第四章:

中纬度的经向环流系统(II)

- Ferrel cell, baroclinic eddies and the westerly jet

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第四章:

中纬度的经向环流系统(II)

- Ferrel cell, baroclinic eddies and the westerly jet

Reference reading:
Vallis Chapter 6.5-6.7; Charney 1947; Eady 1949

Observations

- Summary:
  - Zonal-mean flow:
    - Ferrel Cell: an indirect cell centered at 40-60 degree, with strong seasonal variation in N.H.
    - Westerly jet: surface westerlies centered at 40-60 degree
  - Eddies: transient eddies are dominant with stationary eddies only obvious in N.H.
    - Kinetic energy
    - Momentum flux
    - Heat flux

The Ferrel Cell

eddy-zonal flow interaction (I)

- The simplified equations:
  - Momentum equation:

$$\frac{\partial [u]}{\partial t} = -\frac{\partial([u^* v^*])}{\partial y} + f[v] + [F_x]$$

  - Continuity equation:

$$\frac{\partial [v]}{\partial y} + \frac{\partial [\omega]}{\partial p} = 0$$

  - Thermodynamic equation:

$$\frac{\partial [\theta]}{\partial t} + [\omega]\frac{\partial \theta_s}{\partial p} = -\frac{\partial([\theta^* v^*])}{\partial y} + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q]}{c_p}$$

$$\left(\frac{d}{dt}\right)_p = \left(\frac{\partial}{\partial t}\right)_p + u\left(\frac{\partial}{\partial x}\right)_p + v\left(\frac{\partial}{\partial y}\right)_p + \omega\frac{\partial}{\partial p}$$

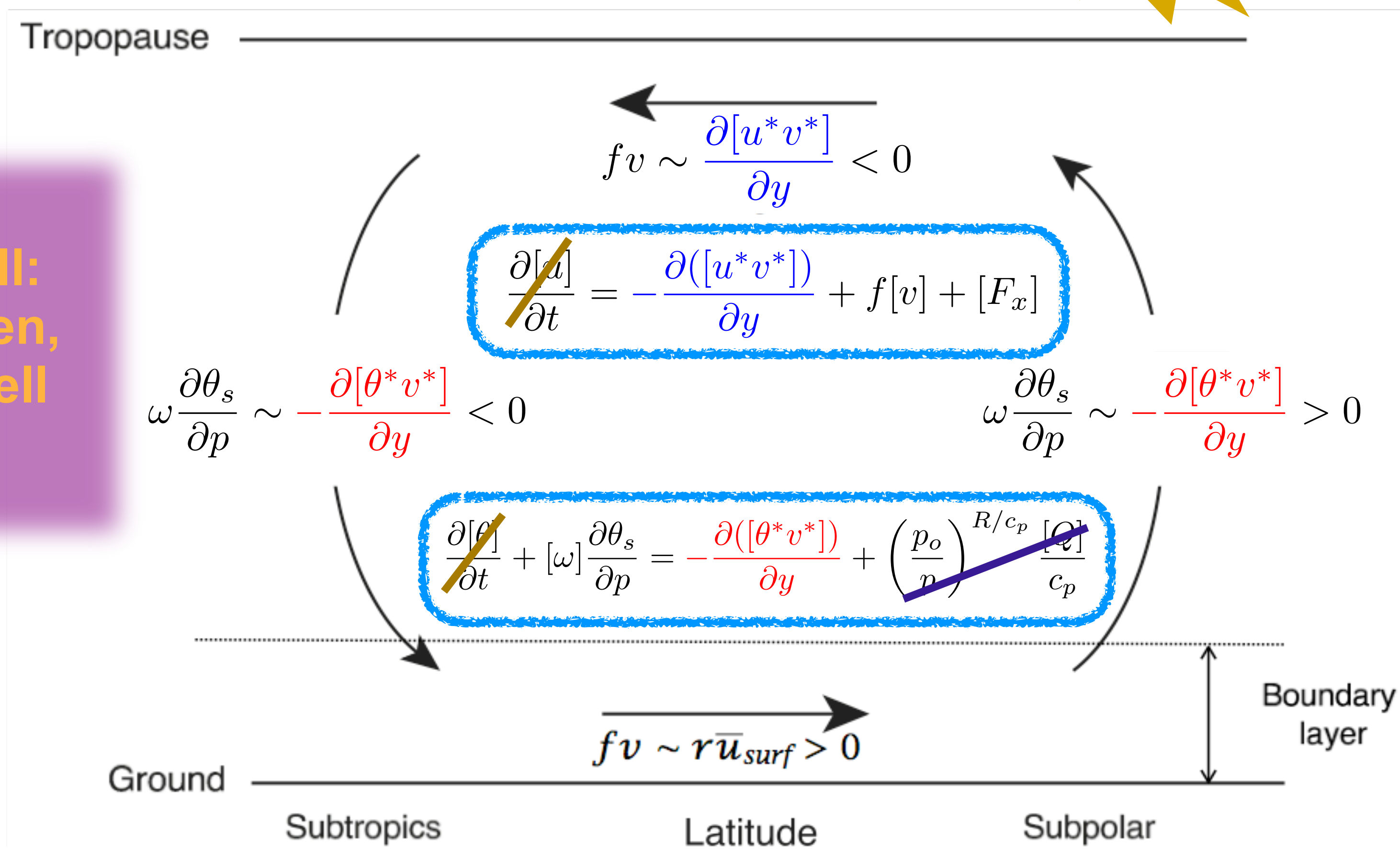
Under the quasi-geostrophic approximation $(R_o \ll 1)$

The Ferrel Cell

Review

- The balance equations:

Ferrel Cell: eddy-driven, indirect cell



Outline

- Observations
- The Ferrel Cell
- Baroclinic eddies
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction, E-P flux
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- Energy cycle

Baroclinic eddies

- baroclinic instability

Review

- Instability:

  - Phenomenon: Given a *basic flow* with *perturbations* at the initial moment, if the perturbation *grows with time*, the basic flow is always taken *unstable*.

  - Mathematics: $P \propto Ae^{\alpha t}, \exists\, \alpha > 0$ (相对于波动解:$P \propto Ae^{i\omega t}$)

  - Energy: 能量源 → 扰动动能

- Linear Instability: the instability that arises in *a linear system*.

Baroclinic eddies

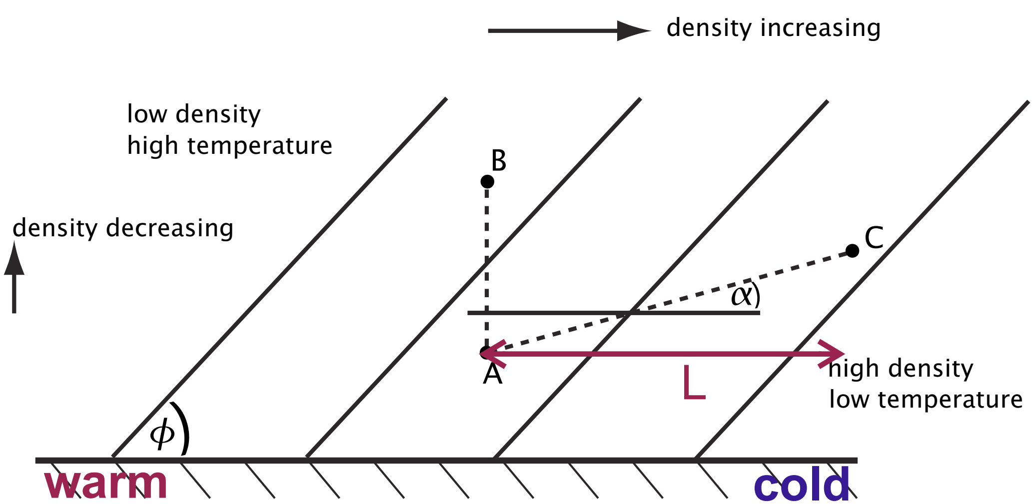
- baroclinic instability

- Baroclinic Instability - "is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*".

Baroclinic eddies

- baroclinic instability

- **Baroclinic Instability** - "is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*".



From A to B: negative buoyant

If A and C are interchanged:

$$PE = \int \rho g dz$$

$$\begin{aligned} \Delta PE &= g(\rho_A z_A + \rho_C z_C - \rho_C z_A - \rho_A z_C) \\ &= g(z_A - z_C)(\rho_A - \rho_C) \\ &= g\Delta\rho\Delta z \end{aligned}$$

$$\Delta PE = g(L\frac{\partial \rho}{\partial y} + L\tan\alpha\frac{\partial \rho}{\partial z})L\tan\alpha$$

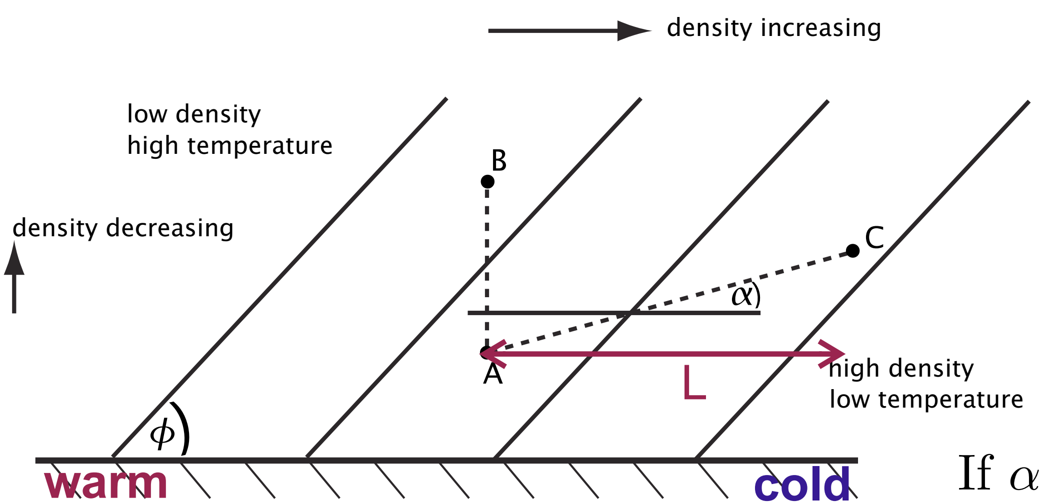
Assume small α and ϕ

$$\Delta PE = gL^2\frac{\partial \rho}{\partial y}\alpha\left(1 - \frac{\alpha}{\phi}\right)$$

Baroclinic eddies

- baroclinic instability

- **Baroclinic Instability** - “is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*”.



From A to B: negative buoyant

If A and C are interchanged:

$$PE = \int \rho g dz$$

$$\Delta PE = gL^2 \frac{\partial \rho}{\partial y} \alpha \left(1 - \frac{\alpha}{\phi}\right)$$

α is called mixing slope.

If $\alpha < \phi$, a loss of potential energy.

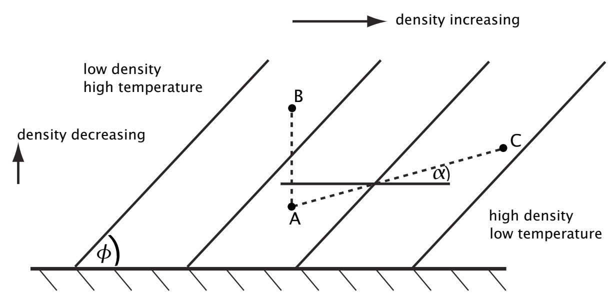
If $\alpha = \frac{1}{2}\phi$, ΔPE is strongest.

Baroclinic eddies

- baroclinic instability

- **Baroclinic Instability** - "is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*".

- Energetics: PE → KE



- Mathematics:
  - Linear Baroclinic Instability
    - Linear baroclinic system →
      - Eady's model (1949)
      - Charney's model (1947)

Baroclinic eddies

- linear baroclinic instability

- Eady's model (1949)
- Charney's model (1947)

Long Waves and Cyclone Waves

By E. T. EADY, Imperial College of Science, London

(Manuscript received 28 Febr. 1949)

Abstract

By obtaining complete solutions, satisfying all the relevant simultaneous differential equations and boundary conditions, representing small disturbances of simple states of steady baroclinic large-scale atmospheric motion it is shown that these simple states of motion are almost invariably unstable. An arbitrary disturbance (corresponding to some inhomogeneity of an actual system) may be regarded as analysed into "components" of a certain simple type, some of which grow exponentially with time. In all the cases examined there exists one particular component which grows faster than any other. It is shown how, by a process analogous to "natural selection", this component becomes dominant in that almost any disturbance tends eventually to a definite size, structure and growth-rate (and to a characteristic life-history after the disturbance has ceased to be "small"), which depends only on the broad characteristics of the initial (unperturbed) system. The characteristic disturbances (forms of breakdown) of certain types of initial system (approximating to those observed in practice) are identified as the ideal forms of the observed cyclone waves and long waves of middle and high latitudes. The implications regarding the ultimate limitations of weather forecasting are discussed.

he present paper aims at developing from principles a quantitative theory of the al stages of development of wave-cyclones long waves. For reasons of space and lability both the argument and the mathe-ics have been rather heavily compressed. ıller and extended treatment of several of points raised will be given in subsequent ers.

The Equations of Motion

wing to the complexity (and non-linearity) he simultaneous partial differential equa-s governing atmospheric motion it is rable to simplify these by the omission of hose terms which do not make a major ribution *to the particular type and scale of*

motion, we may then by successive approxi tion take into account any or all of the te originally omitted. In the present instance are concerned with relatively rapid devel ment, by comparison with which radia processes (or rather their differential effe are slow. For a first approximation there we consider the motion as adiabatic. Also are concerned with the motion of deep la and for a first approximation we neglect effects of internal friction ("turbulence") skin friction. A rough calculation shows the energy dissipated in the surface frict layer is usually much less than the ene supply to the growing disturbance and th probably, in most cases, the major source energy loss. The present paper is concer

Eady, E. 1949. Tellus, 1, 33-52

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THE DYNAMICS OF LONG WAVES IN A BAROCLINIC WESTERLY CURRENT

By J. G. Charney

University of California at Los Angeles[1]

(Manuscript received 9 December 1946)

ABSTRACT

Previous studies of the long-wave perturbations of the free atmosphere have been based on mathematical models which either fail to take properly into account the continuous vertical shear in the zonal current or else neglect the variations of the vertical component of the earth's angular velocity. The present treatment attempts to supply both these elements and thereby to lead to a solution more nearly in accord with the observed behavior of the atmosphere.

By eliminating from consideration at the outset the meteorologically unimportant acoustic and shearing-gravitational oscillations, the perturbation equations are reduced to a system whose solution is readily obtained.

Exact stability criteria are deduced, and it is shown that the instability increases with shear, lapse rate, and latitude, and decreases with wave length. Application of the criteria to the seasonal averages of zonal wind suggests that the westerlies of middle latitudes are a seat of constant dynamic instability.

The unstable waves are similar in many respects to the observed perturbations: The speed of propagation is generally toward the east and is approximately equal to the speed of the surface zonal current. The waves exhibit thermal asymmetry and a westward tilt of the wave pattern with height. In the lower troposphere the maximum positive vertical velocities occur between the trough and the nodal line to the east in the pressure field.

The distribution of the horizontal mass divergence is calculated, and it is shown that the notion of a fixed level of nondivergence must be replaced by that of a sloping surface of nondivergence.

The Rossby formula for the speed of propagation of the barotropic wave is generalized to a baroclinic atmosphere. It is shown that the barotropic formula holds if the constant value used for the zonal wind is that observed in the neighborhood of 600 mb.

CONTENTS

1. Introduction

The large-scale weather phenomena in the extra-tropical zones of the earth are associated with great migratory vortices (cyclones) traveling in the belt of prevailing westerly winds. One of the fundamental problems in theoretical meteorology has been the explanation of the origin and development of these cyclones. The first significant step toward a solution was taken in 1916 by V. Bjerknes [8, p. 785], who advanced the theory, based upon general hydrodynamic considerations, that cyclones originate as dynamically unstable wavelike disturbances in the westerly current. The subsequent discovery of the polar front by J. Bjerknes [2] made possible an empirical confirmation of the theory, for, following this discovery, the synoptic studies of J. Bjerknes and

[1] U.C.L.A. Department of Meteorology, Papers in Meteorology, No. 4.

At present the author is National Research Fellow at the Institute for Theoretical Astrophysics, University of Oslo.

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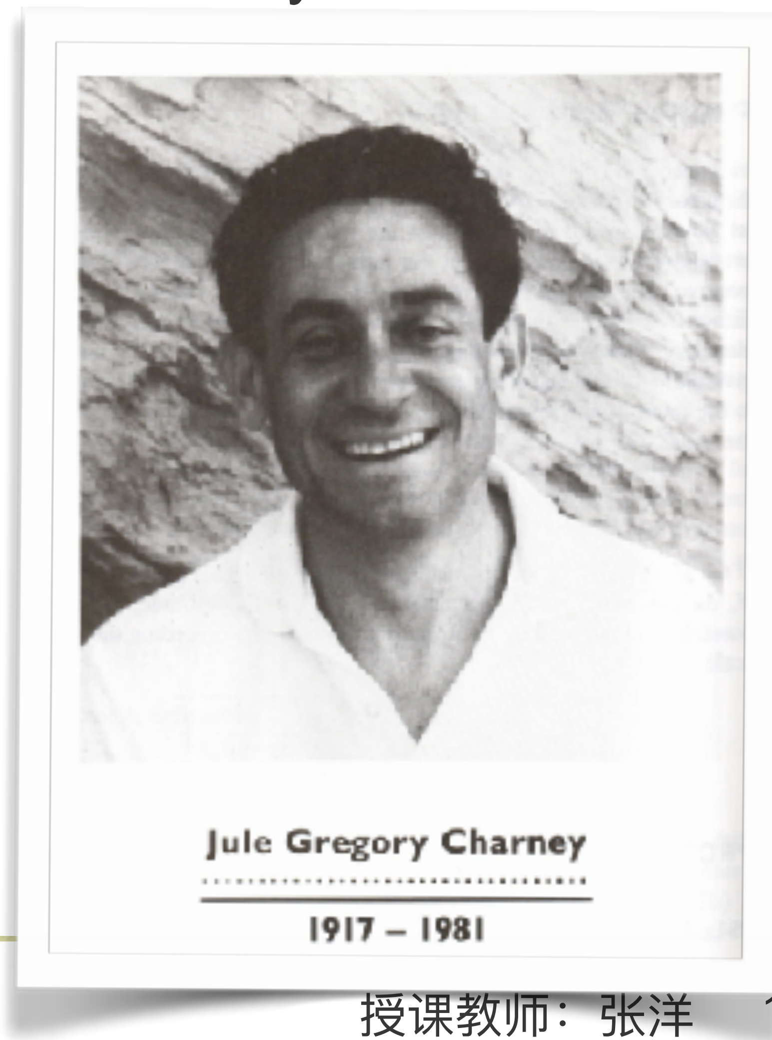
Baroclinic eddies

- linear baroclinic instability

- Eady's model

Eric Thomas Eady
(1915-1966)

- Charney's model



Baroclinic eddies

- linear baroclinic instability

- Charney's model

" **JULE CHARNEY** was one of the dominant figures in atmospheric science in the three decades following World War II. Much of the change in meteorology from an art to a science is due to his scientific vision and his thorough commitment to people and programs in this field."

-- *by Norman Phillips*

Jule Gregory Charney

1917 – 1981

Baroclinic eddies

- linear baroclinic instability

- Eady's model (1949)

a) The basic zonal flow has uniform vertical shear,

$$U_o(Z) = \Lambda Z,\ \ \Lambda \text{ is a constant}$$

b) The fluid is uniformly stratified,

N^2 is a constant.

c) Two rigid lids at the top and bottom, flat horizontal surface, that is

$$\omega = 0 \text{ at } Z = 0 \text{ and } H.$$

d) The motion is on the **f -plane**, that is

$$\beta = 0$$

- Charney's model (1947)

The most distinguished difference with Eady's model is that **beta effect** is considered.

Baroclinic eddies

- linear baroclinic instability

Small amplitude assumption
小扰动

Linear baroclinic system:
Eady model
Charney model

Normal mode assumption
标准波形

Obtain the solutions, e.g.
instability conditions
growth rate
most unstable mode

Variable = Basic state + Perturbation

$$u(\mathbf{x},t) = U(z) + u'(\mathbf{x},t)$$

$$u'(\mathbf{x},t) \ll U(z)$$

Linearized PV equation (q=PV):

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)q' + \frac{\partial \psi}{\partial x}\frac{\partial \bar{q}}{\partial y} = 0$$

$$q' = \frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial y^2} + \frac{f_o^2}{\rho_s}\frac{\partial}{\partial z}\left(\frac{\rho_s}{N^2}\frac{\partial \psi'}{\partial z}\right)$$

$$\bar{q} = \frac{\partial^2 \bar{\psi}}{\partial y^2} + \beta y + \frac{f_o^2}{\rho_s}\frac{\partial}{\partial z}\left(\frac{\rho_s}{N^2}\frac{\partial \bar{\psi}}{\partial z}\right)$$

标准波形法， 带入方程和边界条件：

$$\psi'(\mathbf{x},t) = Ae^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$$

Find the conditions for non-trivial solutions and Ci >0

Baroclinic eddies

- linear baroclinic instability

- Conclusions:

Necessary condition for baroclinic instability: PV gradient changes sign in the interior or boundaries (Charney-stern theory), according to which the midlatitude atmosphere is baroclinic unstable. Different models. i.e. Eady and Charney models have more rigorous conditions.

Growth rate: $\sigma = kc_i \approx 0.3\,\Lambda \dfrac{f_o}{N}$ in both Eady and Charney models!

Most unstable mode:

$$k_{\max}^{-1} \propto L_d^{-1} = \left(\frac{NH}{f_o}\right)^{-1}$$

Eady

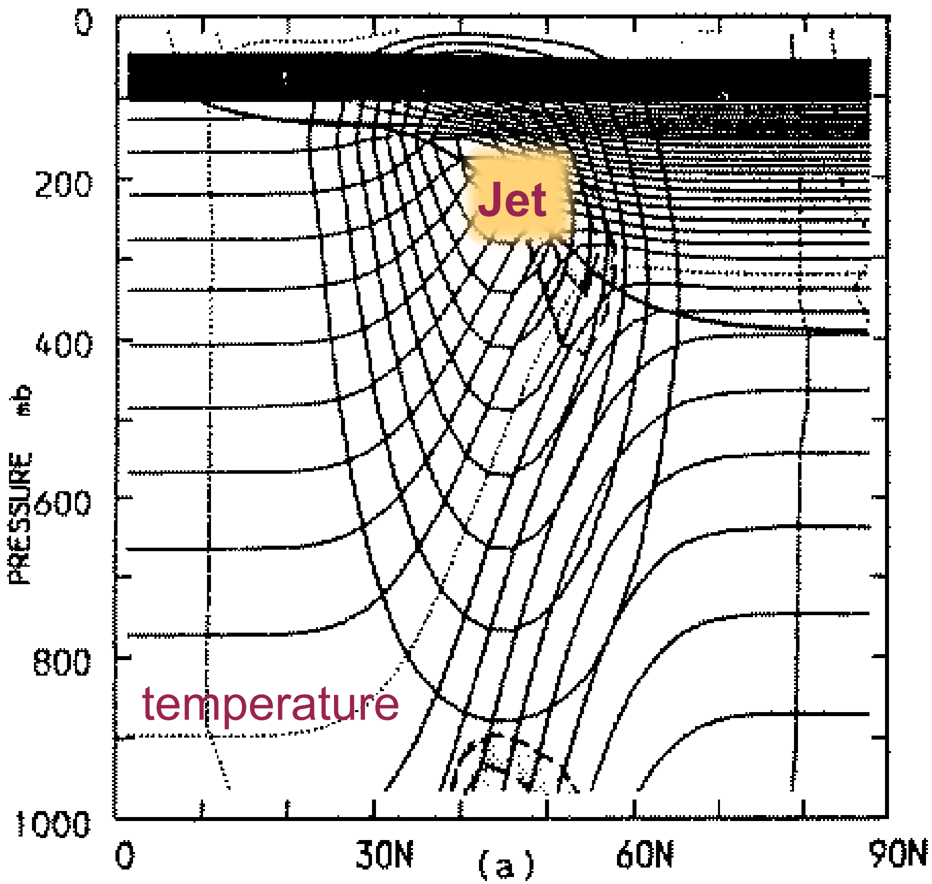
$$k_{\max}^{-1} \propto \Lambda \frac{f_o}{\beta N}$$

Charney

- Discussion
  - Normal mode assumption
  - Small amplitude assumption, linearization
  - Assumption: uniform vertical shear of the zonal flow

Baroclinic eddies

- baroclinic eddy life cycle



Numerical simulations
with idealized GCM:

(Thorncroft et al, 1993, Q.J.R.)

Basic state at the initial moment:
close to real atmosphere.

Results: Capture the synoptic feature of baroclinic eddies.

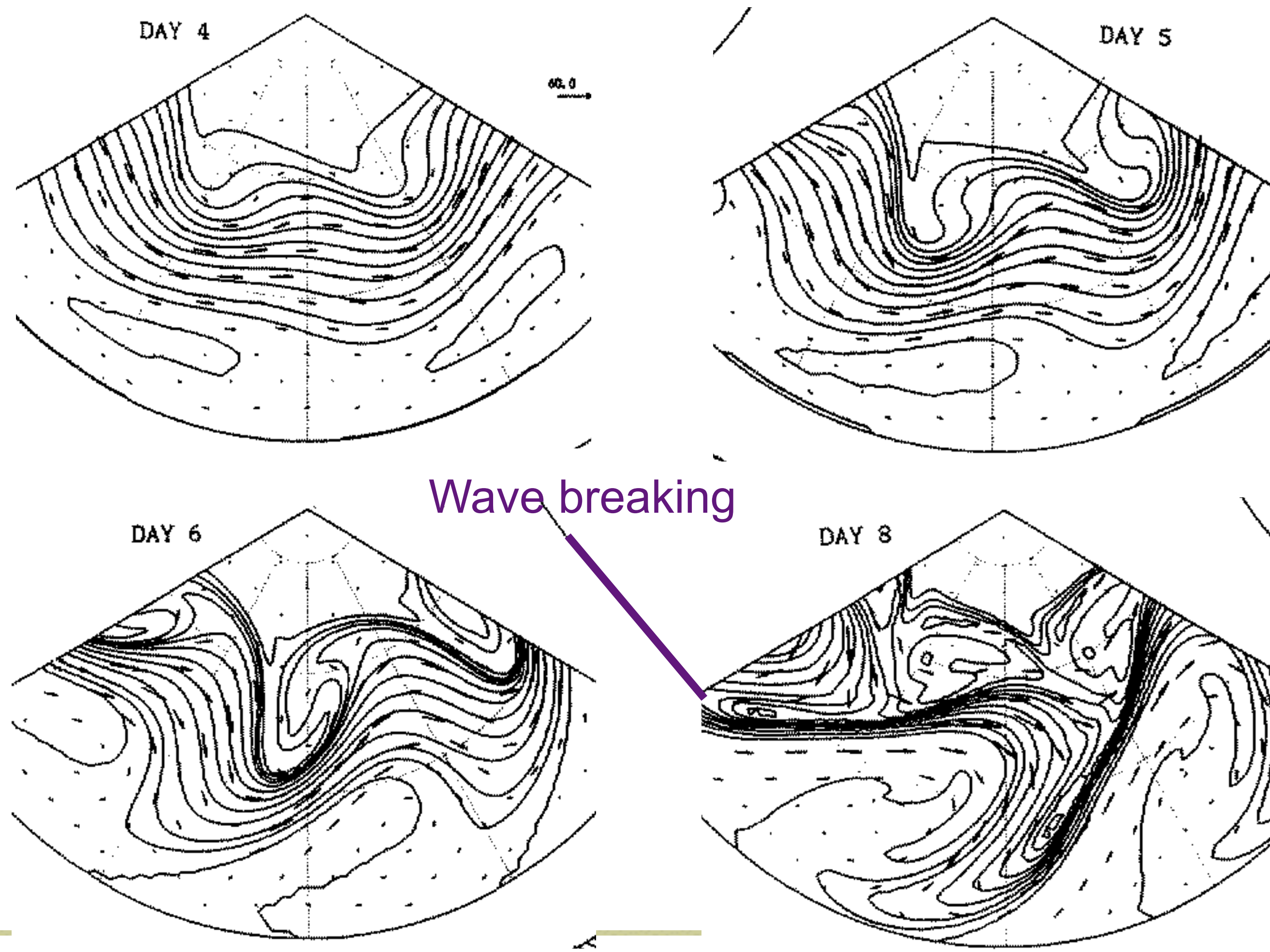
Baroclinic eddies

- baroclinic eddy life cycle

- Eddies' development

Small amplitude perturbations

Finite amplitude perturbations



(Thorncroft et al, 1993, Q.J.R.)

Baroclinic eddies

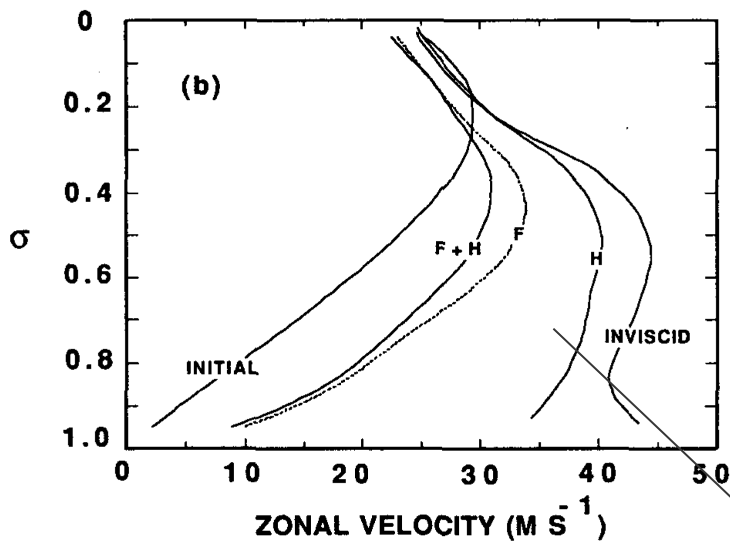
- baroclinic eddy life cycle

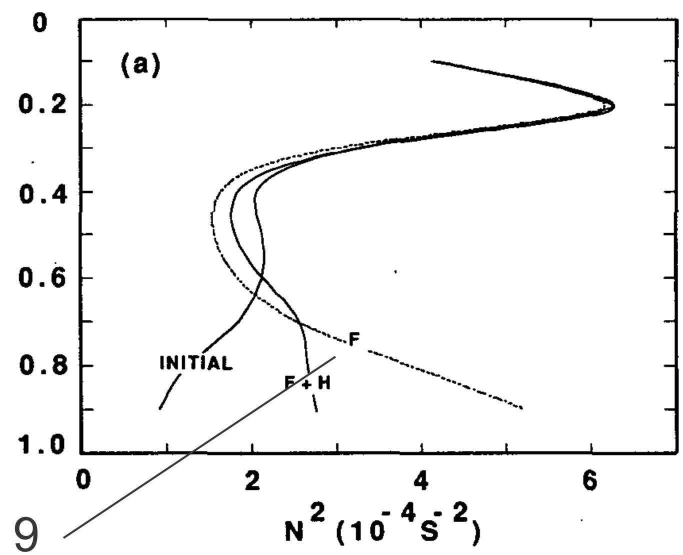
- Mean flow adjustment

Numerical results from

Gutowski et al, 1989, JAS,

where F and H indicate simulations with friction, diabatic heating, respectively





at day 9

Weaker vertical shear mean reduced temperature gradient.

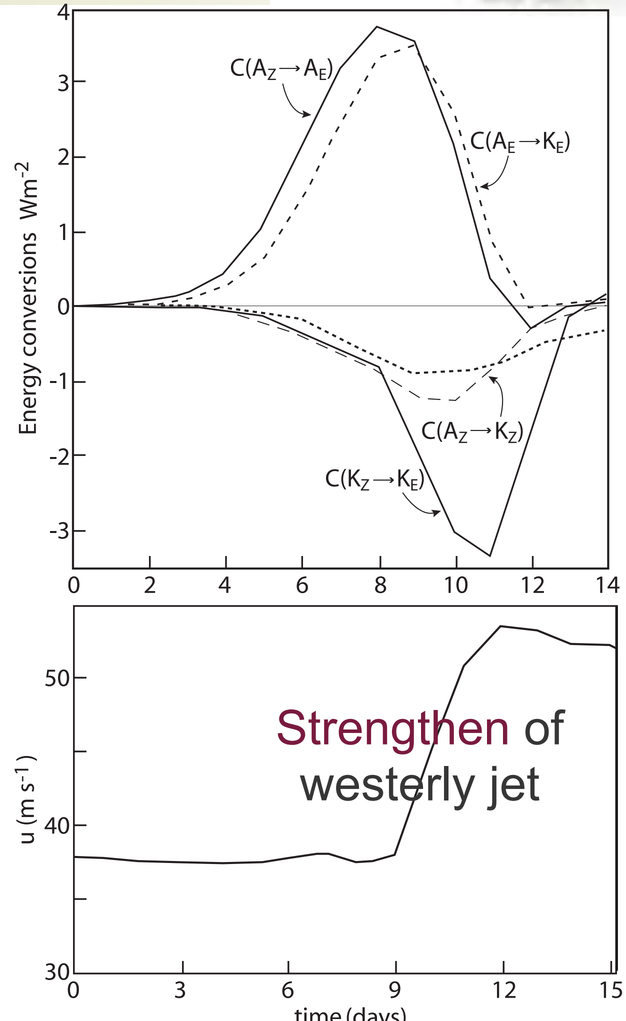
Much more stable stratification in the lower troposphere.

Baroclinic eddies

- baroclinic eddy life cycle

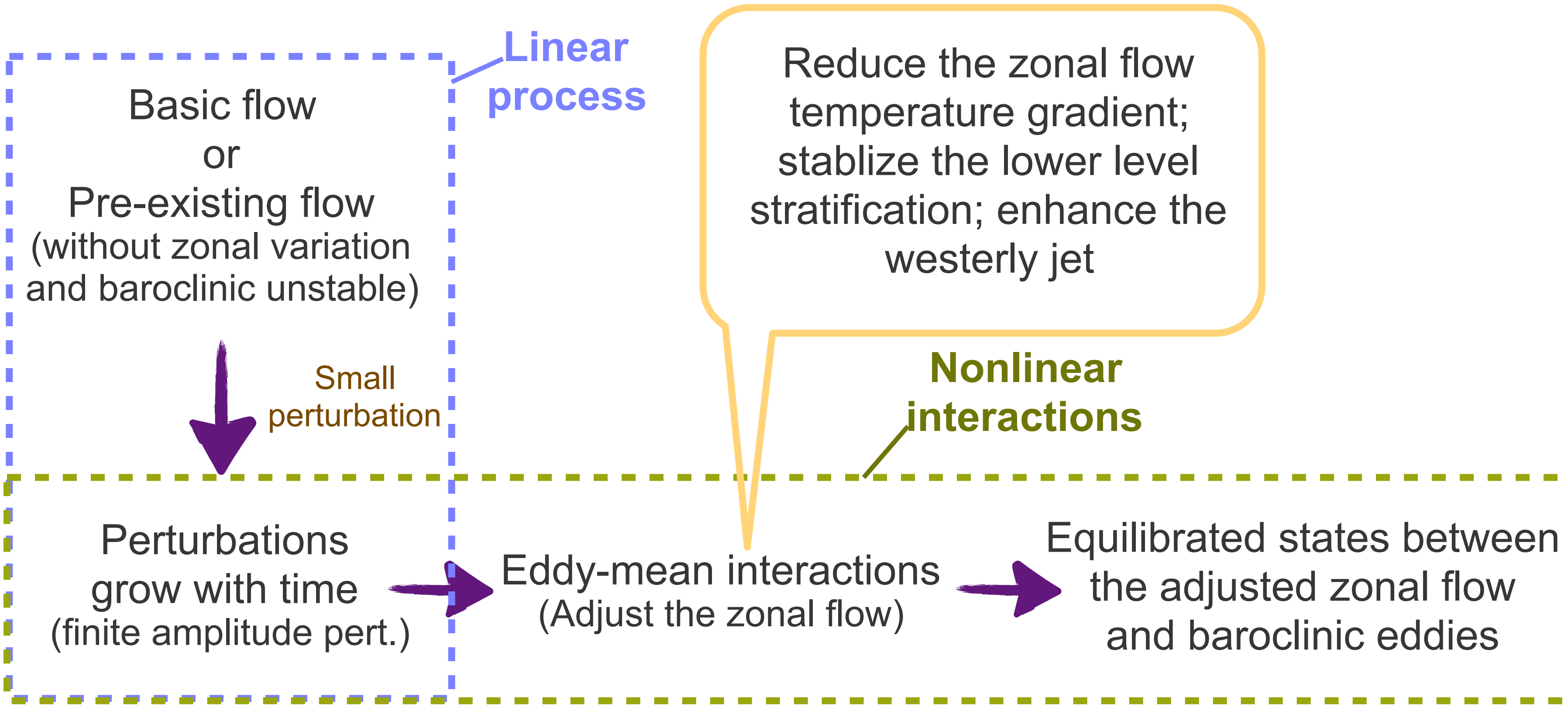
- Westerly jet and energy cycle:

Numerical results from
Simmons and Hoskins, 1978, JAS



Baroclinic eddies

- From linear to nonlinear

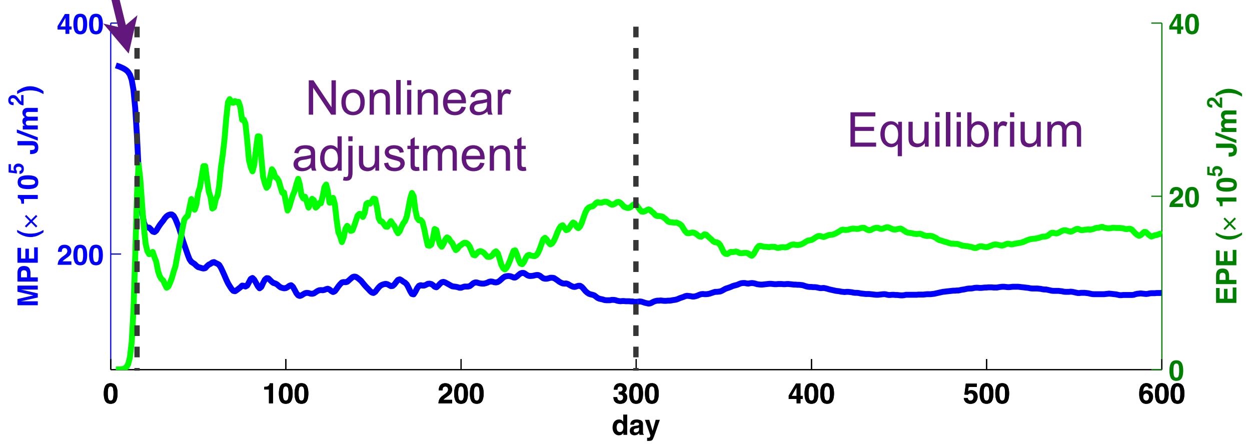


Baroclinic eddies

- From linear to nonlinear

Numerical results from a QG model

(Zhang, 2009)



Basic flow
or
Pre-existing flow
(without zonal variation and baroclinic unstable)

↓ Small perturbation

Perturbations grow with time
(finite amplitude pert.)

→ Eddy-mean interactions
(Adjust the zonal flow)

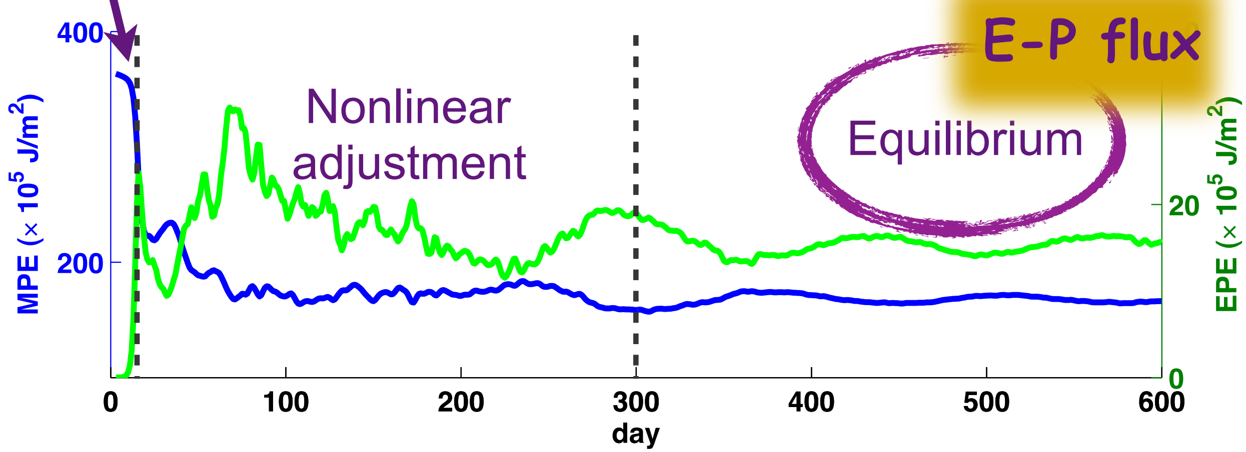
→ Equilibrated states between the adjusted zonal flow and baroclinic eddies

Baroclinic eddies

- From linear to nonlinear

spinup of eddies

Numerical results from a QG model
(Zhang, 2009)



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Perturbations grow with time
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→ Equilibrated states between the adjusted zonal flow and baroclinic eddies

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  - Eddy-mean flow interaction, E-P flux
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- The energy cycle

The Ferrel Cell

eddy-zonal flow interaction (I)

- Start from the equations:
  - Momentum equation: $$\left(\frac{du}{dt}\right)_p - fv = -\left(\frac{\partial \Phi}{\partial x}\right)_p + F_x$$
  - Continuity equation: $$\nabla_p \cdot \boldsymbol{v} + \frac{\partial \omega}{\partial p} = 0$$
  - Thermodynamic equation: $$\left(\frac{d\ln\theta}{dt}\right)_p = \frac{Q}{c_p T}$$

$$\left(\frac{d}{dt}\right)_p = \left(\frac{\partial}{\partial t}\right)_p + u\left(\frac{\partial}{\partial x}\right)_p + v\left(\frac{\partial}{\partial y}\right)_p + \omega\frac{\partial}{\partial p}$$

Decompose into zonal mean and eddy components:

$$A = [A] + A^*$$

The Ferrel Cell

eddy-zonal flow interaction (I)

- The simplified equations:
  - Momentum equation:

$$\frac{\partial [u]}{\partial t} = -\frac{\partial([u^* v^*])}{\partial y} + f[v] + [F_x]$$

  - Continuity equation:

$$\frac{\partial [v]}{\partial y} + \frac{\partial [\omega]}{\partial p} = 0$$

  - Thermodynamic equation:

$$\frac{\partial [\theta]}{\partial t} + [\omega]\frac{\partial \theta_s}{\partial p} = -\frac{\partial([\theta^* v^*])}{\partial y} + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q]}{c_p}$$

$$\left(\frac{d}{dt}\right)_p = \left(\frac{\partial}{\partial t}\right)_p + u\left(\frac{\partial}{\partial x}\right)_p + v\left(\frac{\partial}{\partial y}\right)_p + \omega\frac{\partial}{\partial p}$$

Under the quasi-geostrophic approximation $(R_o \ll 1)$

Baroclinic eddies

- E-P flux

- In a **steady**, **adiabatic** and **frictionless** flow:

- Momentum equation:

$$\frac{\partial [u]}{\partial t} = -\frac{\partial([u^* v^*])}{\partial y} + f[v] + [F_x]$$

- Continuity equation:

$$\frac{\partial [v]}{\partial y} + \frac{\partial [\omega]}{\partial p} = 0$$

- Thermodynamic equation:

$$\frac{\partial [\theta]}{\partial t} + [\omega]\frac{\partial \theta_s}{\partial p} = -\frac{\partial([\theta^* v^*])}{\partial y} + \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p}$$

Baroclinic eddies

- E-P flux

- In a QG, **steady**, **adiabatic** and **frictionless** flow:

- Momentum equation:

$$f[v] - \frac{\partial([u^*v^*])}{\partial y} = 0$$

- Continuity equation:

$$\frac{\partial[v]}{\partial y} + \frac{\partial[\omega]}{\partial p} = 0 \Rightarrow \nabla \cdot \mathcal{F} = 0$$

- Thermodynamic equation:

$$[\omega]\frac{\partial \theta_s}{\partial p} + \frac{\partial([\theta^*v^*])}{\partial y} = 0$$

$$[v] = \frac{1}{f}\frac{\partial}{\partial y}([u^*v^*])$$

$$[\omega] = -\frac{\partial}{\partial y}\left(\frac{[\theta^*v^*]}{\partial \theta_s/\partial p}\right)$$

Define *Eliassen-Palm flux:*

$$\mathcal{F} \equiv -[u^*v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial \theta_s/\partial p}\,\mathbf{k}$$

Baroclinic eddies

- E-P flux

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

- In a QG, **steady**, **adiabatic** and **frictionless** flow:

$$[v] = \frac{1}{f}\frac{\partial}{\partial y}([u^* v^*]) \qquad [\omega] = -\frac{\partial}{\partial y}\left(\frac{[\theta^* v^*]}{\partial\theta_s/\partial p}\right) \qquad \nabla\cdot\mathcal{F} = 0$$

- In a QG, **steady** flow:

$$f[v] - \frac{\partial([u^* v^*])}{\partial y} + [F_x] = 0$$

$$[\omega]\frac{\partial\theta_s}{\partial p} + \frac{\partial([\theta^* v^*])}{\partial y} - \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p} = 0$$

The **meridional overturning flow**, in addition to the **eddy forcing**, has to balance the **external forcing**.