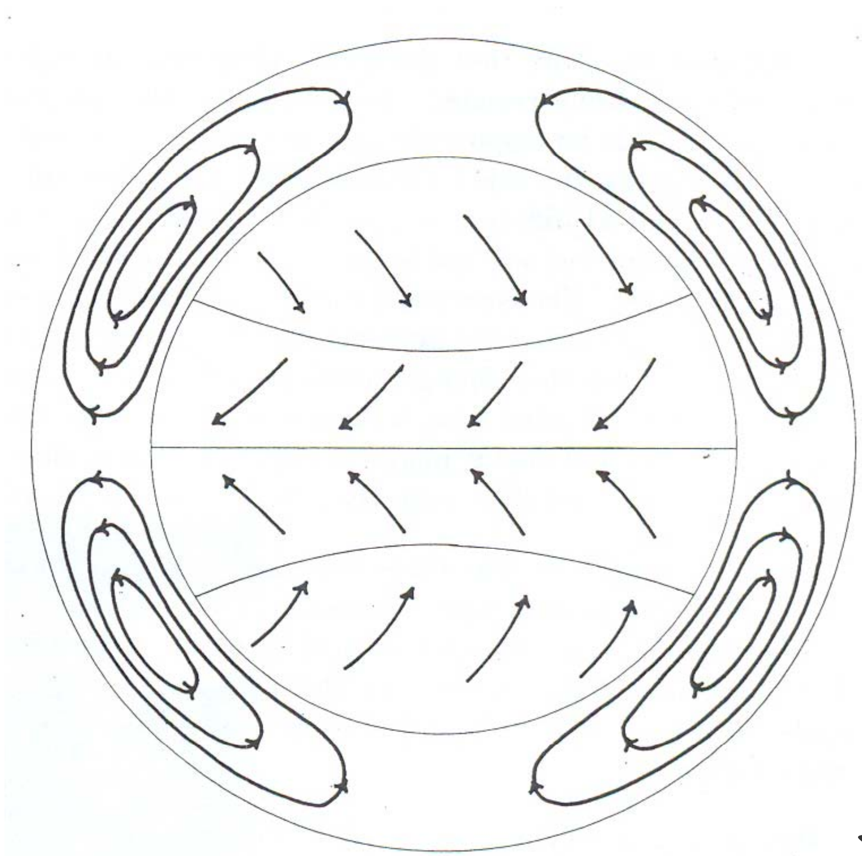
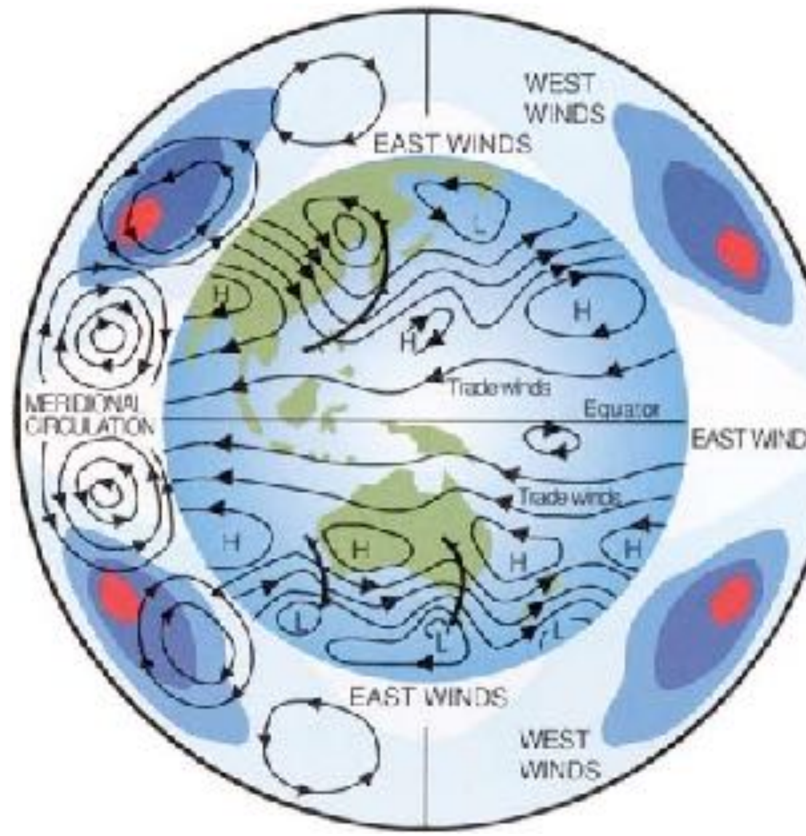
第三章:

Hadley 环流（二）





授课教师：张洋

2022. 10. 20

Hadley Cell

- Observations

Review

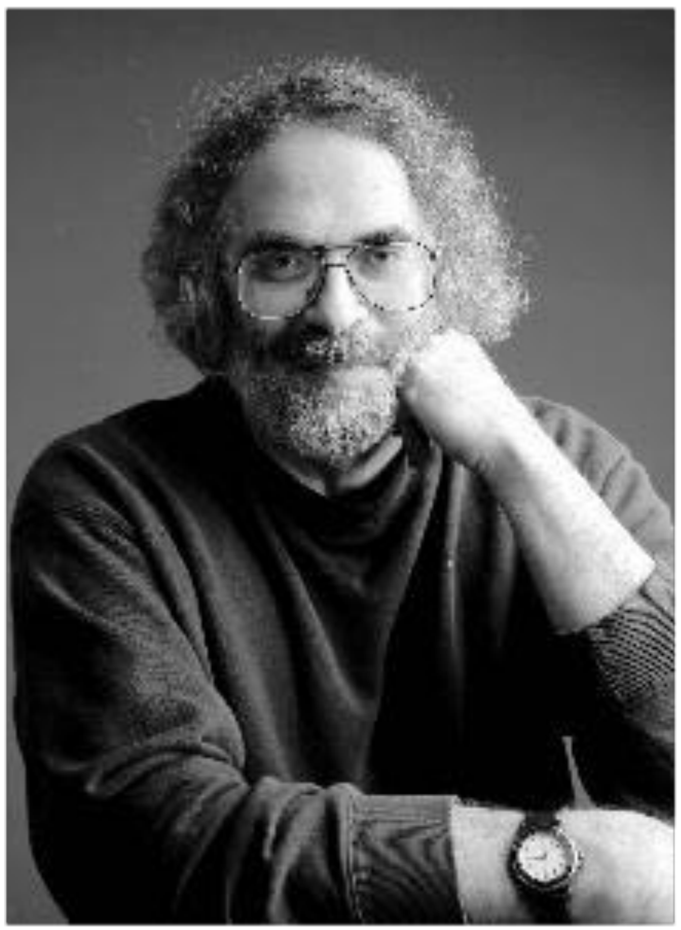
- Summary （小结）
  - **Temperature field:** the equator-pole temperature gradient is much smaller than the RE temperature gradient.
  - **Wind fields: meridional winds** strongest at tropopause and surface; **vertical velocity** strongest at mid-level of the troposphere.
  - **Jets (zonal winds):** strong subtropical jet at **upper level** with its maximum in the latitudes at the edge or just poleward of the descending branch of the Hadley cell; **surface winds**-easterlies near the equator and westerlies in the extratropics.
  - **Strong seasonal variations:** in summer or winter, Hadley cell always appears as a strong single cell across the equator with the ascending branch in the tropics of the summer hemisphere.

Hadley Cell

- Theories

Review

- **Held-Hou model (1980)**



Isaac M. Held

Nonlinear Axially Symmetric Circulations in a Nearly Inviscid Atmosphere

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(Manuscript received 23 July 1979, in final form 16 October 1979)

ABSTRACT

The structure of certain axially symmetric circulations in a stably stratified, differentially heated, rotating Boussinesq fluid on a sphere is analyzed. A simple approximate theory [similar to that introduced by Schneider (1977)] is developed for the case in which the fluid is sufficiently inviscid that the poleward flow in the Hadley cell is nearly angular momentum conserving. The theory predicts the width of the Hadley cell, the total poleward heat flux, the latitude of the upper level jet in the zonal wind, and the distribution of surface easterlies and westerlies. Fundamental differences between such nearly inviscid circulations and the more commonly studied viscous axisymmetric flows are emphasized. The theory is checked against numerical solutions to the model equations.

1. Introduction

The importance of mixing induced by large-scale baroclinic or barotropic instabilities for the general circulation of the atmosphere can best be appreciated by artificially suppressing these instabilities and examining the circulation which develops in their absence. This is most easily accomplished in the idealized case for which radiative forcing and the lower boundary condition are both axially symmetric (independent of longitude). The flow of interest in this case is the large-scale axisymmetric flow consistent with radiative forcing and whatever small-scale mixing is still present in the atmosphere after the large-scale instabilities have been suppressed.

Such axisymmetric circulations have not received as much attention in the meteorological literature as one might expect, given what would appear to be their natural position as first approximations to the general circulation. Reasons for this neglect are not hard to find. It is the accepted wisdom that large-scale zonally asymmetric baroclinic instabilities are

atmospheres (e.g., Dickinson, 1971; Leovy, 1964), the meridional circulation is effectively determined by the parameterized small-scale frictional stresses in the zonal momentum equation. Detailed analyses of such models do not promise to be very fruitful as long as theories for small-scale momentum mixing are themselves not very well developed.

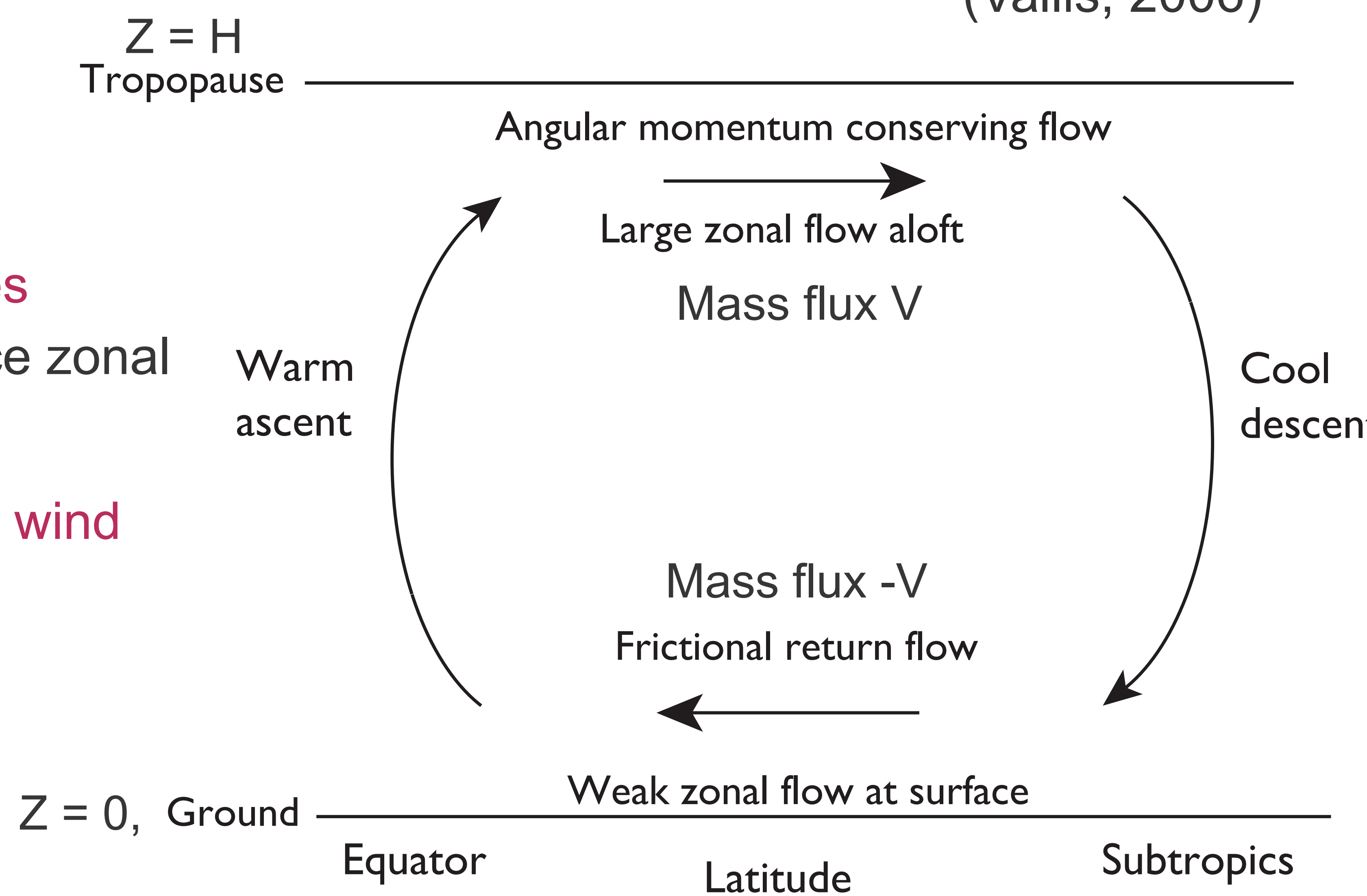
Schneider and Lindzen have recently computed some axisymmetric flows forced by small-scale fluxes of heat and momentum that do bear some resemblance to the observed circulation (Schneider and Lindzen, 1977; Schneider, 1977). Using simple theories for moist convective as well as boundary and radiative fluxes, Schneider obtains a Hadley cell which terminates abruptly at more or less the right latitude, a very strong subtropical jet at the poleward boundary of the Hadley cell, strong trade winds in the tropics, and a shallow Ferrel cell and surface westerlies poleward of the trades. Nakamura (1978) describes an effectively axisymmetric calculation (with heating and frictional formulations differ-

Held-Hou model (1980)

Make assumptions:

- the circulation is steady;
- the upper branch conserves angular momentum; surface zonal winds are weak;
- the circulation is in thermal wind balance.

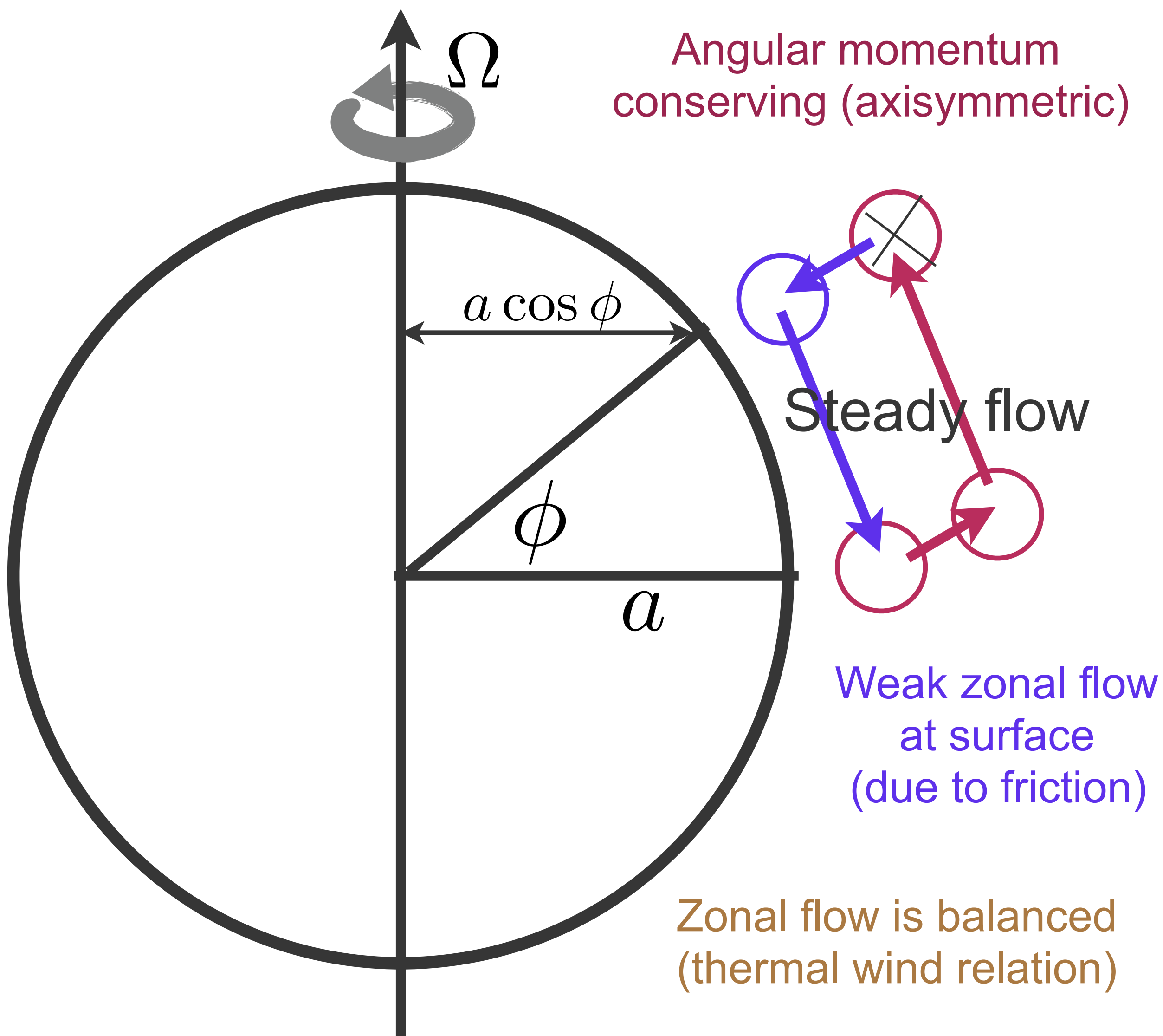
(Vallis, 2006)



- Theories

Review

- **Held-Hou model (1980)**



Meet the model (diagram)
↓
Conservation of angular momentum
↓
Thermal wind balance
↓
Distribution of temperature
↓
Latitude extent of Hadley Cell
↓
Strength of Hadley Cell
↓
Distribution of upper westerly
↓
Distribution of surface winds

Held-Hou model

-Angular momentum

Review

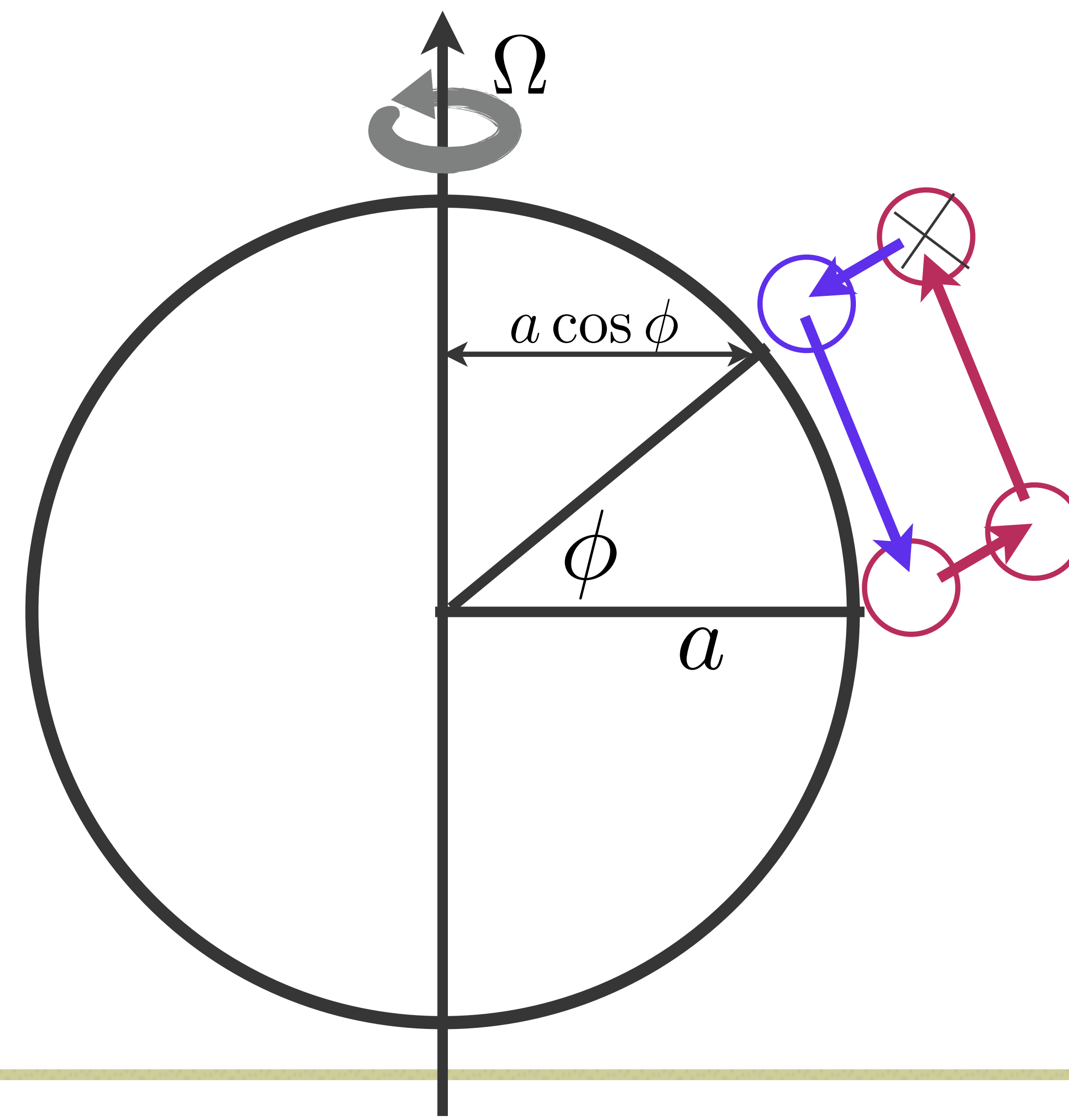
- The absolute angular momentum per unit mass is

$$M = (\Omega a \cos\phi + u) a \cos\phi$$

Due to earth's solid rotation

Deviation from the solid rotation

a is the radius of the earth



$$\frac{D}{Dt} M = -\frac{1}{\rho}\frac{\partial p}{\partial \lambda} + a\cos\phi F_\lambda$$

In an **axisymmetric flow** ([M]=M)

$$\frac{D}{Dt}[M] = a\cos\phi [F_\lambda]$$

In an inviscid (frictionless), **axisymmetric** flow, the angular momentum is conserved.

Held-Hou model

-Angular momentum

Review

$$[M] = (\Omega a \cos\phi + [u]) a \cos\phi$$

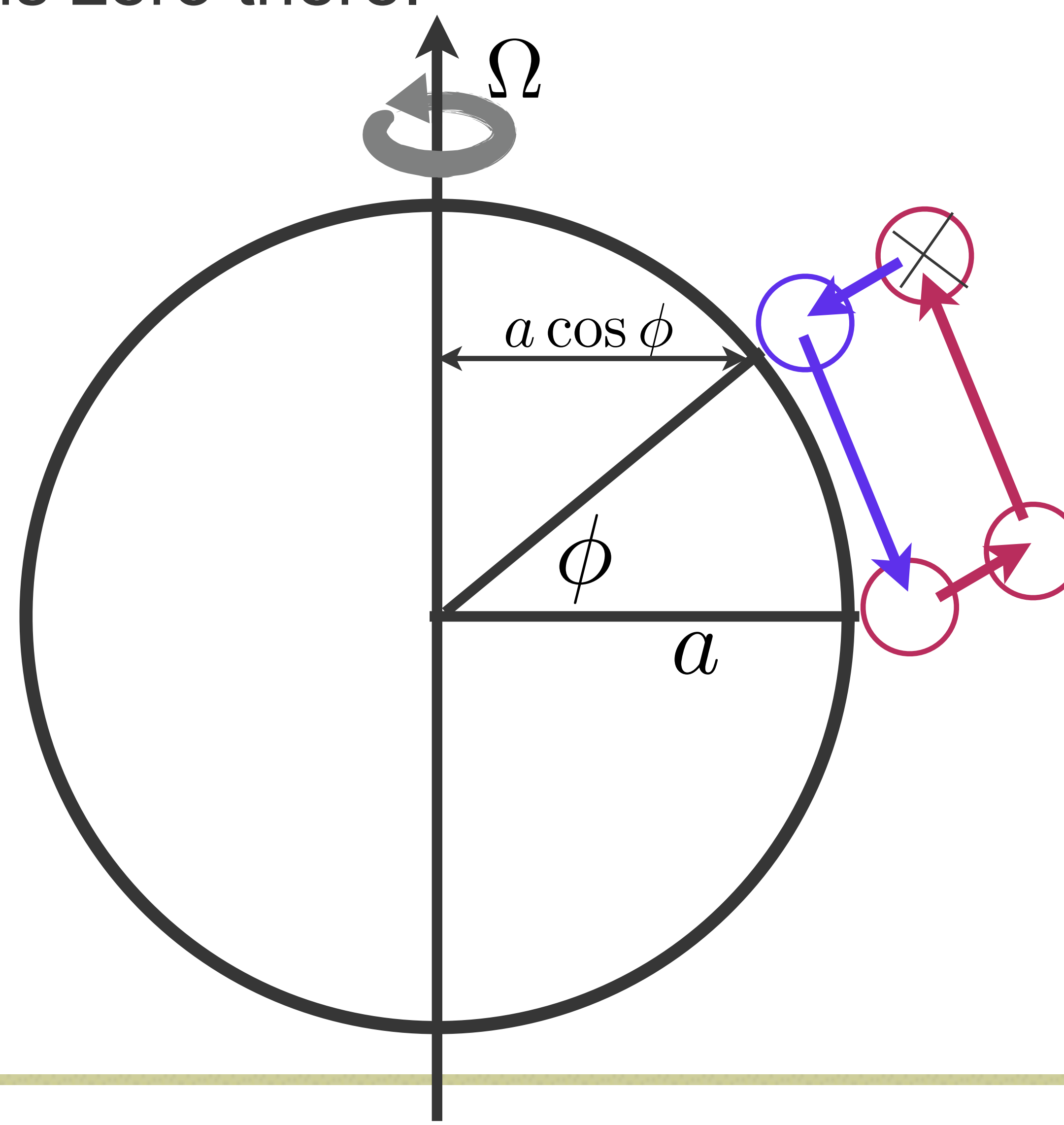
- At the equator, as the parcels rise from the surface, where the flow is weak, we assume that the zonal flow is zero there.

$$[u] = \Omega a \frac{\sin^2\phi}{\cos\phi} \equiv U_M$$

Then, what is the U_M at 10, 20, 30 degree?

Answers: 14, 57, 134 m/s, respectively

Combined with the weak surface flow, this indicates strong vertical shear of the zonal wind.



Held-Hou model
-Temperature distribution

Review

- Angular momentum:

$$[u] = \Omega a \frac{\sin^2\phi}{\cos\phi} \equiv U_M$$

- Thermal wind relation:

$$f[u(H) - u(0)] + \frac{\tan\phi}{a}[u^2(H) - u^2(0)] = -\frac{gH}{a\Theta_o}\frac{\partial\tilde{\Theta}}{\partial\phi}$$

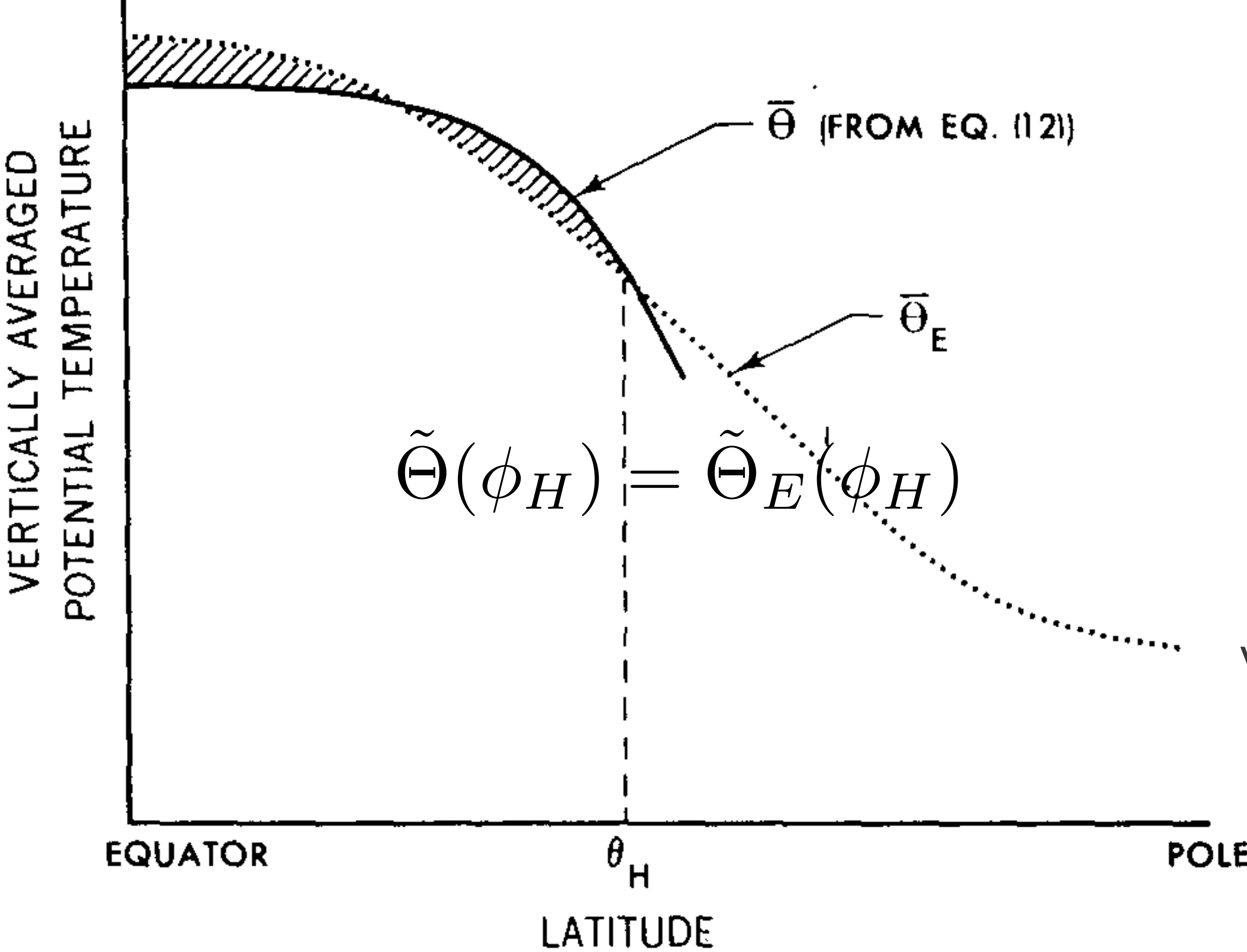
$$\frac{\tilde{\Theta}(0) - \tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2}{2gH}\frac{\sin^4\phi}{\cos^2\phi}$$

Conservation of angular momentum and the maintenance of thermal wind completely determine the variation of temperature within the Hadley Cell !

Held-Hou model
-Extent of Hadley Cell

Review

$$\int_0^{\phi_H} \tilde{\Theta} \cos\phi d\phi = \int_0^{\phi_H} \tilde{\Theta}_E \cos\phi d\phi$$



$$\tilde{\Theta}(\phi_H) = \tilde{\Theta}_E(\phi_H)$$

- Radiative equilibrium temperature

$$\frac{\Theta_E(\phi, z)}{\Theta_o} = 1 - \frac{2}{3}\Delta_H P_2(\sin\phi) + \Delta_v(\frac{z}{H} - \frac{1}{2})$$

Δ_H - fractional temperature difference between equator and pole

Δ_v - fractional temperature difference between ground and top

P_2 - second Legendre polynomial, $P_2(x) = \frac{1}{2}(3x^2 - 1)$

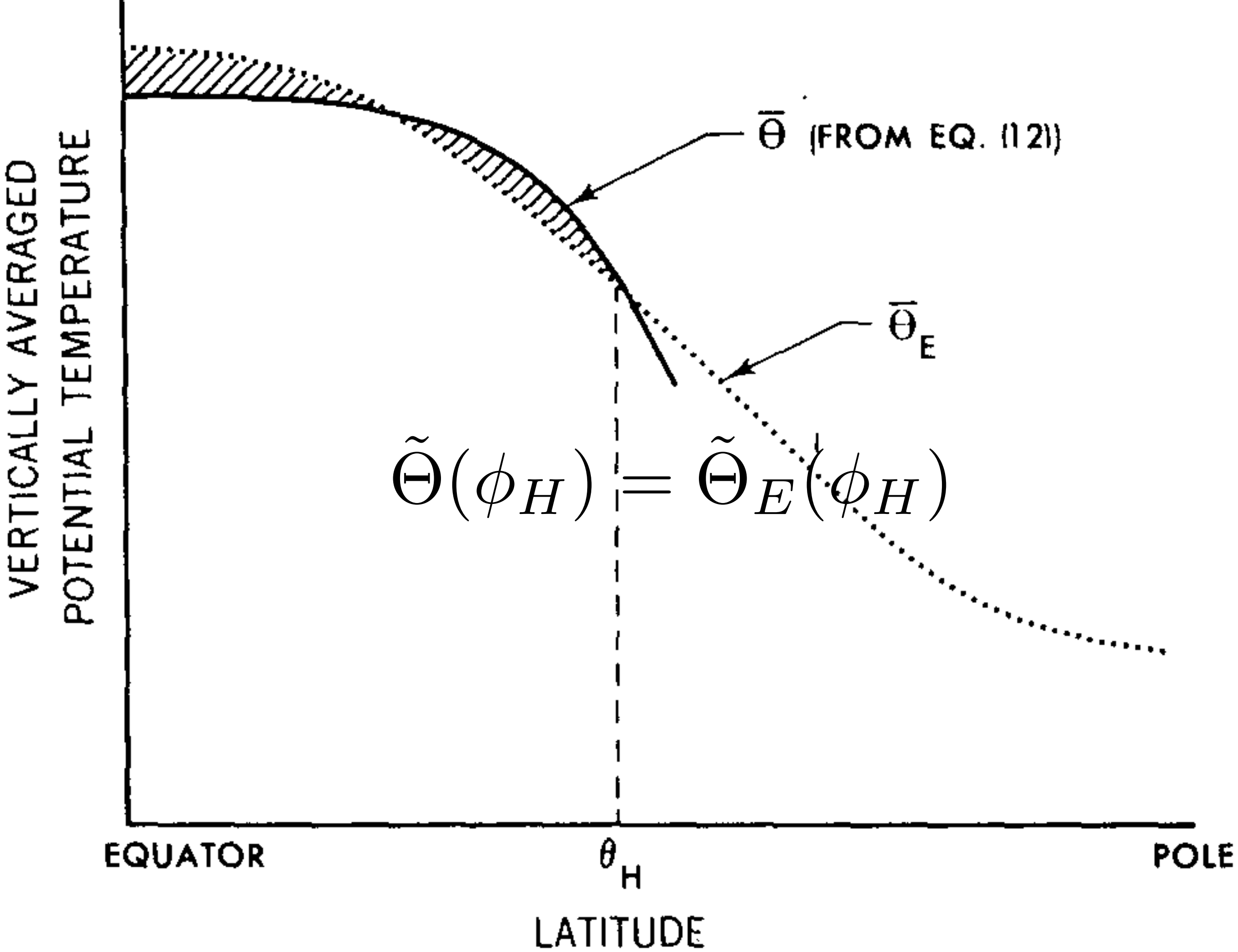
Vertical average:

$$\frac{\tilde{\Theta}_E(\phi, z)}{\Theta_o} = 1 - \frac{2}{3}\Delta_H P_2(\sin\phi)$$

Held-Hou model
-Extent of Hadley Cell

Review

$$\int_0^{\phi_H} \tilde{\Theta} \cos\phi d\phi = \int_0^{\phi_H} \tilde{\Theta}_E \cos\phi d\phi$$



$$\tilde{\Theta}(\phi_H) = \tilde{\Theta}_E(\phi_H)$$

Assume small ϕ, $\sin\phi \sim \phi$

$$\frac{\tilde{\Theta}(0)}{\Theta_o} \approx \frac{\tilde{\Theta}_E(0)}{\Theta_o} - \frac{5}{18}\frac{gH\Delta_H^2}{\Omega^2 a^2}$$

$$\phi_H = \left(\frac{5}{3}\frac{gH\Delta_H}{\Omega^2 a^2}\right)^{1/2}$$

set $R = \dfrac{gH\Delta_H}{\Omega^2 a^2}$, then, $\phi_H = \left(\dfrac{5}{3}R\right)^{1/2}$

Held-Hou model
-Strength of Hadley Cell

- Thermodynamic equation **at equator** and steady state:

$$\frac{D\Theta}{Dt} = \frac{\Theta_E - \Theta}{\tau} \quad \Rightarrow \quad w\frac{\partial \Theta}{\partial z} \approx \frac{\Theta_E - \Theta}{\tau}$$

- If the static stability is mostly determined by the forcing instead of meridional circulation:

$$\frac{1}{\Theta_o}\frac{\partial \Theta}{\partial z} \approx \frac{\Delta_V}{H}$$

$$w \approx \frac{H}{\Theta_o \Delta_V}\frac{\Theta_E - \Theta}{\tau} = \frac{5g\Delta_H^2 H^2}{18a^2\tau\Omega^2\Delta_V}$$

- Using mass continuity:

$$v \sim \frac{(gH)^{3/2}\,\Delta_H^{5/2}}{a^2\Omega^3\tau\Delta_V}$$

$$\tau_d = \frac{H}{w}$$

Characteristic overturning time scale can be estimated.

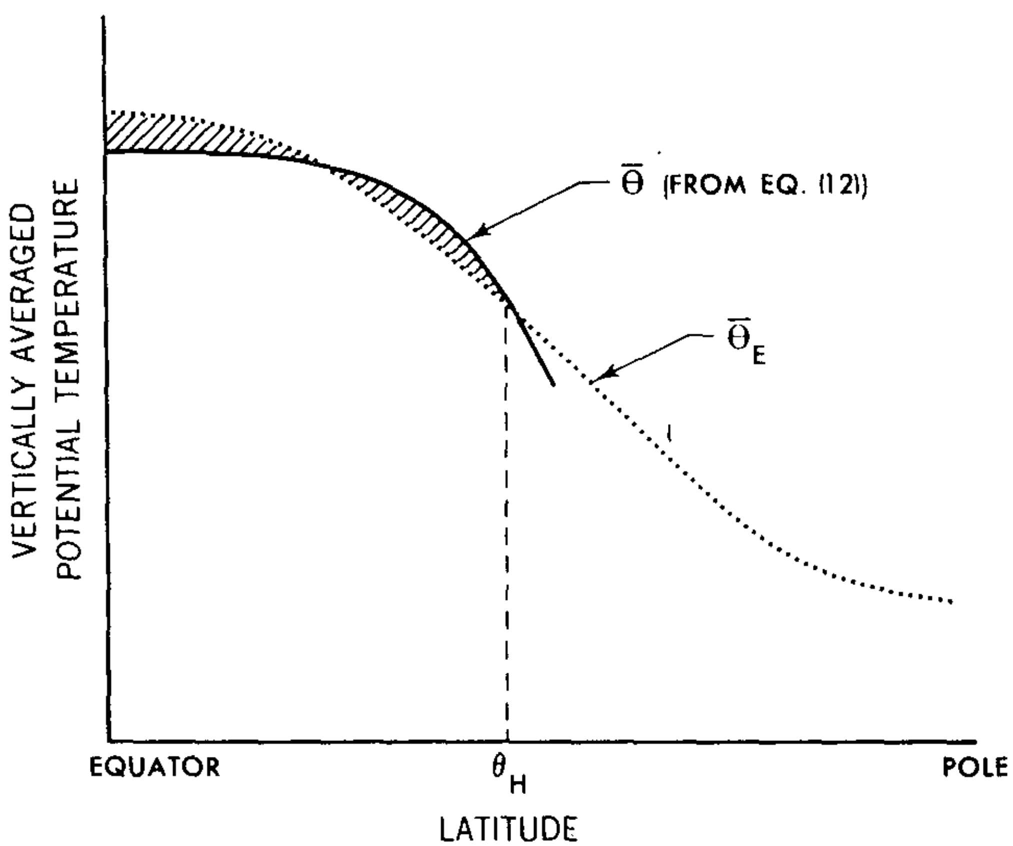
Held-Hou model

-Upper jet

- Angular momentum conservation:

$$[u] = \Omega a \frac{\sin^2 \phi}{\cos \phi} \equiv U_M$$

- Radiative equilibrium:

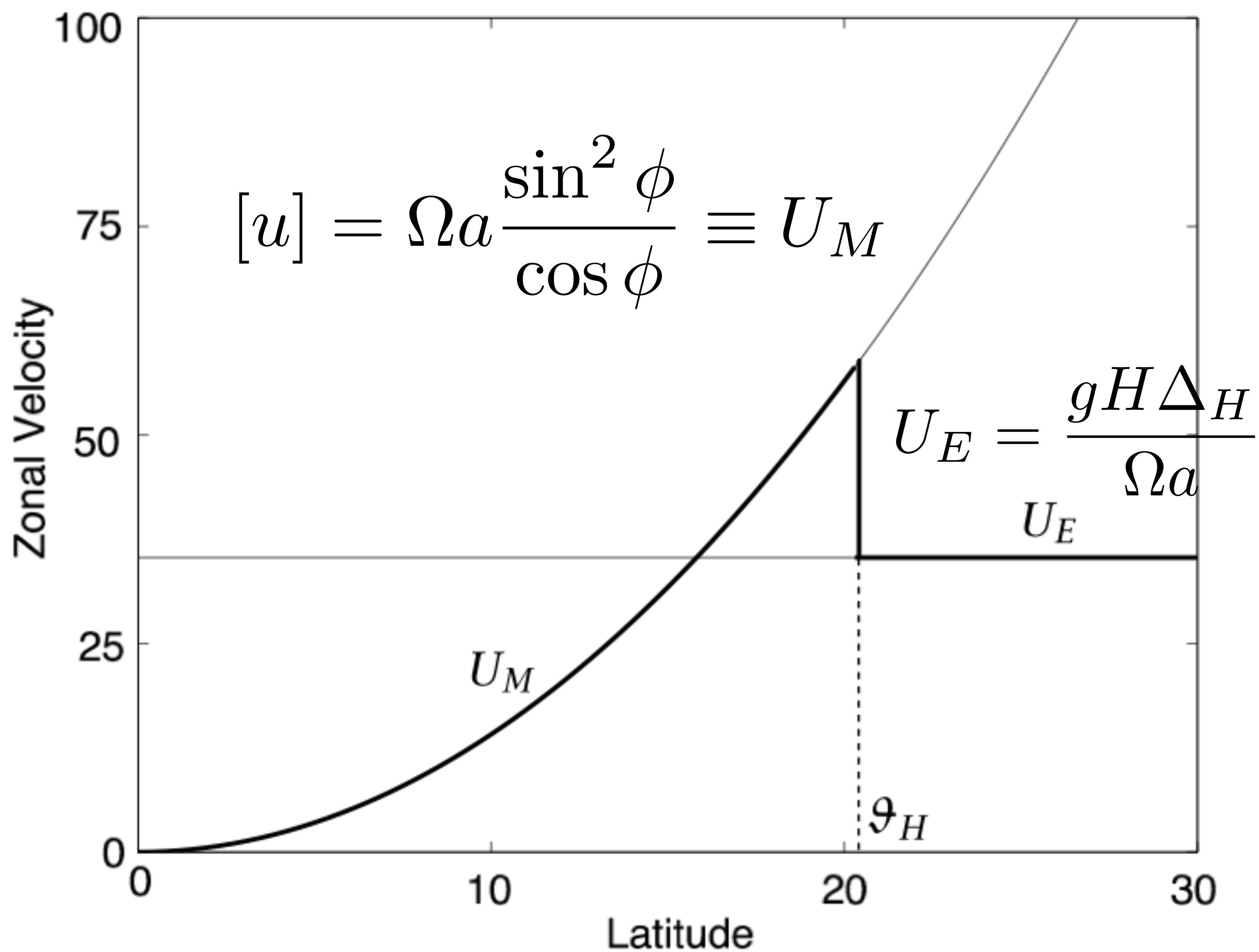


From thermal wind relation and assuming surface zonal wind is weak:

$$U_E = \frac{gH\Delta_H}{\Omega a}$$

Held-Hou model

-Upper jet



Held-Hou model

-Summary

- Distribution of temperature constrained the cor angular momentum: and thermal wind balance

$$\frac{\tilde{\Theta}(0)-\tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2}{2gH}\frac{\sin^4\phi}{\cos^2\phi}$$

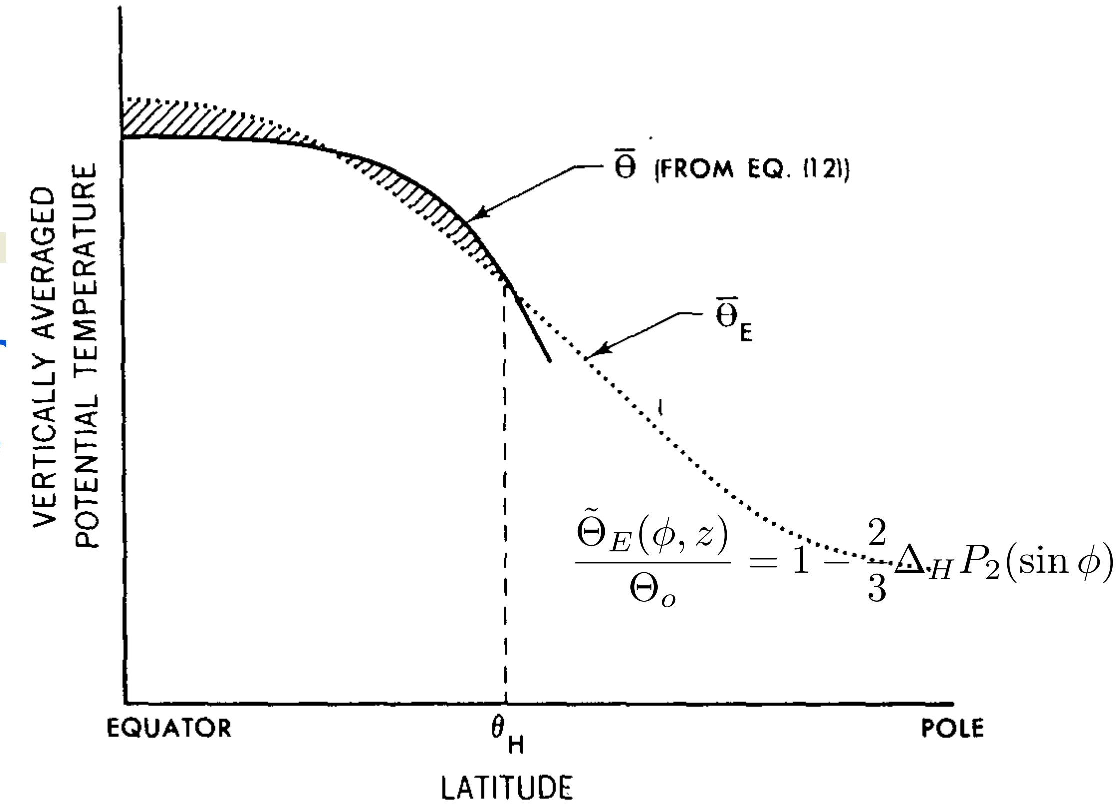
Smaller than the RE temp gradient

- Extent of Hadley Cell:

$$\phi_H = \left(\frac{5}{3}\frac{gH\Delta_H}{\Omega^2 a^2}\right)^{1/2}$$

- Upper jet

$$[u] = \Omega a \frac{\sin^2\phi}{\cos\phi} \equiv U_M$$



Use parameters:

$$\begin{aligned} H &= 10km, \\ \Delta_H &= 50K, \\ \Theta_o &= 300K, \\ \tilde{\Theta}_E(0) &= 303K \end{aligned}$$

Then: $\phi_H = 20.4^o$

$$\tilde{\Theta}(0) - \Theta(\tilde{\phi}_H) \sim 5K$$

Roughly consistent with observations

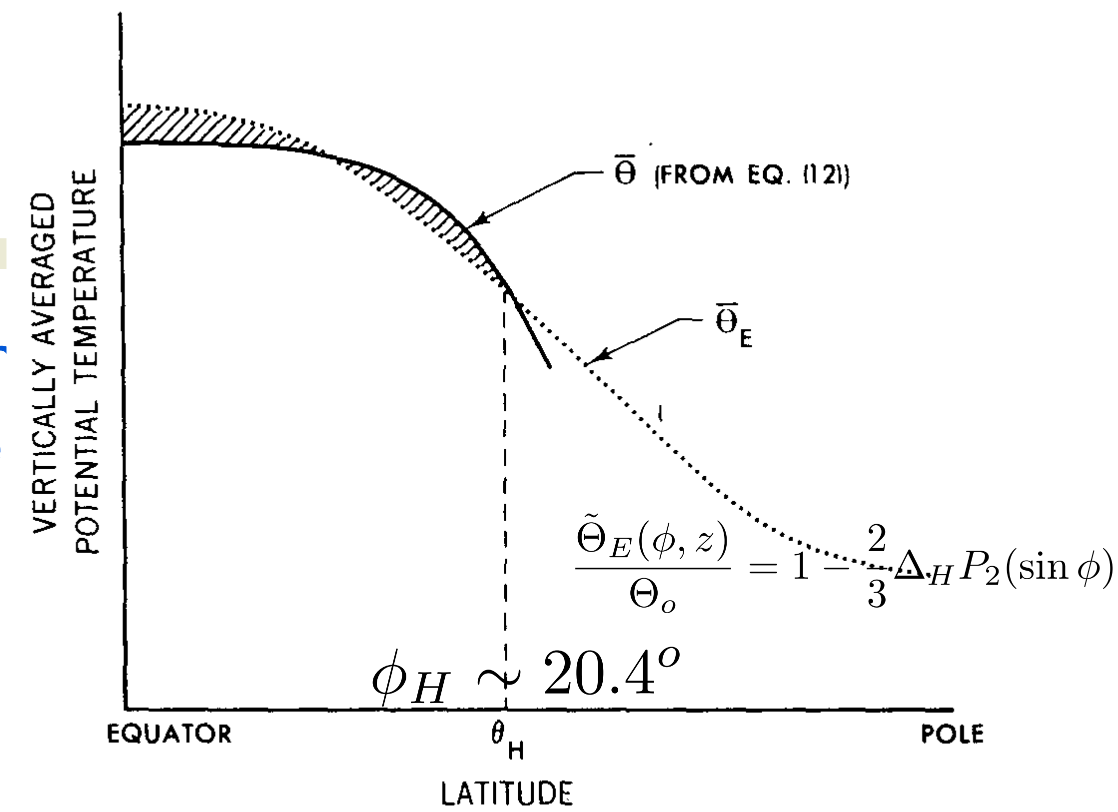
Held-Hou model

-Summary

- Distribution of temperature constrained the cor angular momentum: and thermal wind balance

$$\frac{\tilde{\Theta}(0) - \tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2}{2gH} \frac{\sin^4 \phi}{\cos^2 \phi}$$

Smaller than the RE temp gradient



- Extent of Hadley Cell:

$$\phi_H = \left(\frac{5}{3}\frac{gH\Delta_H}{\Omega^2 a^2}\right)^{1/2}$$

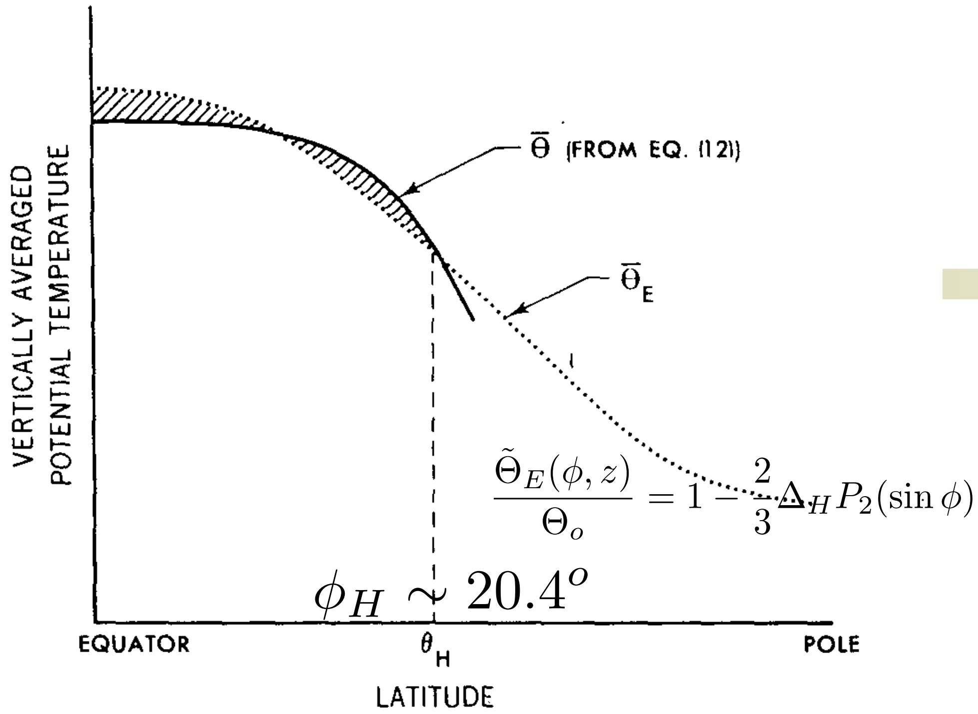
- Strength of Hadley Cell:

$$v \sim \frac{(gH)^{3/2}\,\Delta_H^{5/2}}{a^2\Omega^3\tau\Delta_V}$$

- Upper jet $$[u] = \Omega a \frac{\sin^2 \phi}{\cos \phi} \equiv U_M$$

- Surface winds ?

-Surface winds



- Radiative equilibrium temperature

$$\frac{\Theta_E(\phi,z)}{\Theta_o} = 1 - \frac{2}{3}\Delta_H P_2(\sin\phi) + \Delta_v\left(\frac{z}{H} - \frac{1}{2}\right)$$

Δ_H - fractional temperature difference between equator and pole

Δ_v - fractional temperature difference between ground and top

P_2 - second Legendre polynomial, $P_2(x) = \frac{1}{2}(3x^2 - 1)$

Vertical average:

$$\frac{\tilde{\Theta}_E(\phi,z)}{\Theta_o} = 1 - \frac{2}{3}\Delta_H P_2(\sin\phi)$$

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$$\left.\begin{aligned}
0 &= -\nabla\cdot(\mathbf{v}u) + fv + \frac{uv\tan\theta}{a} + \frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right)\\
0 &= -\nabla\cdot(\mathbf{v}v) - fu - \frac{u^2\tan\theta}{a} - \frac{1}{a}\frac{\partial\Phi}{\partial\theta} + \frac{\partial}{\partial z}\left(\nu\frac{\partial v}{\partial z}\right)\\
0 &= -\nabla\cdot(\mathbf{v}\Theta) - (\Theta - \Theta_E)\tau^{-1} + \frac{\partial}{\partial z}\left(\nu\frac{\partial\Theta}{\partial z}\right)\\
0 &= -\nabla\cdot\mathbf{v}\\
\frac{\partial\Phi}{\partial z} &= g\Theta/\Theta_0
\end{aligned}\right\}, \quad (1)$$

with boundary conditions

$$\left.\begin{aligned}
&\text{at } z = H: \quad w = 0; \quad \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial\Theta}{\partial z} = 0\\
&\text{at } z = 0: \quad w = 0; \quad \frac{\partial\Theta}{\partial z} = 0;\\
&\nu\frac{\partial u}{\partial z} = Cu; \quad \nu\frac{\partial v}{\partial z} = Cv
\end{aligned}\right\}. \quad (1a)$$

Held-Hou model

-Surface winds

- From the zonal momentum equation

$$\frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos^2\phi\int_0^H uvdz\right) = -Cu(0)$$

- *Assumptions:*
  - meridional flow is confined to thin boundary layer adjacent to the upper and lower boundaries;
  - stratification is almost fixed or not modified by the Hadley cell.

$$\frac{1}{\Theta_o}\int_0^H v\Theta dz \approx V\Delta_V$$

V is the mass flux in the boundaries

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$$\left.\begin{aligned}
0 &= -\nabla\cdot(\mathbf{v}u) + fv + \frac{uv\tan\theta}{a} + \frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right)\\
0 &= -\nabla\cdot(\mathbf{v}v) - fu - \frac{u^2\tan\theta}{a} - \frac{1}{a}\frac{\partial\Phi}{\partial\theta} + \frac{\partial}{\partial z}\left(\nu\frac{\partial v}{\partial z}\right)\\
0 &= -\nabla\cdot(\mathbf{v}\Theta) - (\Theta - \Theta_E)\tau^{-1} + \frac{\partial}{\partial z}\left(\nu\frac{\partial\Theta}{\partial z}\right)\\
0 &= -\nabla\cdot\mathbf{v}\\
\frac{\partial\Phi}{\partial z} &= g\Theta/\Theta_0
\end{aligned}\right\}, \quad (1)$$

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&\text{at } z = 0: \quad w = 0; \quad \frac{\partial\Theta}{\partial z} = 0;\\
&\qquad \nu\frac{\partial u}{\partial z} = Cu; \quad \nu\frac{\partial v}{\partial z} = Cv
\end{aligned}\right\}. \quad (1a)$$

Held-Hou model

-Surface winds

- From the zonal momentum equation

$$\frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos^2\phi\int_0^H uv dz\right) = -Cu(0)$$

$$\frac{1}{\Theta_o}\int_0^H v\Theta dz \approx V\Delta_V$$

$$\frac{1}{H}\int_0^H \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(v\Theta\cos\phi)dz = \frac{\tilde{\Theta}_E - \tilde{\Theta}}{\tau}$$

Then, mass flux V can be solved. Similarly, we have the momentum flux,

$$\int_0^H uv dz \approx VU_m$$

$$0 = -\nabla\cdot(\mathbf{v}u) + fv + \frac{uv\tan\theta}{a} + \frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right)$$

$$0 = -\nabla\cdot(\mathbf{v}v) - fu - \frac{u^2\tan\theta}{a} - \frac{1}{a}\frac{\partial\Phi}{\partial\theta} + \frac{\partial}{\partial z}\left(\nu\frac{\partial v}{\partial z}\right) \quad (1)$$

$$0 = -\nabla\cdot(\mathbf{v}\Theta) - (\Theta - \Theta_E)\tau^{-1} + \frac{\partial}{\partial z}\left(\nu\frac{\partial\Theta}{\partial z}\right)$$

$$0 = -\nabla\cdot\mathbf{v}$$

$$\frac{\partial\Phi}{\partial z} = g\Theta/\Theta_0$$

with boundary conditions

at $z = H$: $w = 0$; $\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial\Theta}{\partial z} = 0$

at $z = 0$: $w = 0$; $\frac{\partial\Theta}{\partial z} = 0$; (1a)

$\nu\frac{\partial u}{\partial z} = Cu$; $\nu\frac{\partial v}{\partial z} = Cv$

Held-Hou model

-Surface winds

- From the zonal momentum equation

$$\frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos^2\phi\int_0^H uv dz\right) = -Cu(0)$$

$$Cu(0) \approx -\frac{25}{18}\frac{g^2H^3\Delta_H^3}{a^3\Omega^3\tau\Delta_V}\left[\left(\frac{\phi}{\phi_H}\right)^2 - \frac{10}{3}\left(\frac{\phi}{\phi_H}\right)^4 + \frac{7}{3}\left(\frac{\phi}{\phi_H}\right)^6\right]$$

Surface easterlies

$$\phi < \left(\frac{3}{7}\right)^{1/2}\phi_H$$

Surface westerlies

$$\left(\frac{3}{7}\right)^{1/2}\phi_H < \phi < \phi_H$$

Held-Hou model

-Summary

- Distribution of temperature constrained by the conservation of angular momentum and thermal wind balance.

$$\frac{\tilde{\Theta}(0)-\tilde{\Theta}(\phi)}{\Theta_o}=\frac{\Omega^2 a^2}{2gH}\frac{\sin^4\phi}{\cos^2\phi}$$

Smaller than the RE temp gradient

- Extent of Hadley Cell:

$$\phi_H=\left(\frac{5}{3}\frac{gH\Delta_H}{\Omega^2 a^2}\right)^{1/2}$$

- Strength of Hadley Cell:

$$v\sim\frac{(gH)^{3/2}\,\Delta_H^{5/2}}{a^2\Omega^3\tau\Delta_V}$$

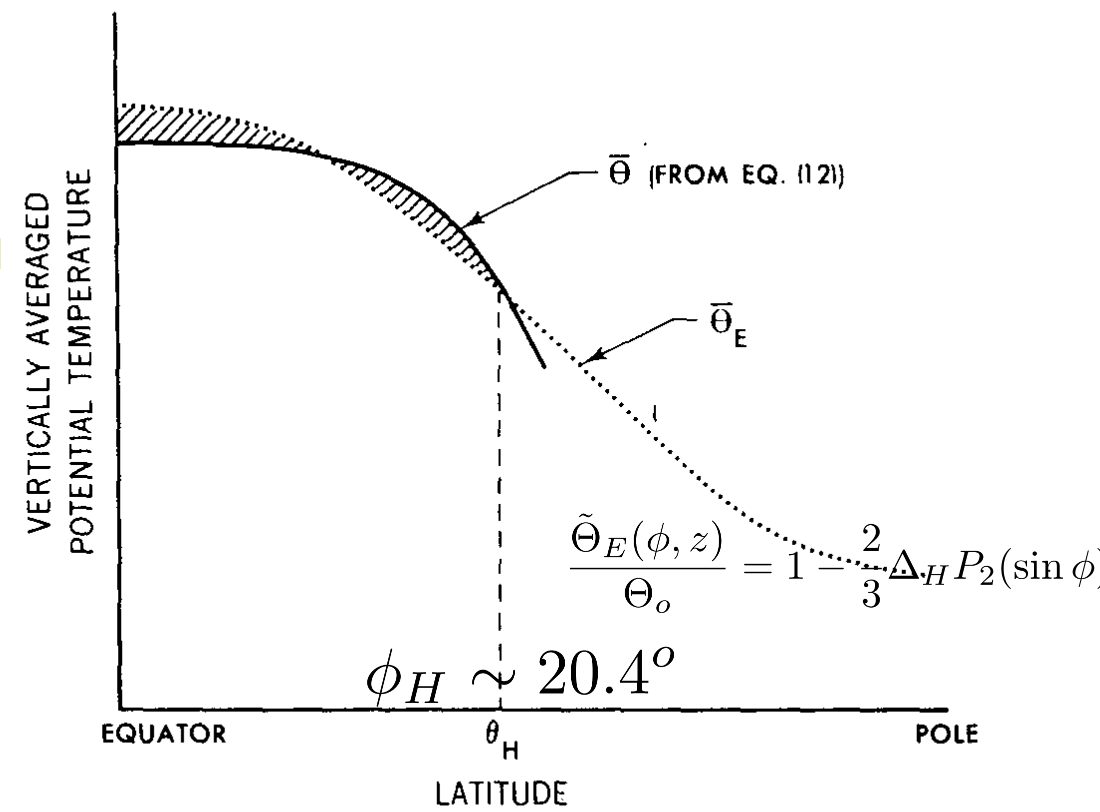
- Upper jet:

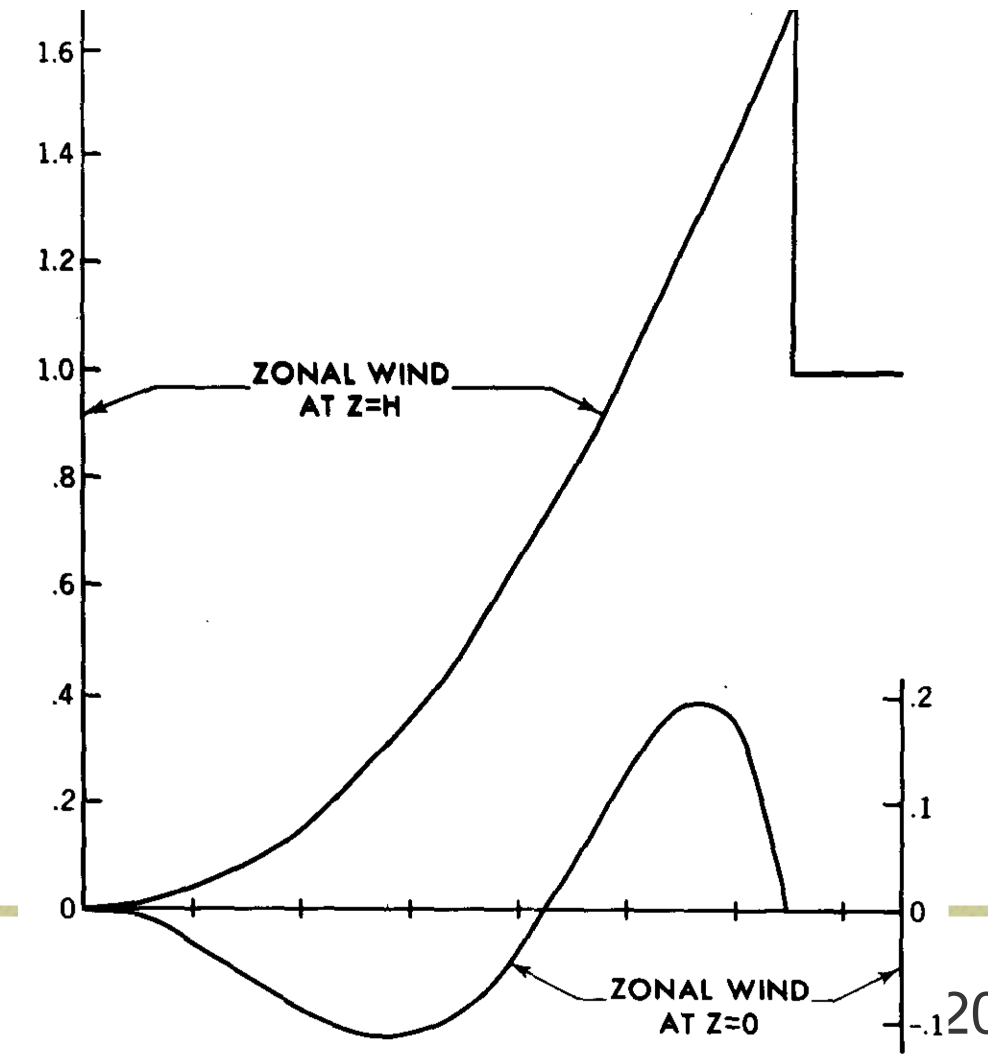
$$[u]=\Omega a\frac{\sin^2\phi}{\cos\phi}\equiv U_M$$

- Surface winds:

$$Cu(0)\approx-\frac{25}{18}\frac{g^2H^3\Delta_H^3}{a^3\Omega^3\tau\Delta_V}\left[\left(\frac{\phi}{\phi_H}\right)^2-\frac{10}{3}\left(\frac{\phi}{\phi_H}\right)^4+\frac{7}{3}\left(\frac{\phi}{\phi_H}\right)^6\right]$$

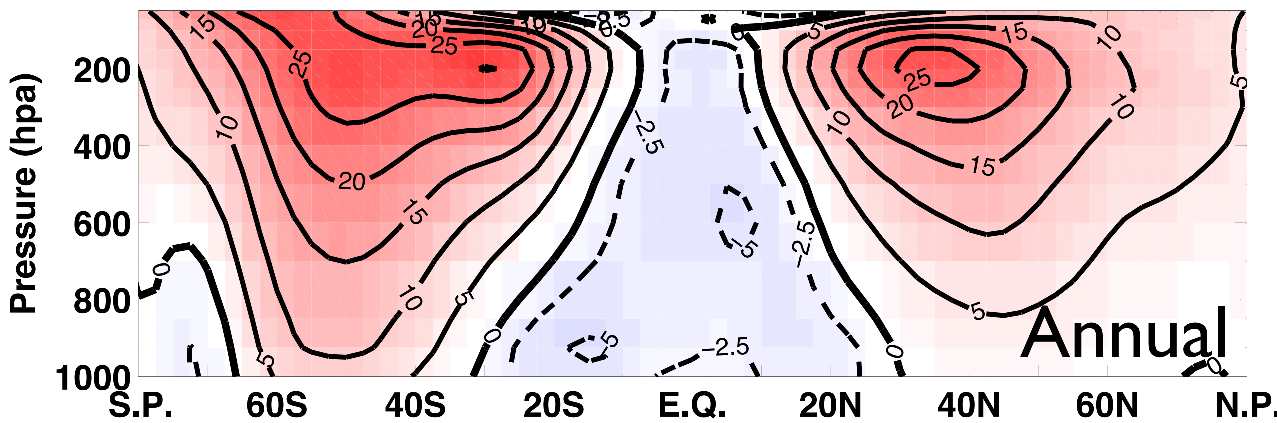
surface easterlies $\phi<\left(\frac{3}{7}\right)^{1/2}\phi_H$

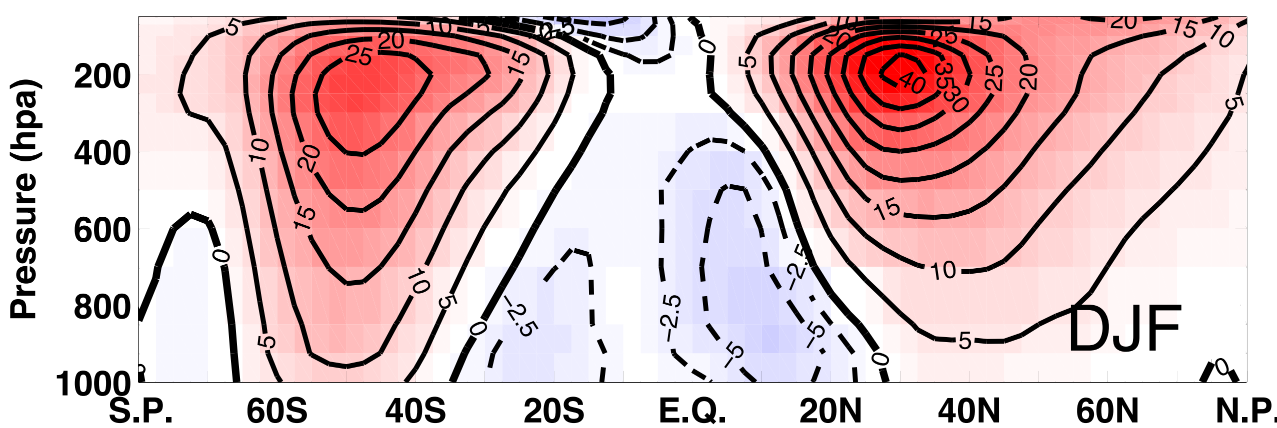


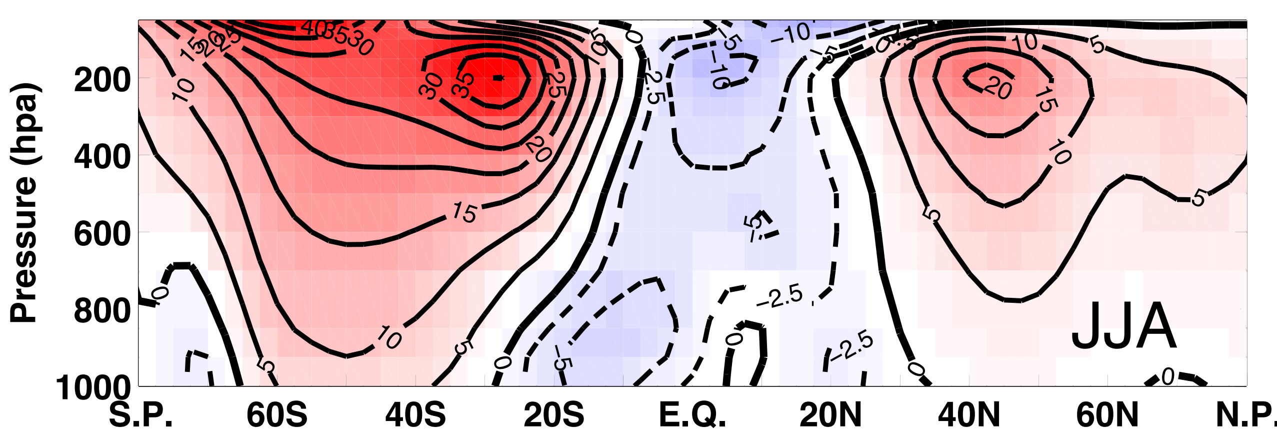


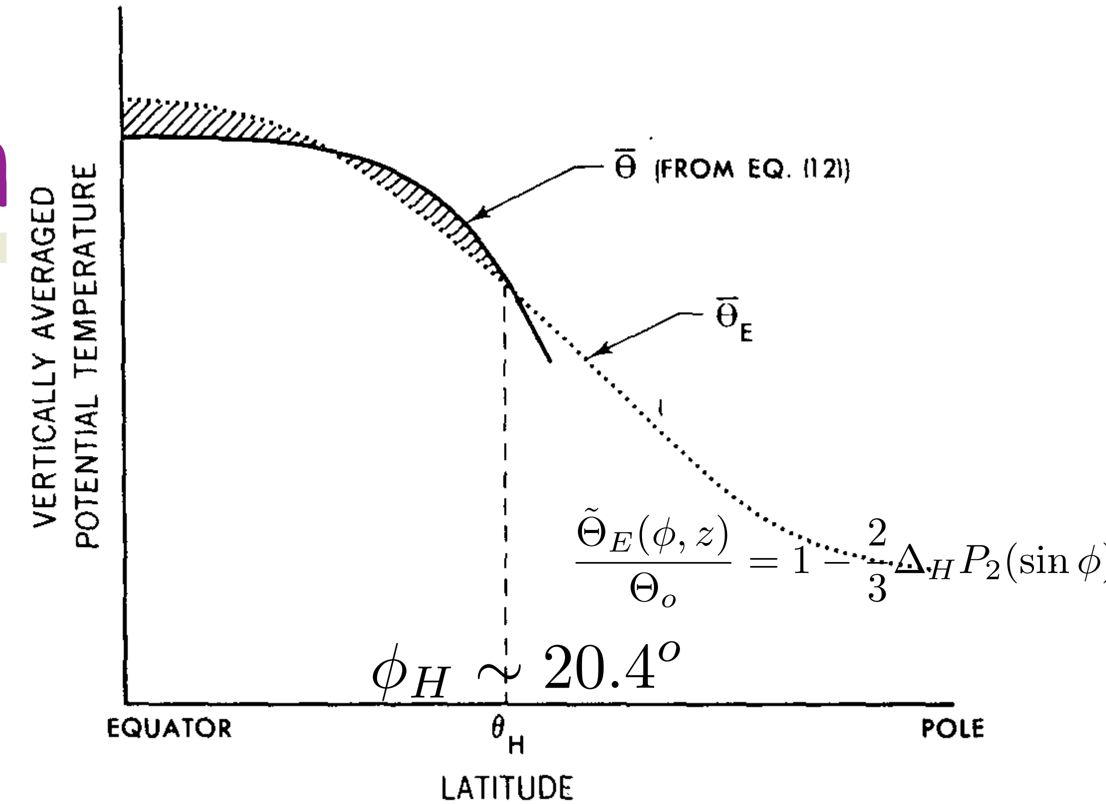
Held-Hou model (review)

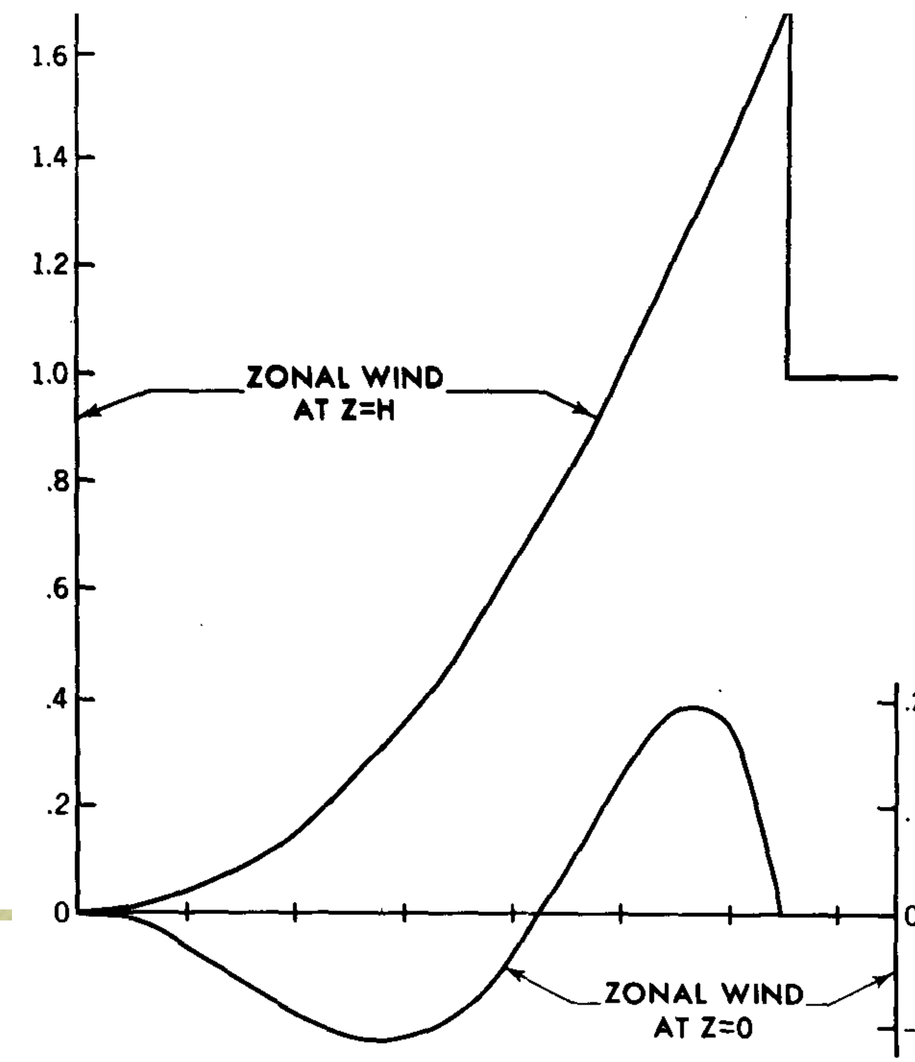
-Discussion









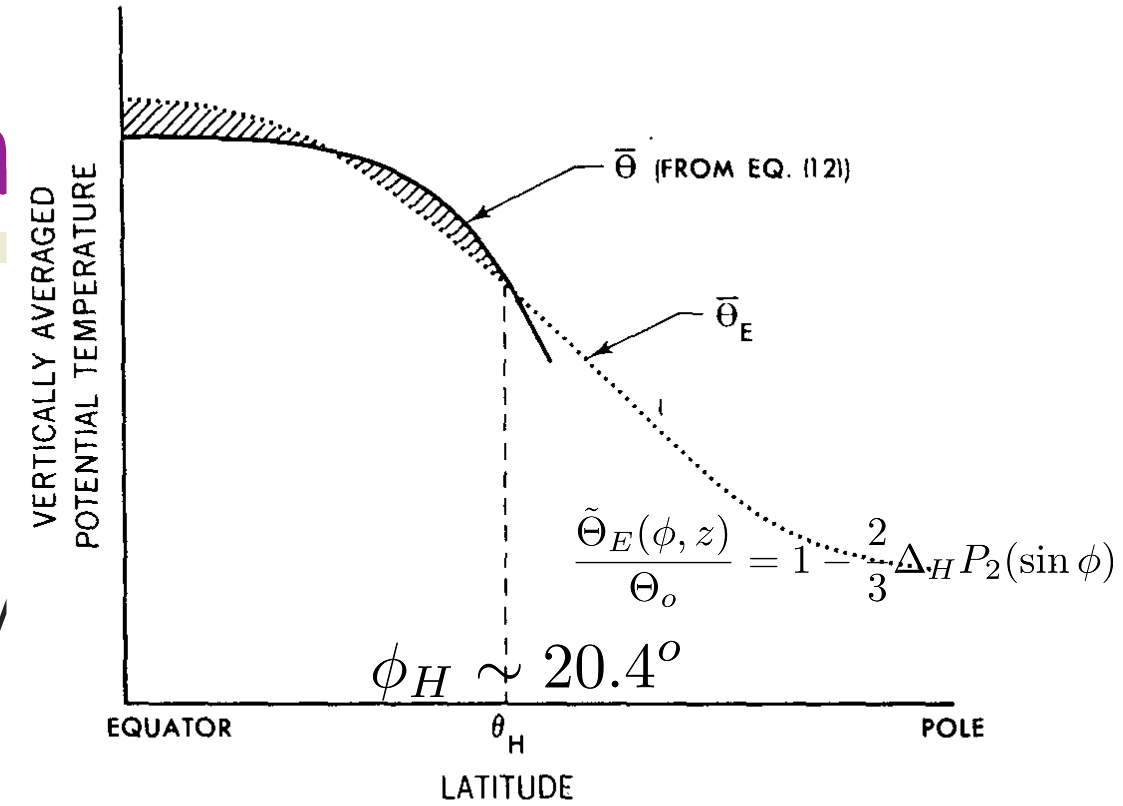


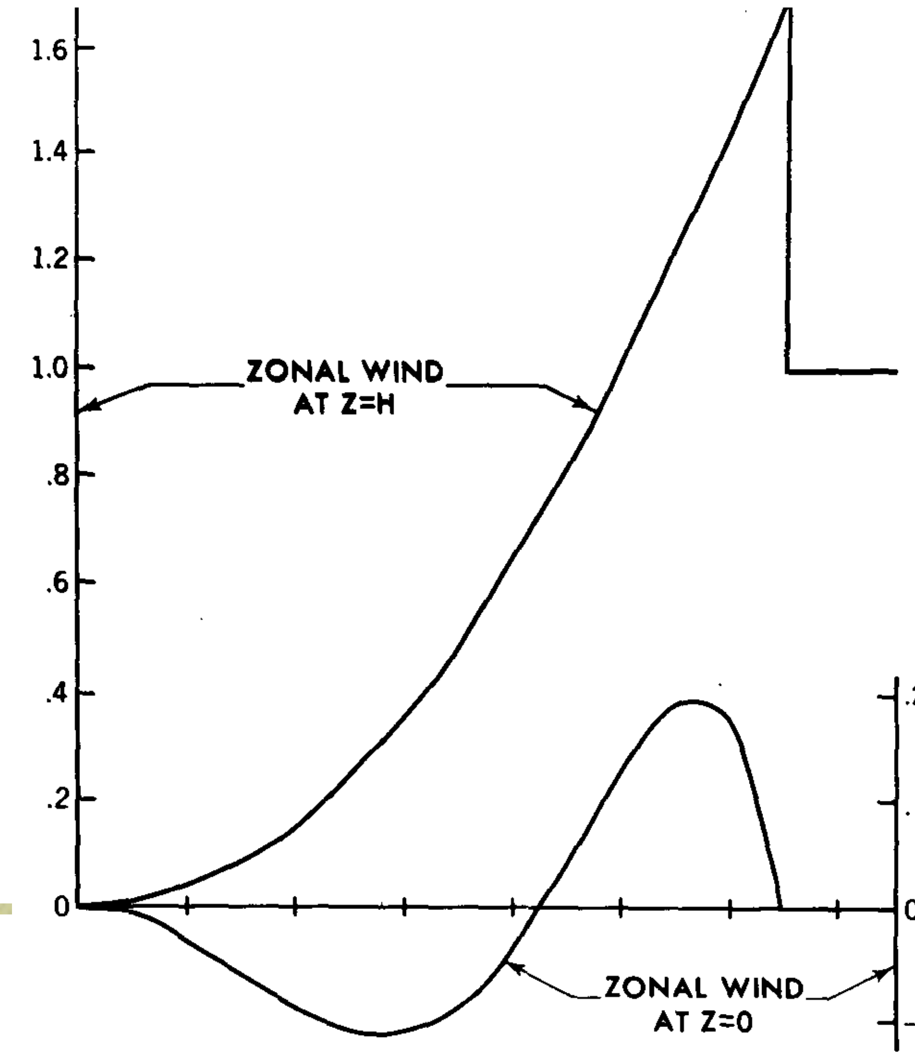
Held-Hou model (review)

-Discussion

- Upper jet: right place, but too large and discontinue.
- Extent of Hadley Cell: only a finite extent. Hadley cell cannot carry heat from equator to the pole, thus cannot be responsible to the observed equator-pole temperature difference. So does the wind, momentum and heat flux distribution.

- Axisymmetric flow: roles of eddies are neglected.
- Moisture effect is neglected.
- Seasonal variation and asymmetry on the equator?



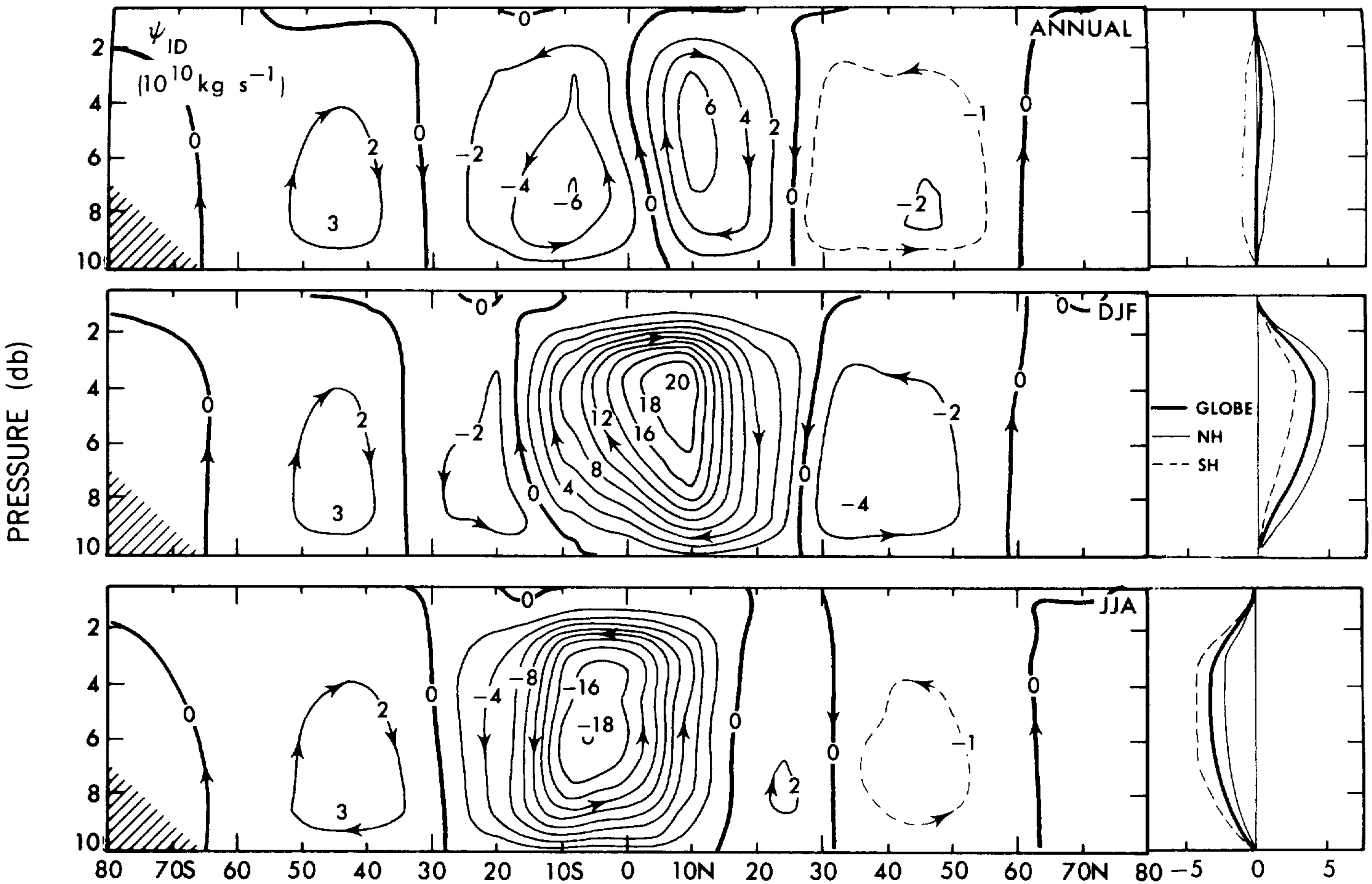


- Outline

- Observations
- Held-Hou theory (axisymmetric flow, a model that is symmetric about the equator)
- Lindzen-Hou theory (axisymmetric flow, a model that is asymmetry about the equator)
- The role of eddies
- Moisture effects
- Discussions

Hadley Cell - Theory:
Asymmetry about the equator*

- Stream function (流函数)



Hadley Cell - Theory: Asymmetry about the equator*

- **Lindzen-Hou (1988)**

Richard S. Lindzen

Arthur Hou

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Hadley Circulations for Zonally Averaged Heating Centered off the Equator

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(Manuscript received 15 October 1987, in final form 29 February 1988)

ABSTRACT

Consistent with observations, we find that moving peak heating even 2 degrees off the equator leads to profound asymmetries in the Hadley circulation, with the winter cell amplifying greatly and the summer cell becoming negligible. It is found that the annually averaged Hadley circulation is much larger than the circulation forced by the annually averaged heating. Implications for the general circulation are discussed, as are implications for Milankovitch forcing of climate variations and for tropical meteorology and oceanography.

1. Introduction

Understanding the zonally averaged circulation of the atmosphere is, arguably, the oldest scientific problem in dynamic meteorology (Lorenz 1967). The zonally averaged meridional circulation rising from the tropics is known as the Hadley circulation. Beginning with Hadley (1735) and continuing through the 19th Century (Ferrel 1856, Thomson 1857), no serious distinction was made between the zonally averaged circulation and the axially symmetric circulation forced by axially symmetric heating. By the early part of this century (Jeffreys 1926), the idea was being put forth that the zonally averaged circulation might, in large measure, be forced by eddies, and by the post-World War II period, Starr (1948) was going so far as to suggest that the symmetric circulation was inconsequential. These developments are summarized by Lorenz (1967). Surprisingly, despite all arguments, there, in surface winds. At least as concerns the subtropical jets, these results strongly suggested that the net effect of eddies would be to reduce these jets—rather than to maintain them. Moreover, Schneider (1977) and Held and Hou (1980, HH hereafter) showed that detailed numerical results could be replicated by the application of some simple balances for angular momentum and thermal energy. The simple theory will be reviewed in section 2 of the present paper.

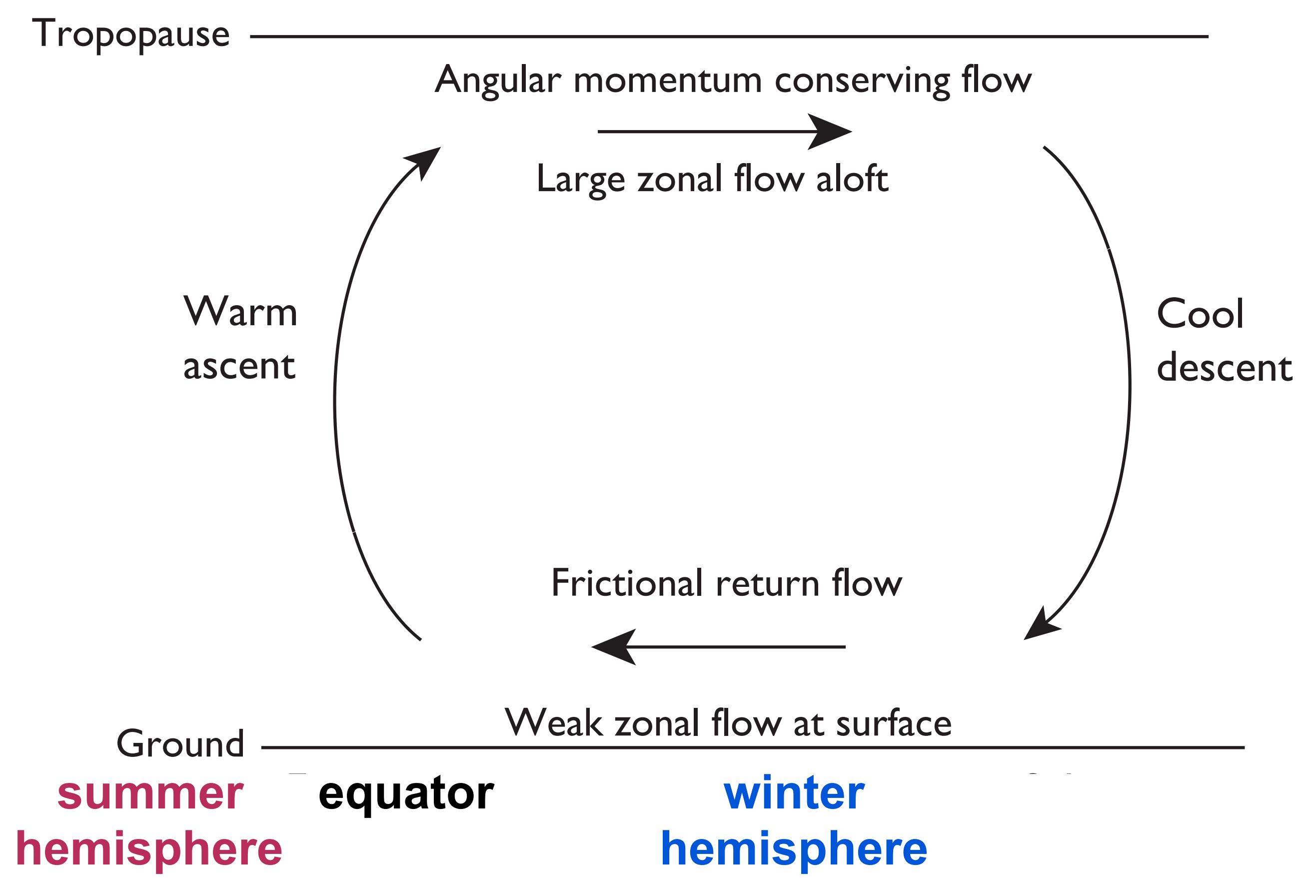
All the aforementioned calculations of the symmetric circulation used (following Hadley 1735; Thomson 1857; Ferrel 1856) heating distributions symmetric about the equator. Implicit in this was the assumption that the annually averaged symmetric circulation would correspond to the circulation forced by annually averaged heating. In addition, there was a question of what to use for the heating. It was argued by Lindzen (1978), that given the fact that the adjustment time

Hadley Cell - Theory: Asymmetry about the equator*

- **Lindzen-Hou (1988)**

Still make the assumptions:

- the circulation is steady or quasi-steady (the flow adjusts to a steady circulation on a timescale faster than that on which the solar radiation varies);
- the upper branch conserves angular momentum; surface zonal winds are weak;
- the circulation is in thermal wind balance;



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$$\frac{D\Theta}{Dt} = \frac{\Theta_E - \Theta}{\tau}$$

Newtonian cooling: the cooling rate linearly depends on the local temperature perturbation

- Radiative equilibrium temperature

$$\frac{\Theta_E(\phi, z)}{\Theta_o} = 1 - \frac{2}{3}\Delta_H P_2(\sin\phi - \sin\phi_0) + \Delta_v(\frac{z}{H} - \frac{1}{2})$$

Δ_H, Δ_V - fractional temperature difference between equator and pole, ground and top of the flow

P_2 - second Legendre polynomial, $P_2(x) = \frac{1}{2}(3x^2 - 1)$

Θ_o - reference potential temperature, equivalent to RE temp at equator

Vertical average: $$\frac{\tilde{\Theta}_E(\phi, z)}{\Theta_o} = 1 - \frac{2}{3}\Delta_H P_2(\sin\phi - \sin\phi_0)$$

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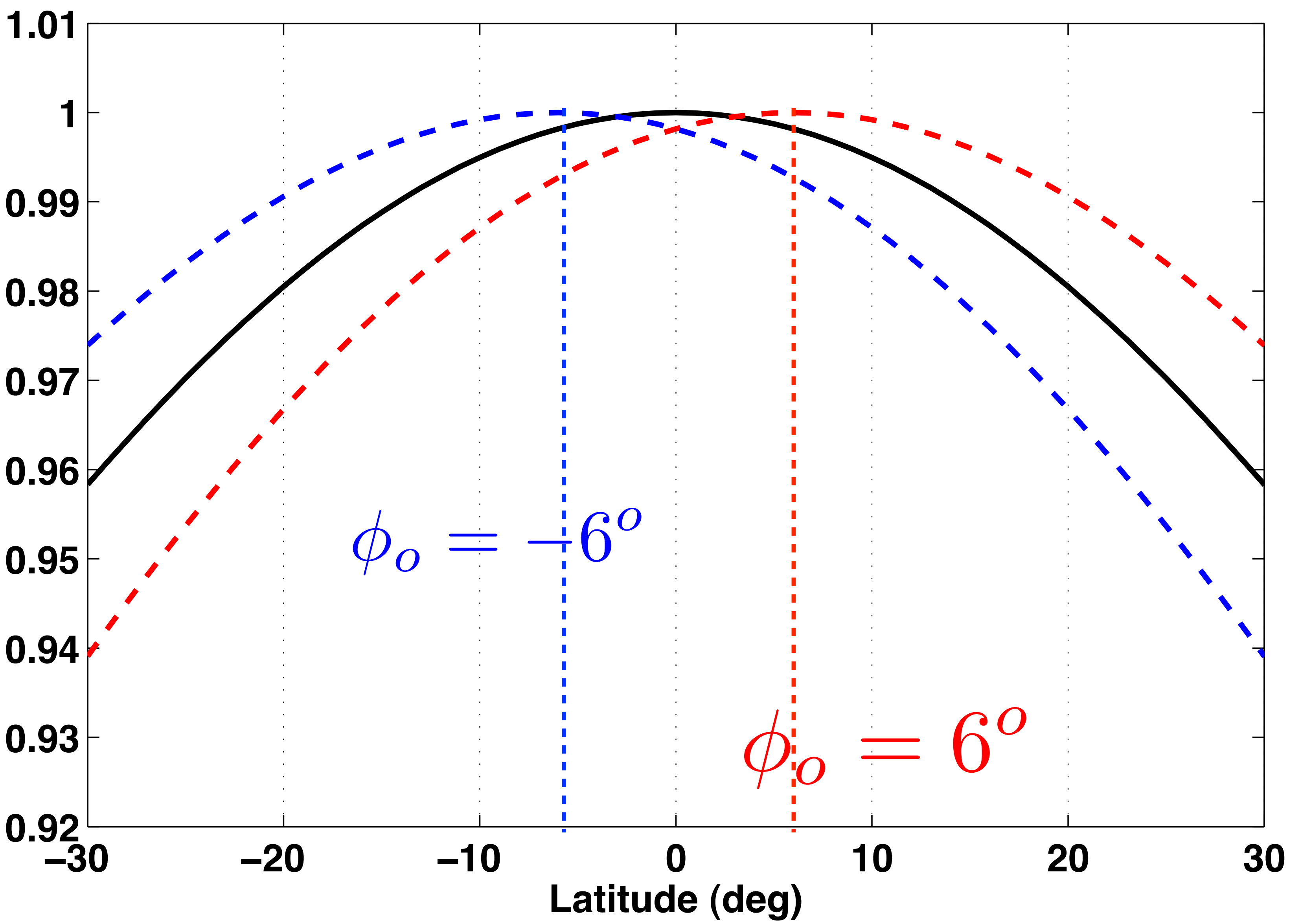

Asymmetry about the equator*
-Angular momentum

- In an inviscid, **axisymmetric** flow, the angular momentum is conserved for the upper branch of Hadley Cell.

$$\begin{aligned} M &= (\Omega a \cos\phi + u) a \cos\phi \\ &= \Omega a^2 \cos^2\phi_1 \end{aligned}$$

$$U_M = \frac{\Omega a(\cos^2\phi_1 - \cos^2\phi)}{\cos\phi}$$

ϕ_1 - latitude of the rising motion

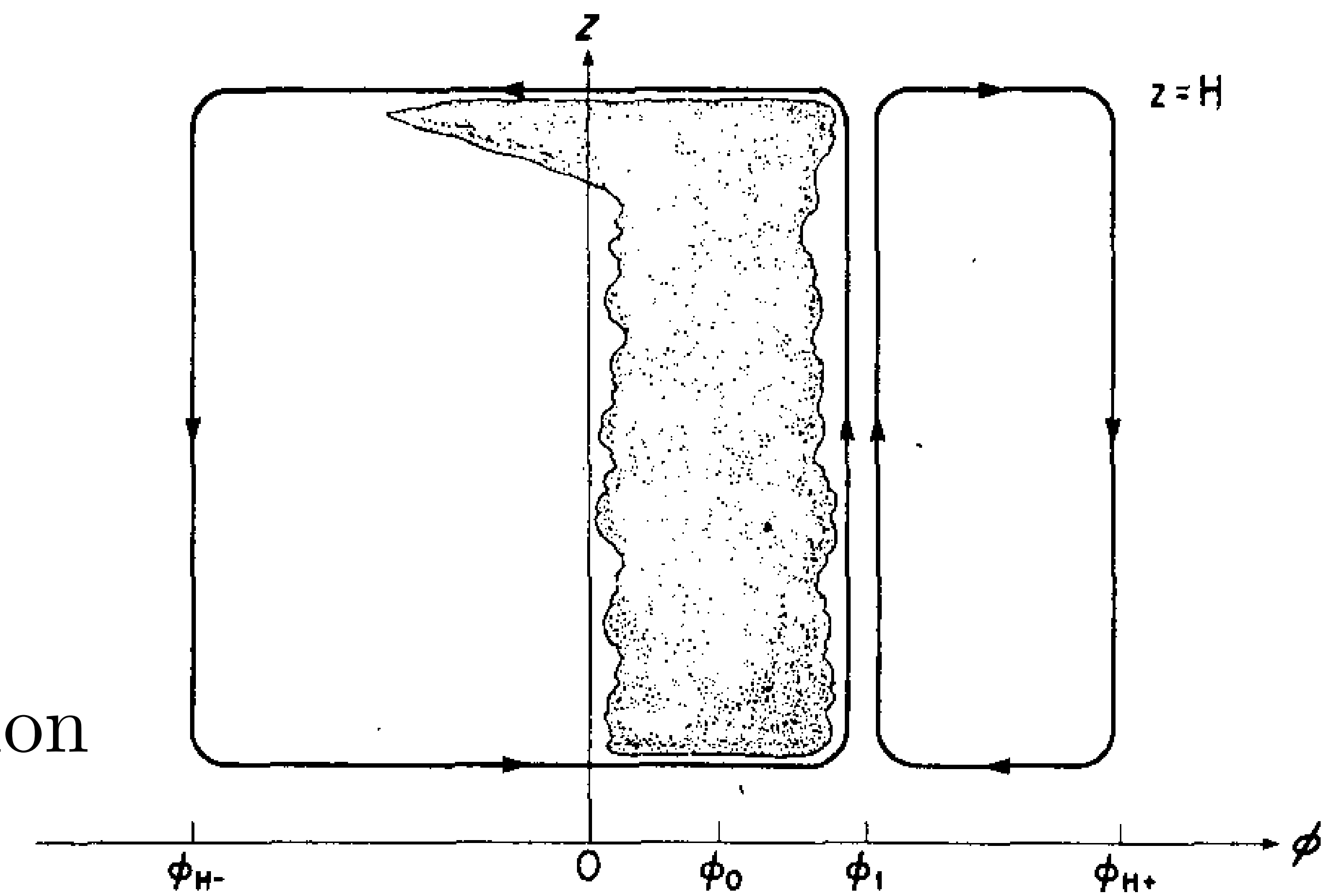

FIG. 3. Schematic illustration of the Hadley circulation.

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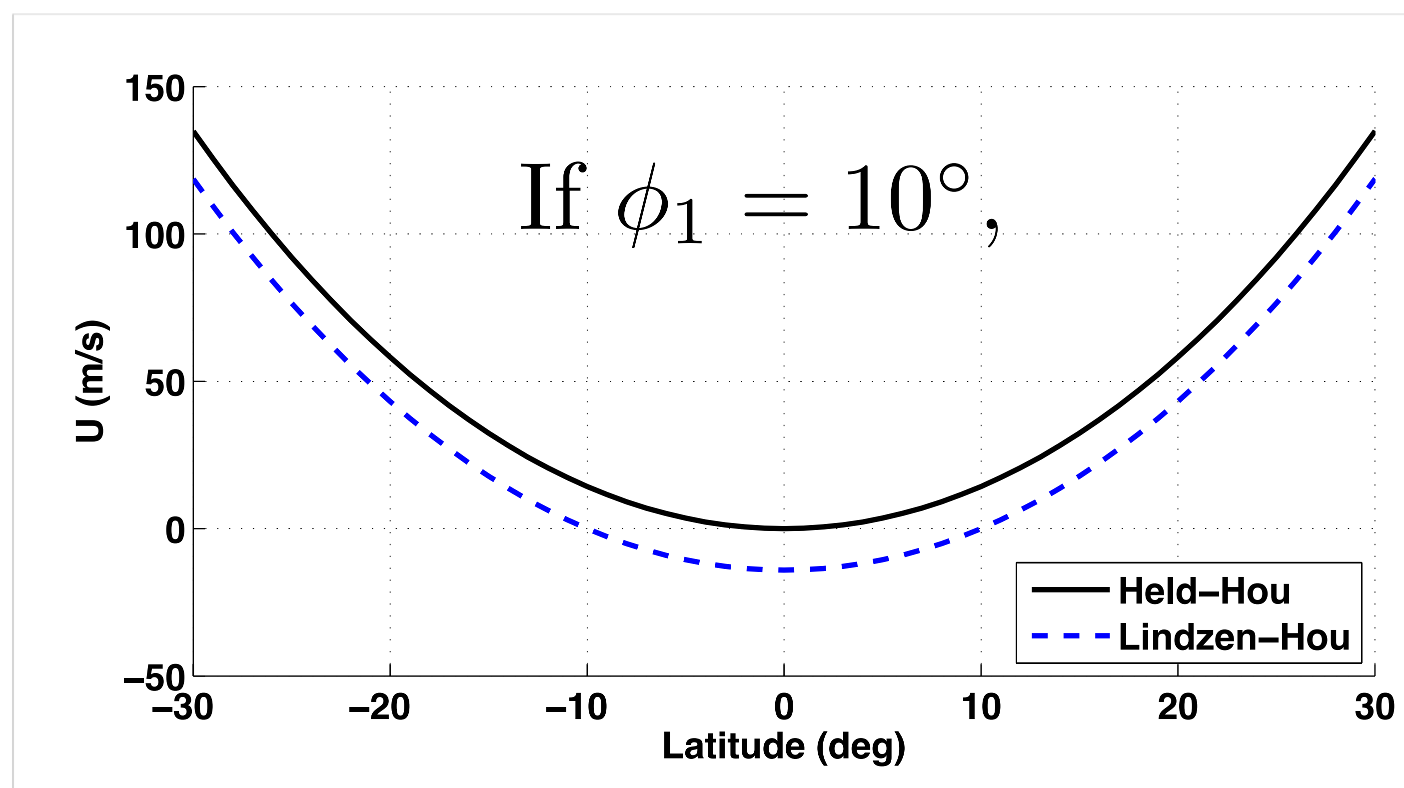


Hadley Cell - Theory: Asymmetry about the equator*

- Angular momentum:

$$U_M = \frac{\Omega a(\cos^2\phi_1 - \cos^2\phi)}{\cos\phi}$$

- Thermal wind relation:

From steady state momentum equation

$$fu + \frac{u^2\tan\phi}{a} = -\frac{1}{a}\frac{\partial\Phi}{\partial\phi} \qquad \Phi = \frac{p}{\rho_s}$$

At z=H and Z=0

$$f[u(H) - u(0)] + \frac{\tan\phi}{a}[u^2(H) - u^2(0)] = -\frac{1}{a}\frac{\partial}{\partial\phi}[\Phi(H) - \Phi(0)]$$

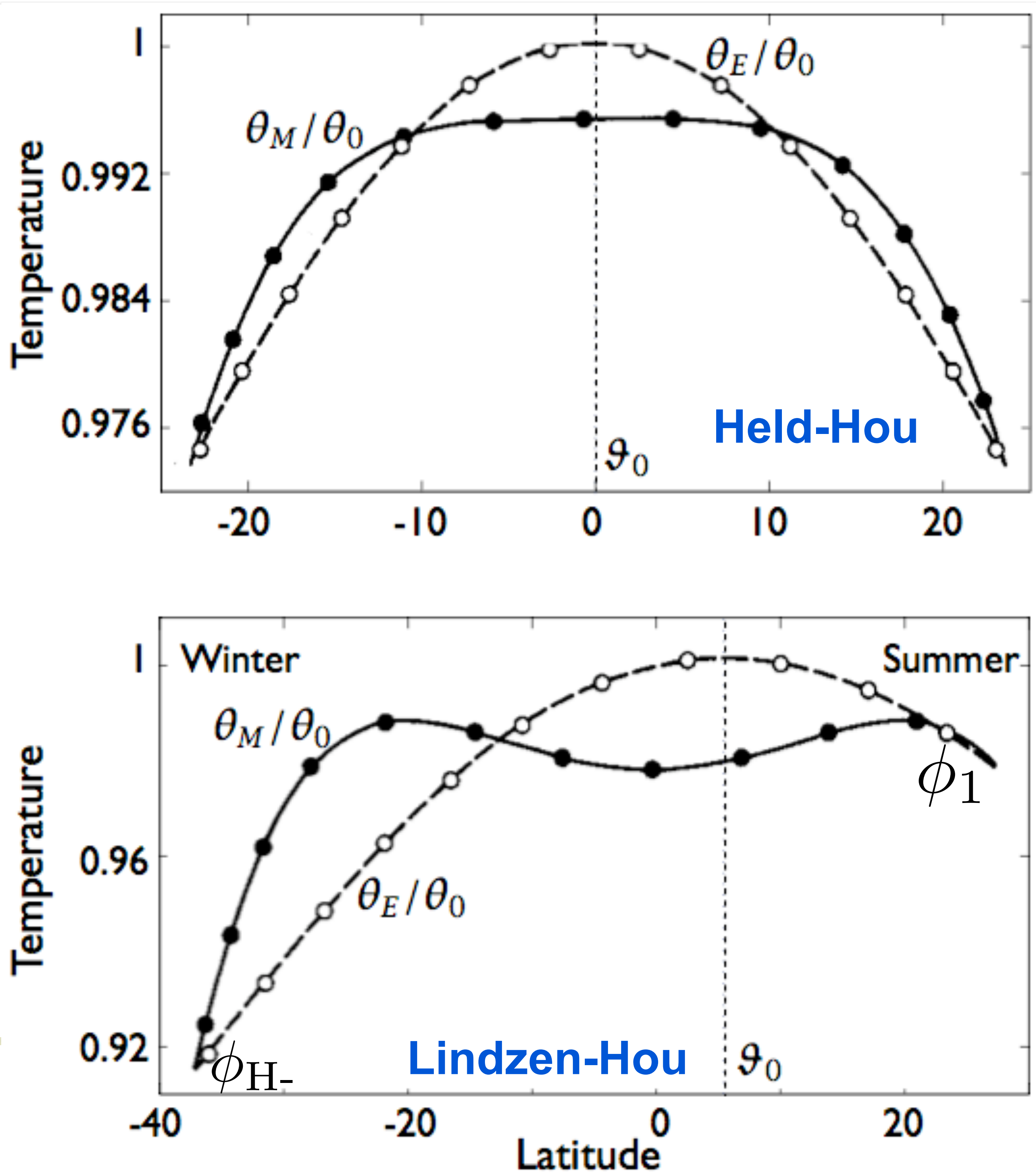
$$f[u(H) - u(0)] + \frac{\tan\phi}{a}[u^2(H) - u^2(0)] = -\frac{gH}{a\Theta_o}\frac{\partial\tilde{\Theta}}{\partial\phi}$$

Integrate with respect to ϕ

- Temperature lat. variation:

$$\frac{\tilde{\Theta}(0) - \tilde{\Theta}(\phi)}{\Theta_o} = \frac{\Omega^2 a^2}{2gH}\frac{(\sin^2\phi - \sin^2\phi_1)^2}{\cos^2\phi}$$

Hadley Cell - Theory: Asymmetry about the equator*



$$\frac{\tilde{\Theta}(0)-\tilde{\Theta}(\phi)}{\Theta_o}=\frac{\Omega^2a^2}{2gH}\frac{(\sin^2\phi-\sin^2\phi_1)^2}{\cos^2\phi}$$

- Extent of Hadley Cell satisfies:
  - Temperature should be continuous at the edge:

$$\tilde{\Theta}(\phi_{\text{H+}})=\tilde{\Theta}_E(\phi_{\text{H+}})$$

$$\tilde{\Theta}(\phi_{\text{H-}})=\tilde{\Theta}_E(\phi_{\text{H-}})$$

$$\text{also continous at } \phi_1, \tilde{\Theta}(\phi_1)=\tilde{\Theta}_E(\phi_1)$$

  - Hadley cell does not produce net heating but just carry heat poleward over the extent of Hadley Cell:

$$\int_{\phi_1}^{\phi_{\text{H+}}}\tilde{\Theta}\cos\phi d\phi=\int_{\phi_1}^{\phi_{\text{H+}}}\tilde{\Theta}_E\cos\phi d\phi$$

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Hadley Cell - Theory:
Asymmetry about the equator*

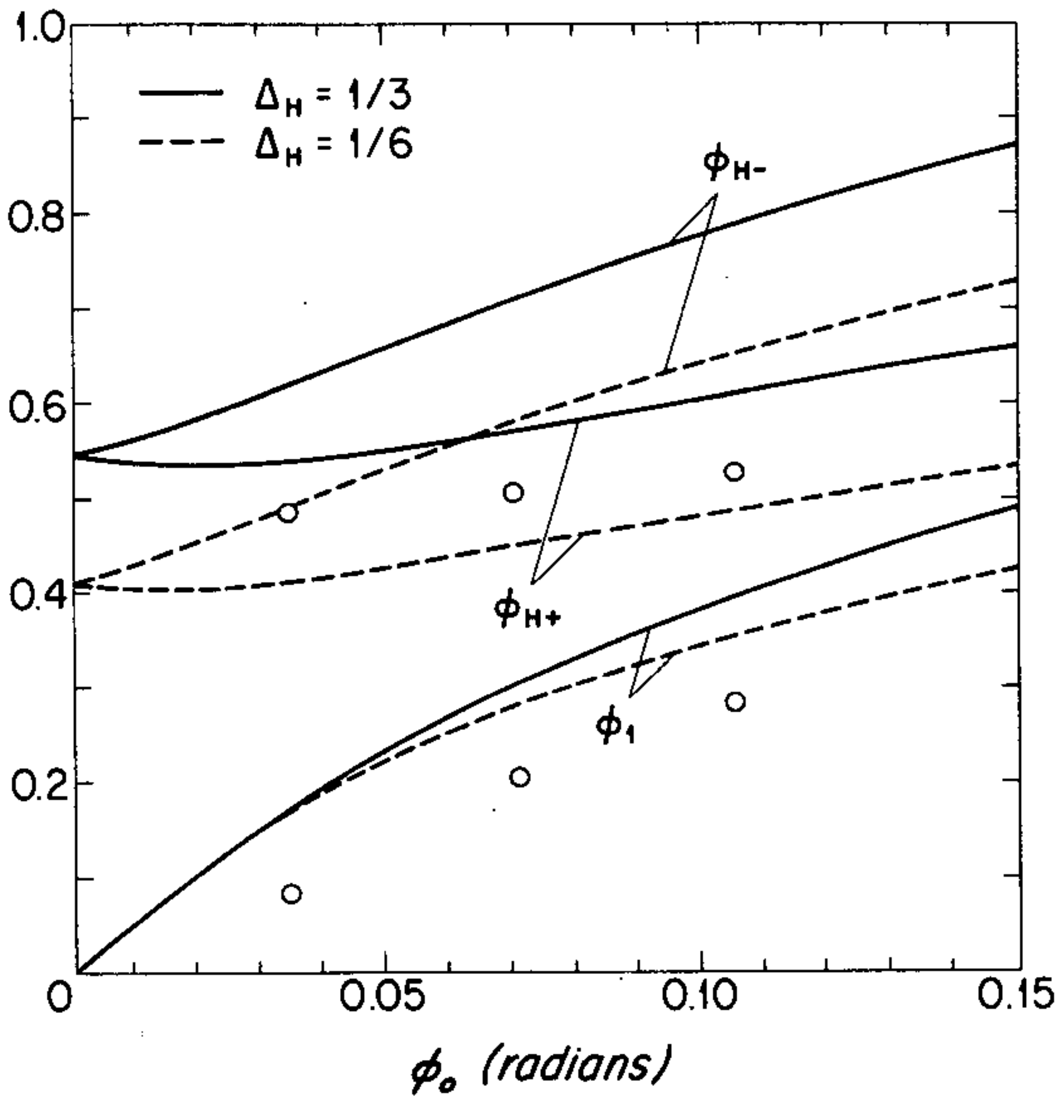

FIG. 4. ϕ_1, ϕ_{H^+} and ϕ_{H^-} as functions of ϕ_0 (see text for definitions). Open circles show results from numerical integration for ϕ_1 and ϕ_{H-1} when $\Delta_H = 1/6$. (Note 1° of latitude ≈ 0.0175 radians.)

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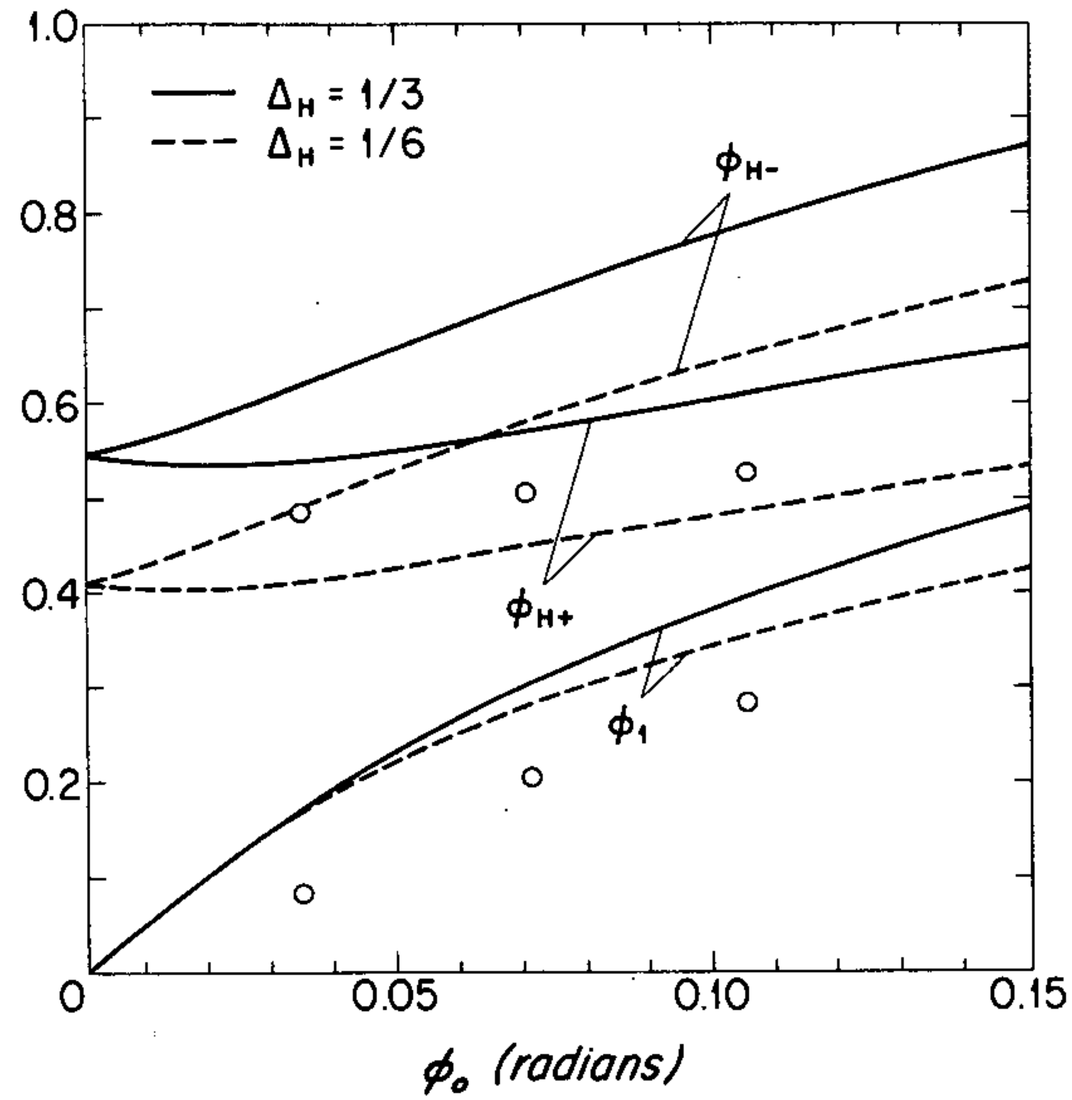

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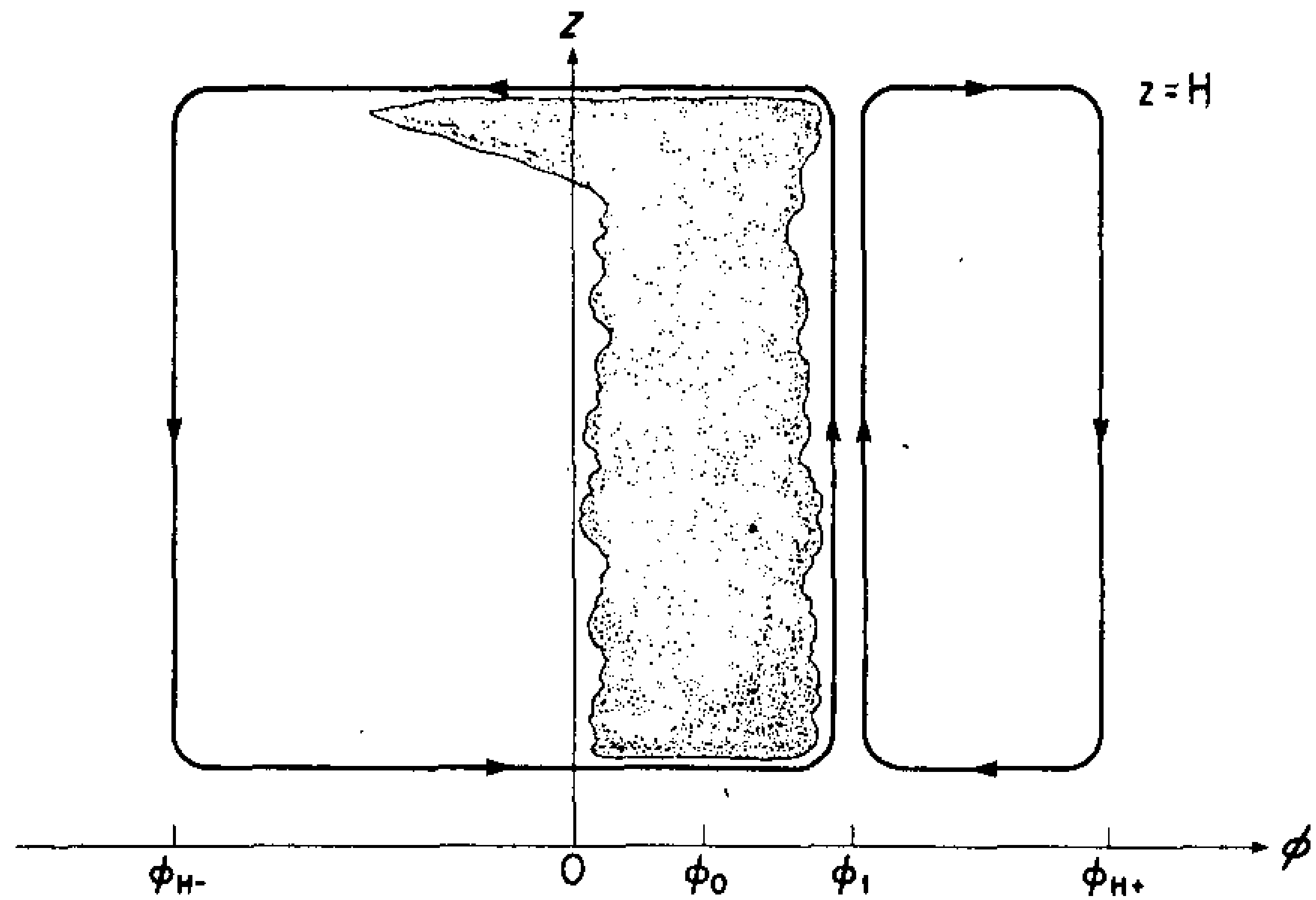

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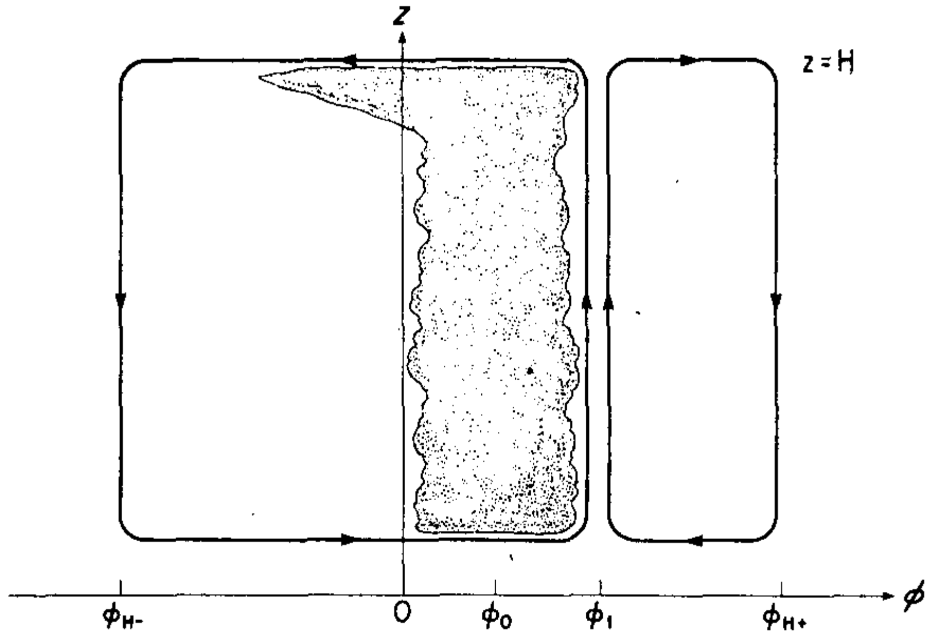
Hadley Cell - Theory: Asymmetry about the ec



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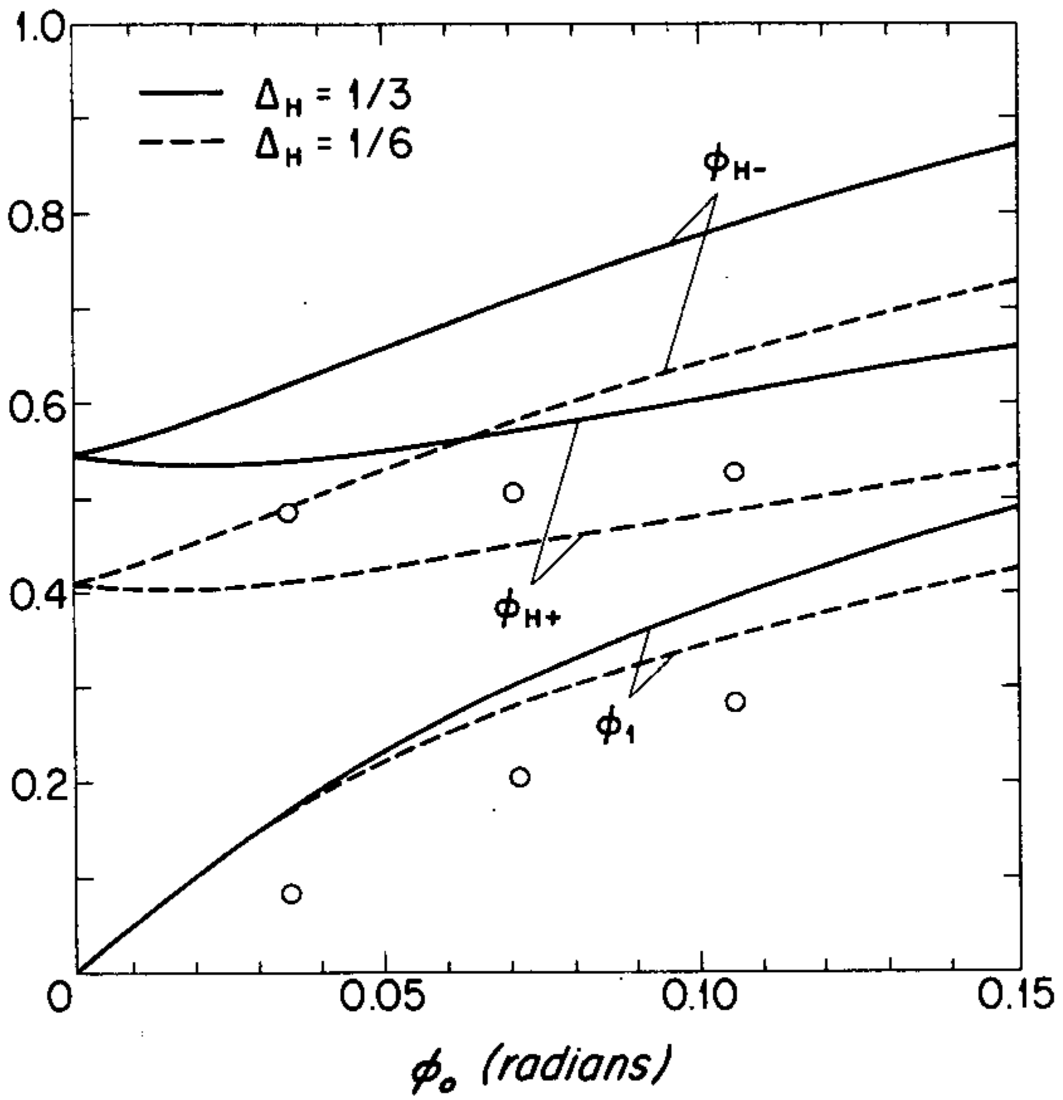

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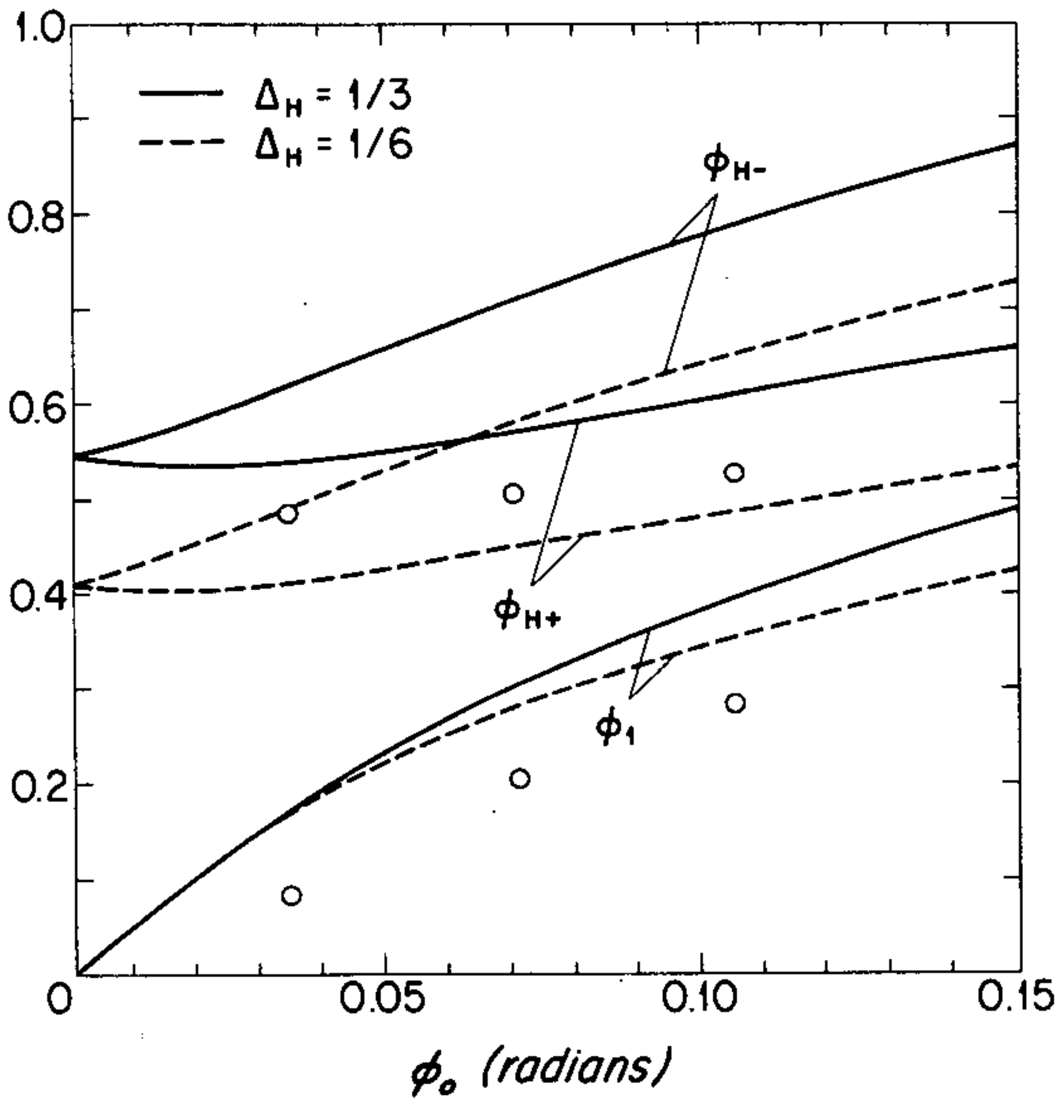
- As the heating **shifts off the equator**, both cells and the center of the raising branch **shift poleward**, with a **wider** Hadley cell **cross the equator** and a **narrower** Hadley cell in **the summer hemisphere**;
- As the diabatic heating varies stronger in the meridional direction, **both cells shift poleward and become wider**.

Hadley Cell - Theory: Asymmetry about t



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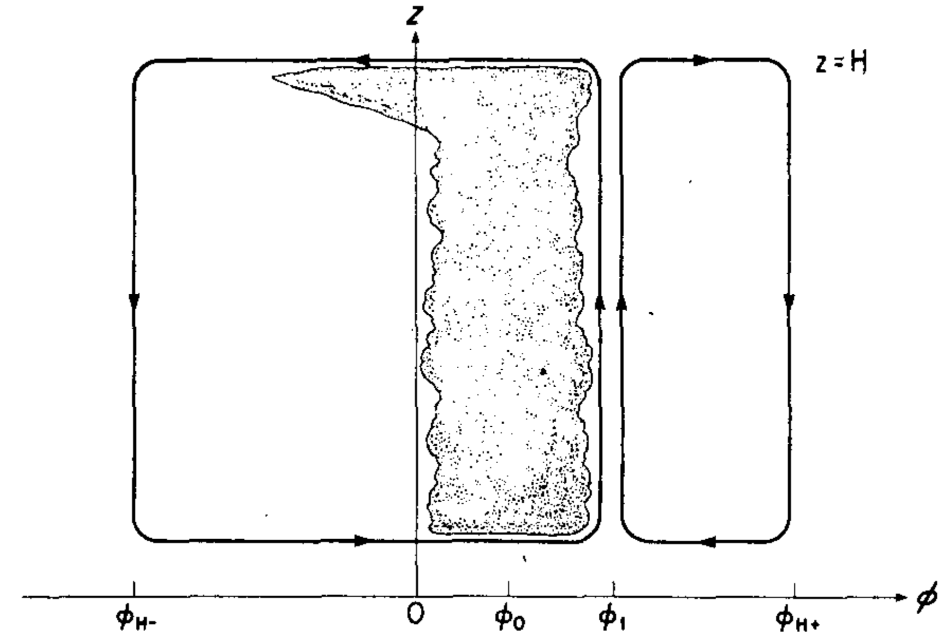

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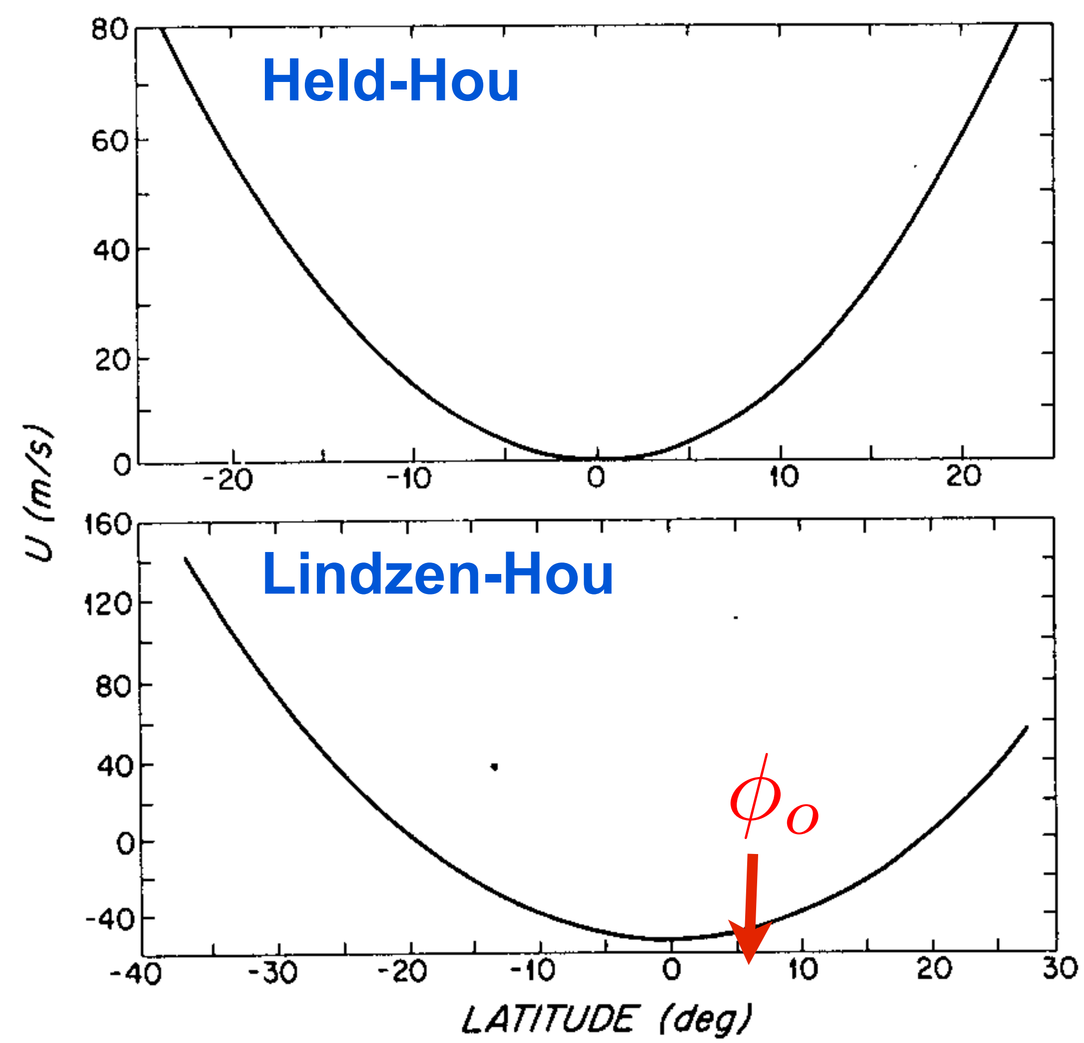
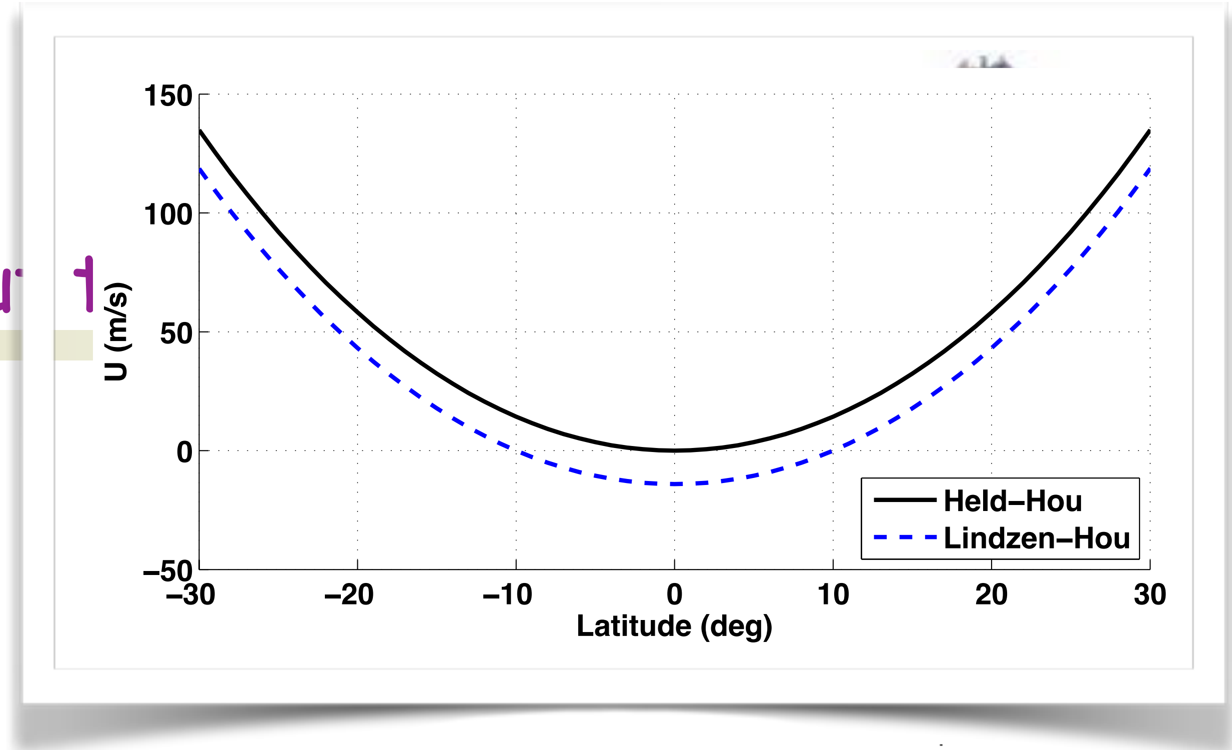
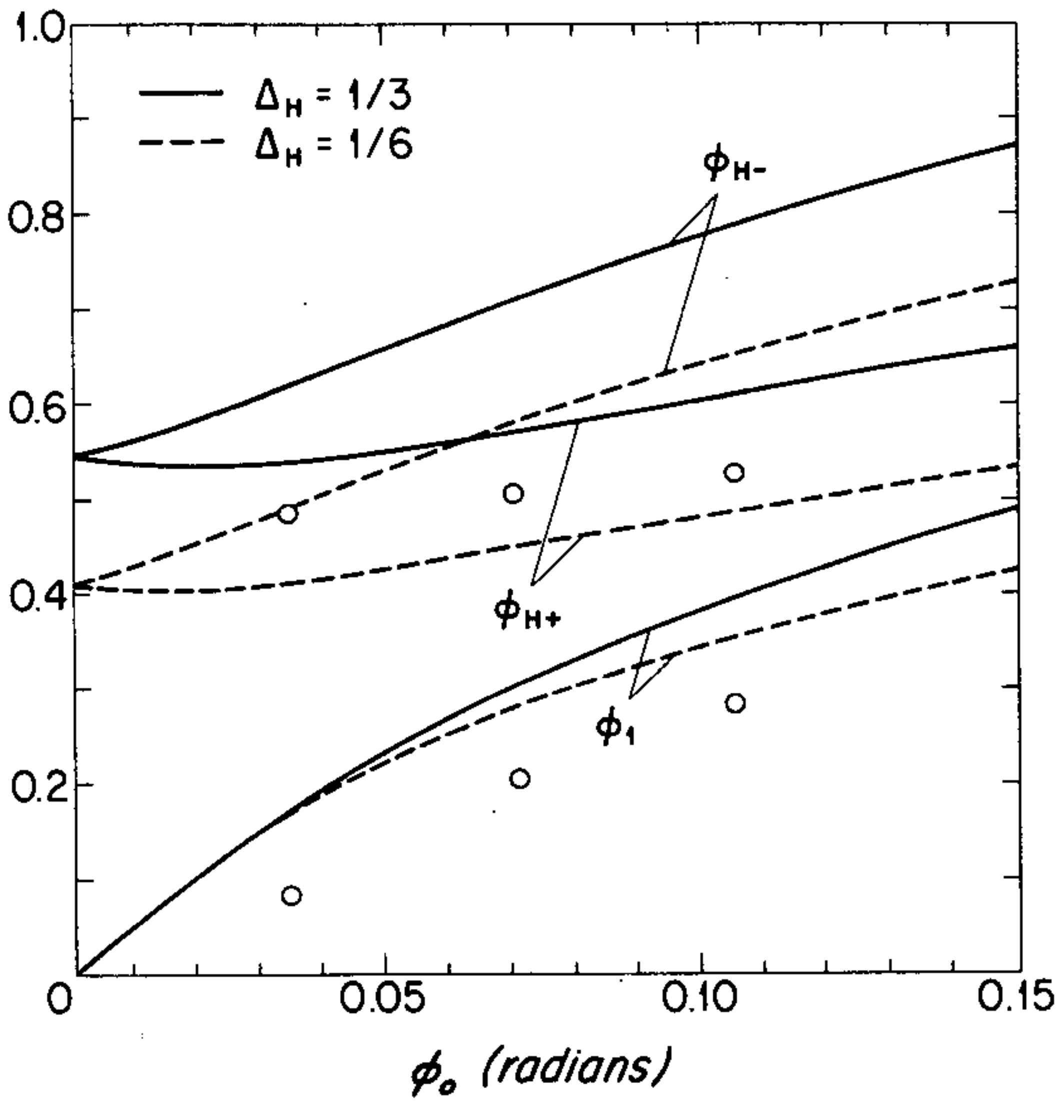


FIG. 6. $u(H, \phi)$ as a function of ϕ using the simple model. (a) $\phi_0 = 0$. (b) $\phi_0 = 6°$.

Hadley Cell - Theory:
Asymmetry abou





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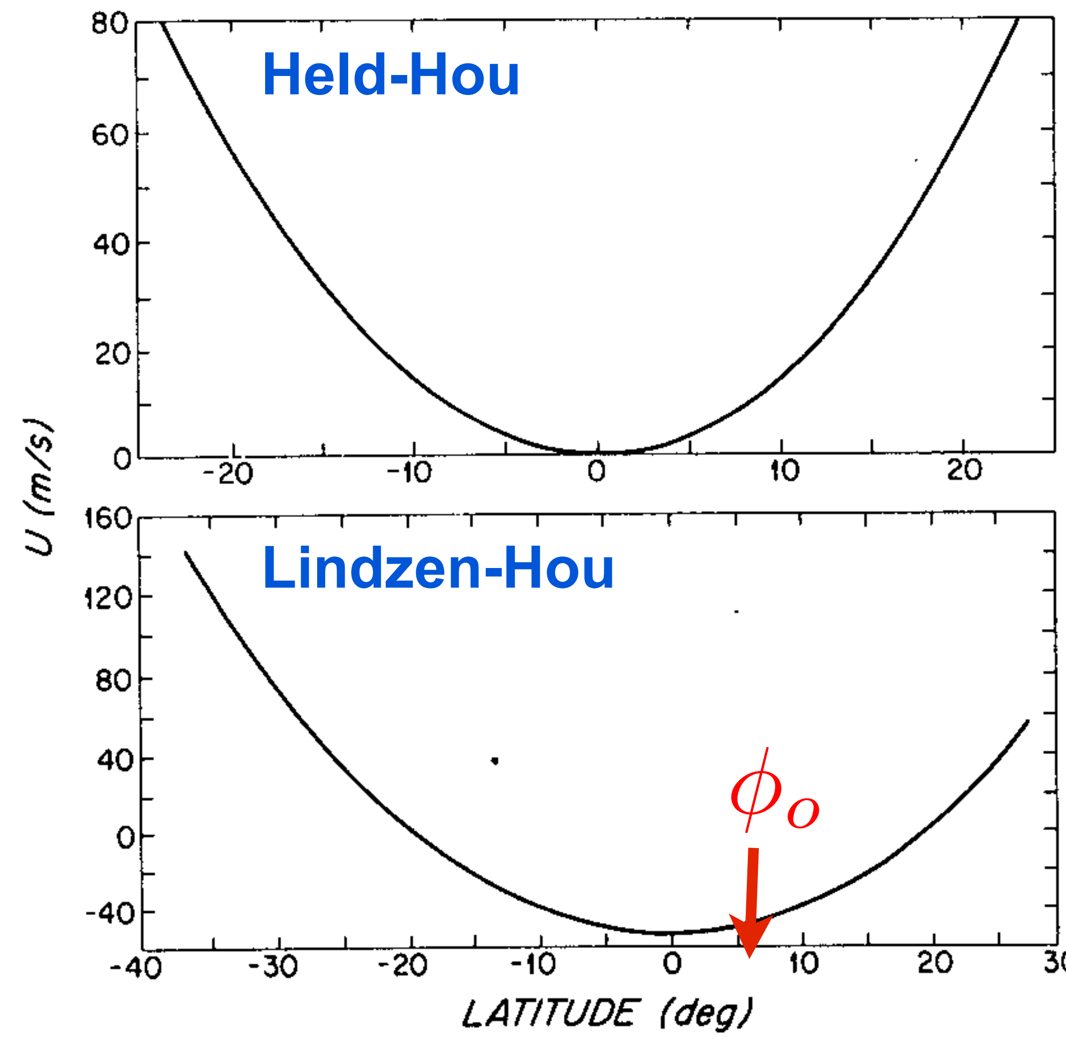

FIG. 6. $u(H, \phi)$ as a function of ϕ using the simple model. (a) $\phi_0 = 0$. (b) $\phi_0 = 6°$.

Hadley Cell - Discussion: Asymmetry about the equator*

- **Lindzen-Hou (1988)**
  - Quasi-steady assumption, however the seasonal cycle is temporally progressing;
  - The lack of angular momentum conservation in reality, especially when the angular momentum transport by **eddies** are significant.
  - The **moisture effect** is still neglected;