第四章:

中纬度的经向环流系统(IV)

- *Ferrel cell, baroclinic eddies and the westerly jet*

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2022. 12. 01

E-P flux, TEM and Residual Circulation
- Summary

Review

- E-P flux: $$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$
- In a **steady**, **adiabatic** and **frictionless** flow:

$$[v] = \frac{1}{f}\frac{\partial}{\partial y}([u^* v^*]) \qquad [\omega] = -\frac{\partial}{\partial y}\left(\frac{[\theta^* v^*]}{\partial\theta_s/\partial p}\right) \qquad \nabla\cdot\mathcal{F} = 0$$

- Residual mean circulations:

$$[\tilde{\omega}] = [\omega] + \frac{\partial}{\partial y}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right), \qquad [\tilde{v}] = [v] - \frac{\partial}{\partial p}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right)$$

- TEM equations: $$\frac{\partial[u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x]\,, \qquad \frac{\partial[\theta]}{\partial t} = -[\tilde{\omega}]\frac{\partial\theta_s}{\partial p} + \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p}$$

E-P flux, TEM and Residual Circulation - Summary

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#1

$$\begin{aligned}[v^*q^*] &= - \frac{\partial}{\partial y}[u^*v^*] + f_o\frac{\partial}{\partial p}\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\\ &= \nabla\cdot\mathcal{F}\end{aligned}$$

#2

$$\frac{\partial\mathcal{A}}{\partial t} + \nabla\cdot\mathcal{F} = 0$$

$$\downarrow$$

#3

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A} \longrightarrow \frac{\partial\mathcal{A}}{\partial t} + \nabla\cdot(\mathcal{A}\vec{\mathbf{c_g}}) = 0$$

- Residual mean circulations:

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Outline

- Observations
- The Ferrel Cell
- Baroclinic eddies
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction, E-P flux
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- The energy cycle

Baroclinic eddy life cycle
- An E-P flux view

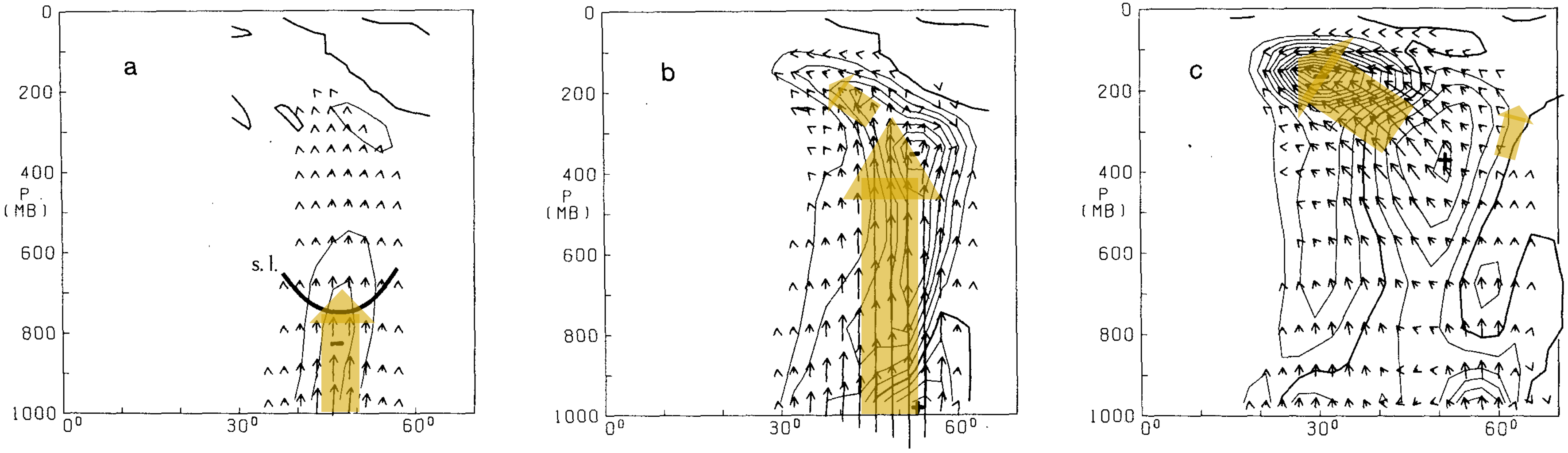
Review

Numerica *om*
Simmons and Hoskins,
1978, JAS

$$\mathcal{F} \equiv -[u^*v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

Eddies: generate at lower level, propagate **upwards** and **away** from the eddy source region



TOTAL E-P FLUX DIVERGENCE
DAY .00

TOTAL E-P FLUX DIVERGENCE
DAY 5.00

TOTAL E-P FLUX DIVERGENCE
DAY 8.00

E-P flux

- The westerly jet

Review

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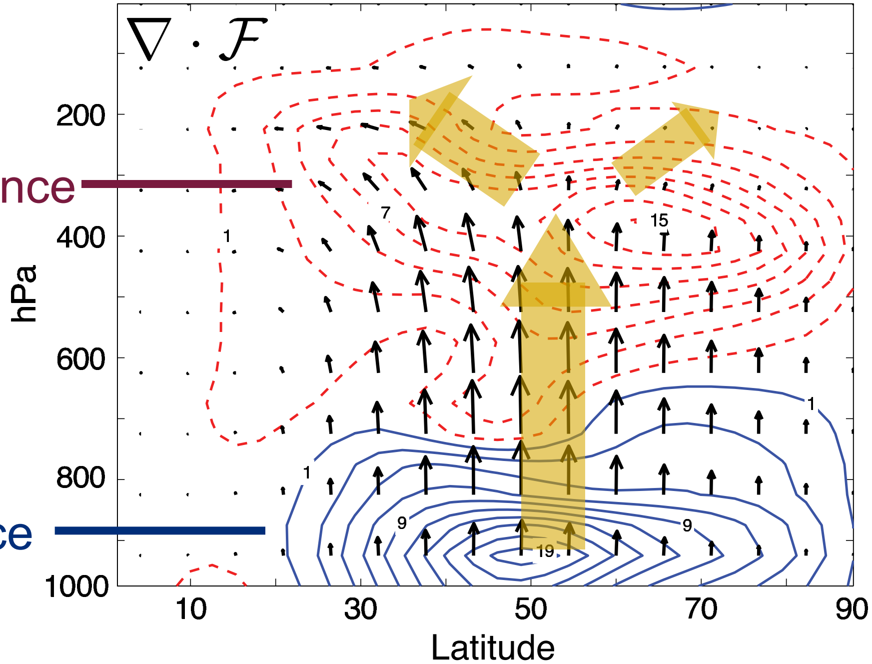
$$\frac{\partial[u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x]$$

Wave energies: propagate **upwards** and **away** from the center of the jet



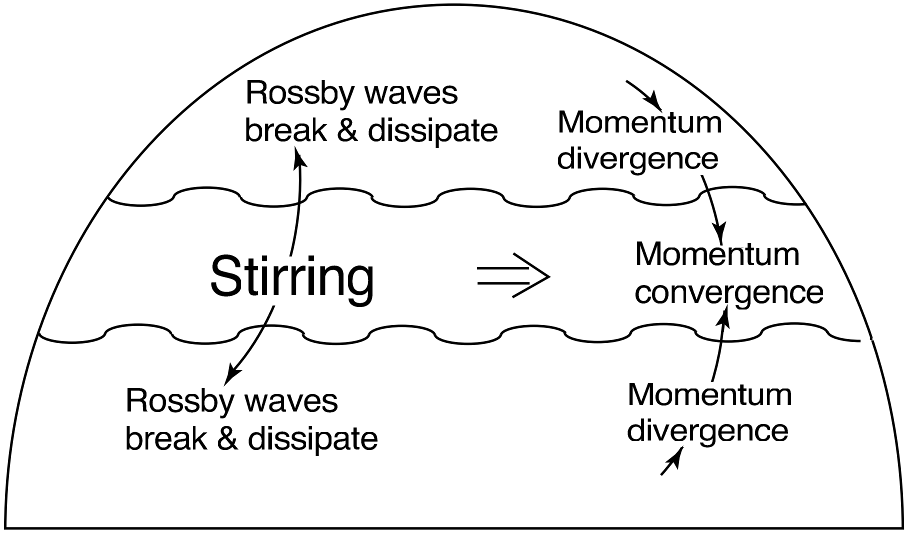
Convergence

Divergence

In the vertical direction:

Accelerating the lower jet
decelerating the upper jet
reduce the vertical shear of U

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$



$$\frac{\partial}{\partial t} < [u] >= -\frac{\partial}{\partial y} < [u^* v^*] > -r[u_{\mathrm{surf}}]$$

< > means vertical average

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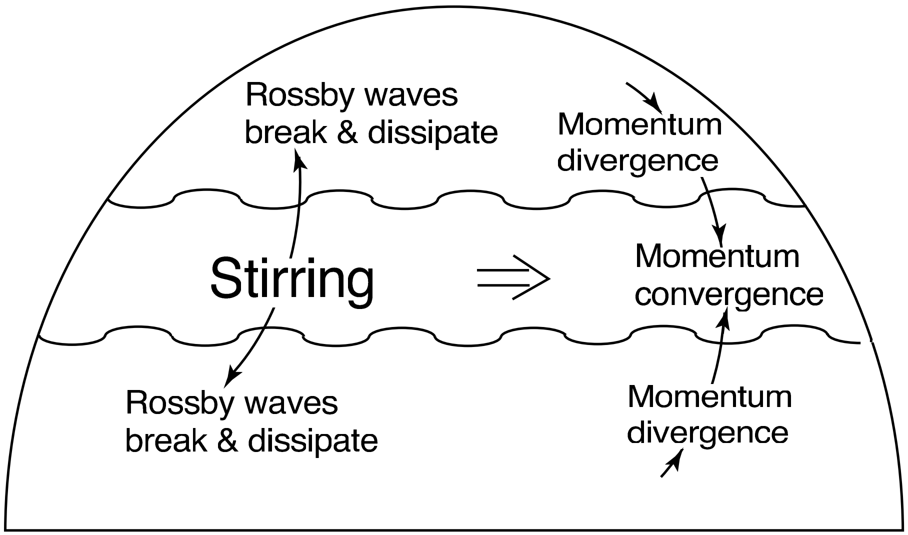

Rossby waves break & dissipate

Momentum divergence

Stirring $\Longrightarrow$

Momentum convergence

Rossby waves break & dissipate

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In equilibrium: $$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

$$r[u_{\text{surf}}] \sim -\frac{\partial}{\partial y} < [u^*v^*] >$$

Review

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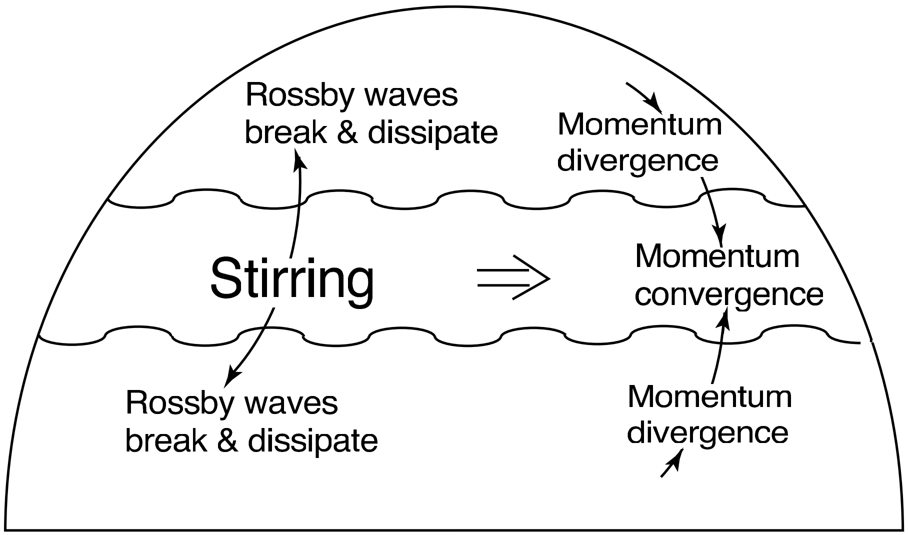

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There MUST be **surface westerlies** at midlatitudes.

Eddy-driven jet:
- the momentum budget

Review

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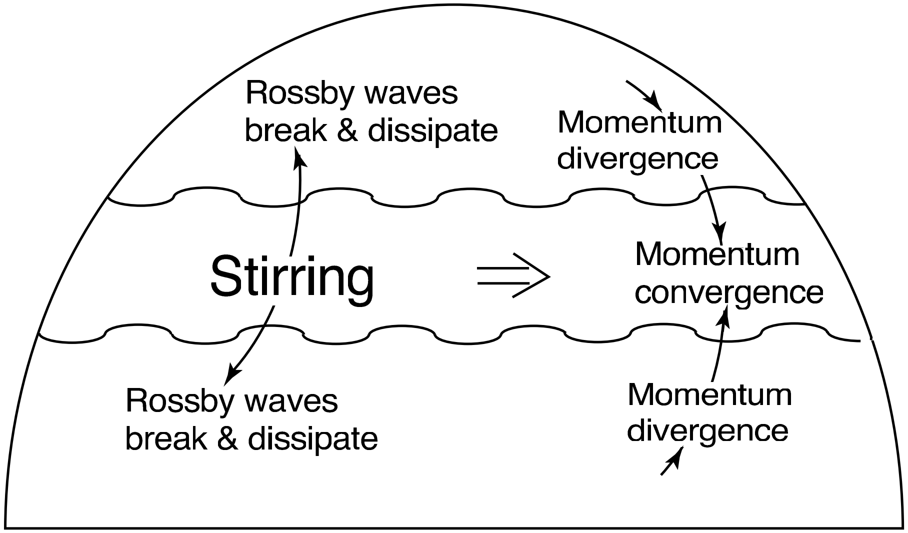

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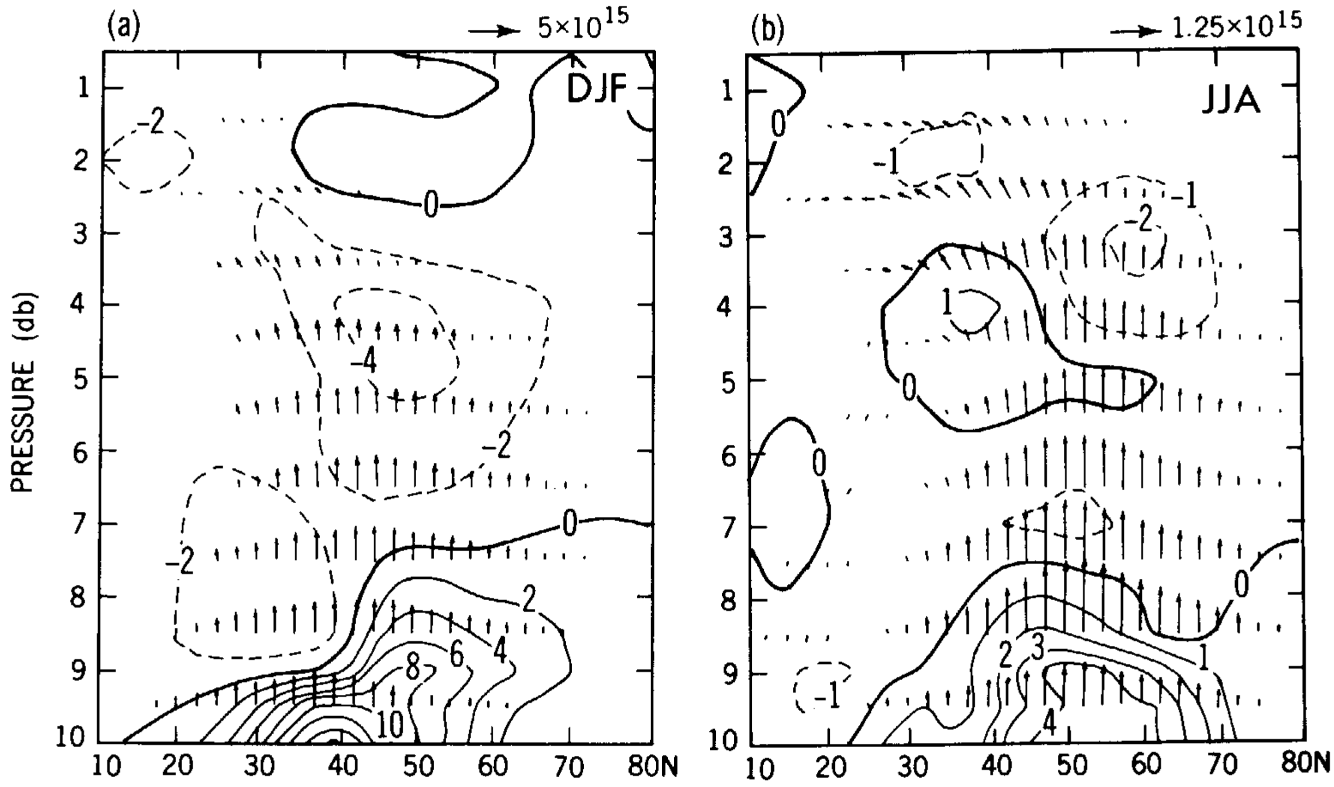
E–P FLUX TRANSIENT EDDIES



E-P flux

- in the real atmosphere

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

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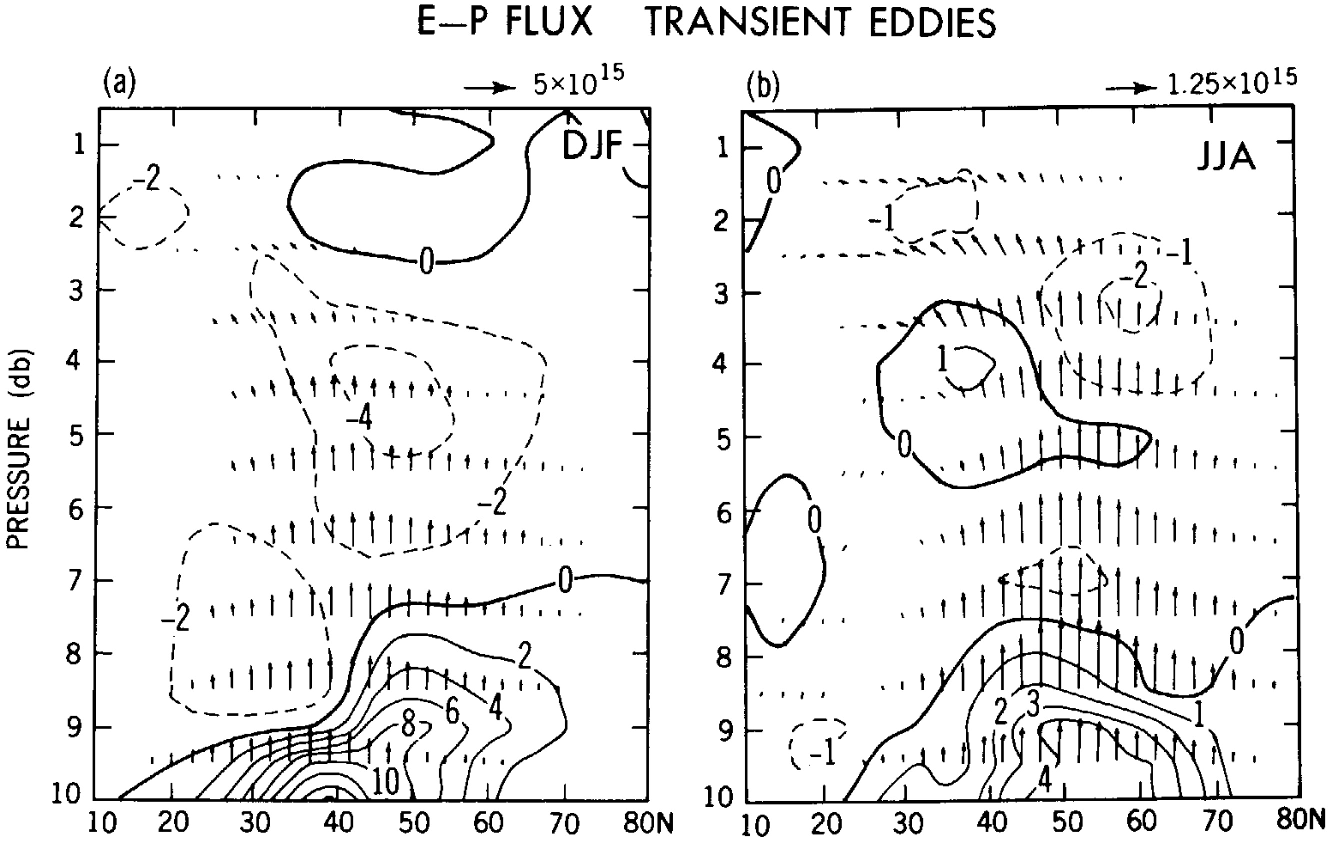

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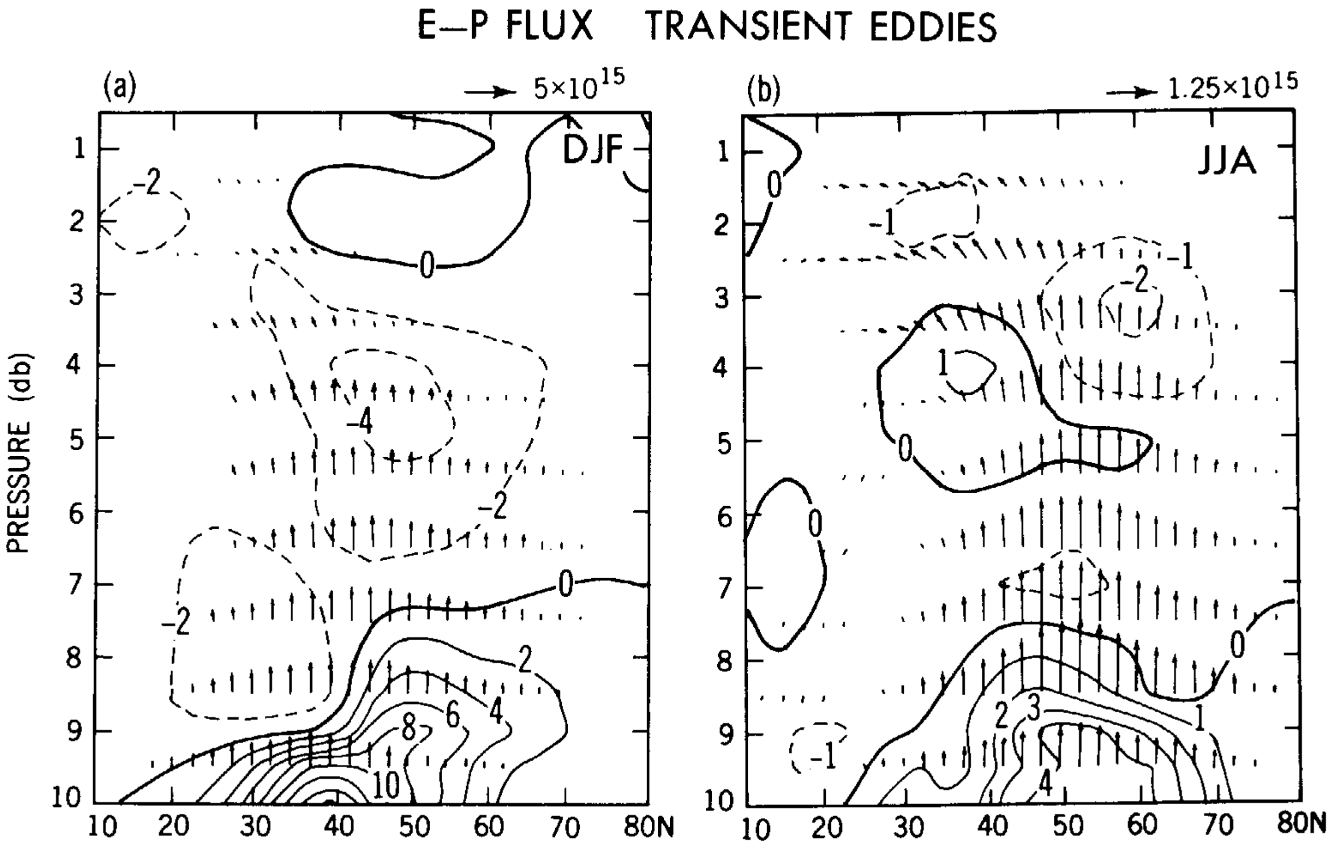
Vertical component is dominant.



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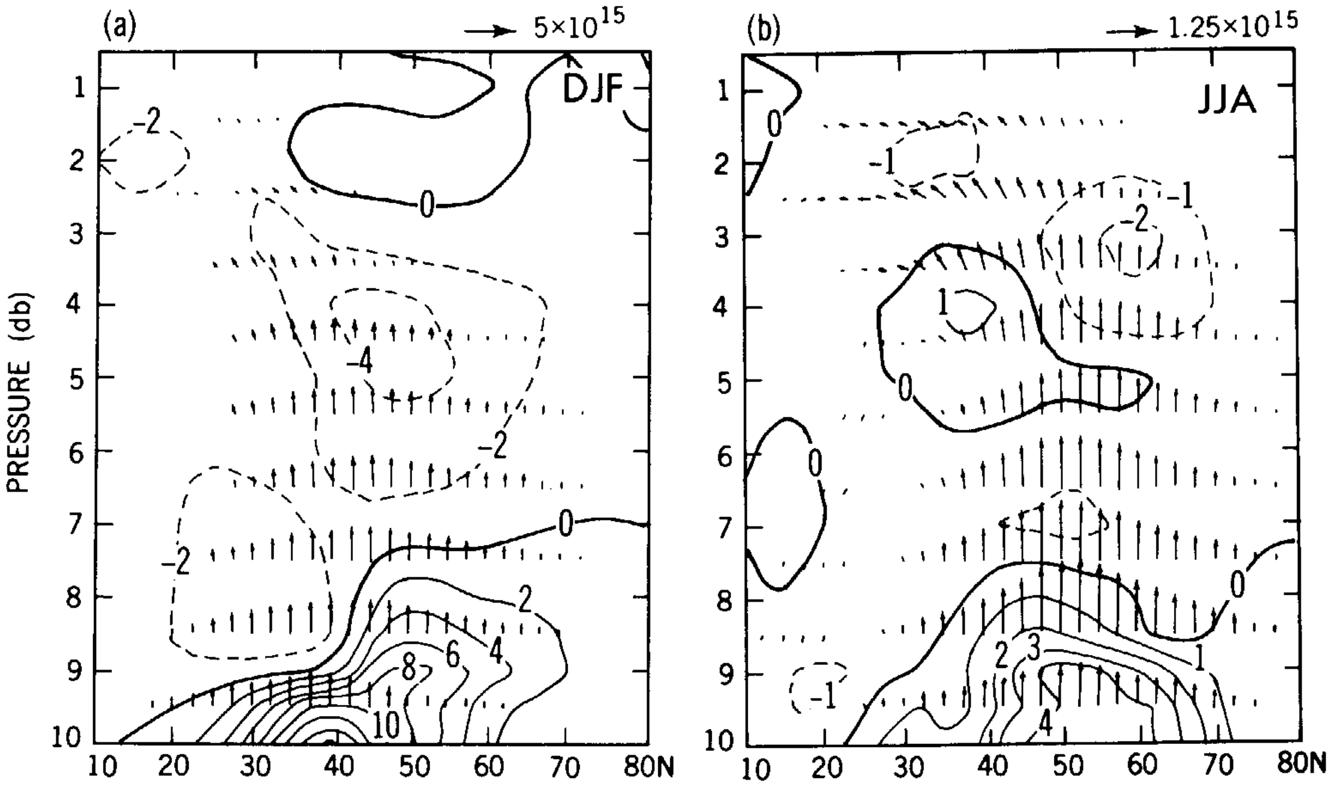
Vertical component is dominant.

EP divergence in the lower layers; convergence in the upper layers.

E—P FLUX TRANSIENT EDDIES



E-P flux and the eddy-driven jet

-summary

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- Momentum budget indicates that there MUST be surface westerlies in the eddy source latitude.

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Energy cycles

in the baroclinic eddy-mean flow interactions

- Basic forms of energy:

Energy cycles

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  - Kinetic energy (动能): $K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2)$

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- Total energy:

$$E = I + \Phi + \mathrm{LH} + K$$

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  - Gravitational-potential energy (位能): $\Phi=gz$
  - Total potential energy:

$$\int_0^\infty \rho(I+\Phi)\mathrm{d}z=\frac{1}{g}\int_0^{p_s}(c_vT+RT)\mathrm{d}p=\frac{1}{g}\int_0^{p_s}c_pT\mathrm{d}p$$

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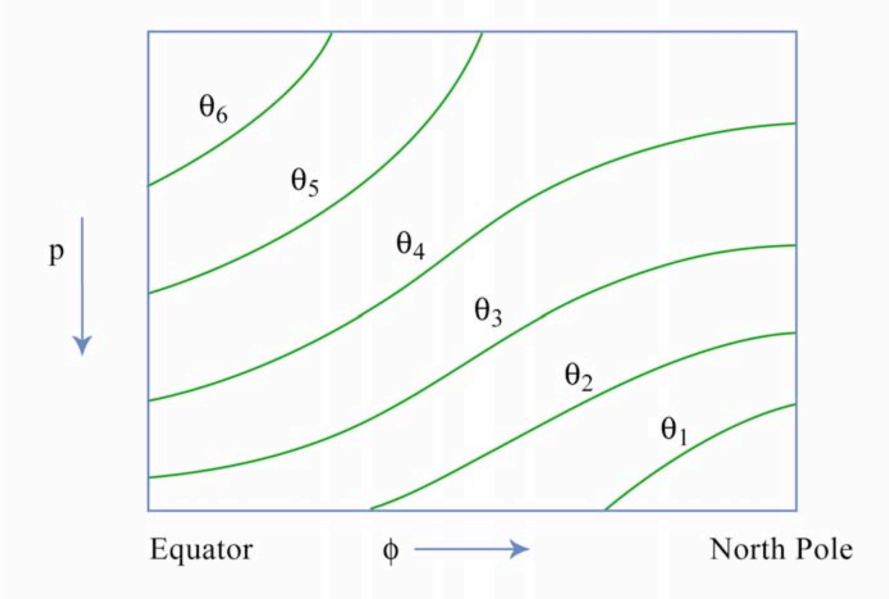

From Stone's class notes

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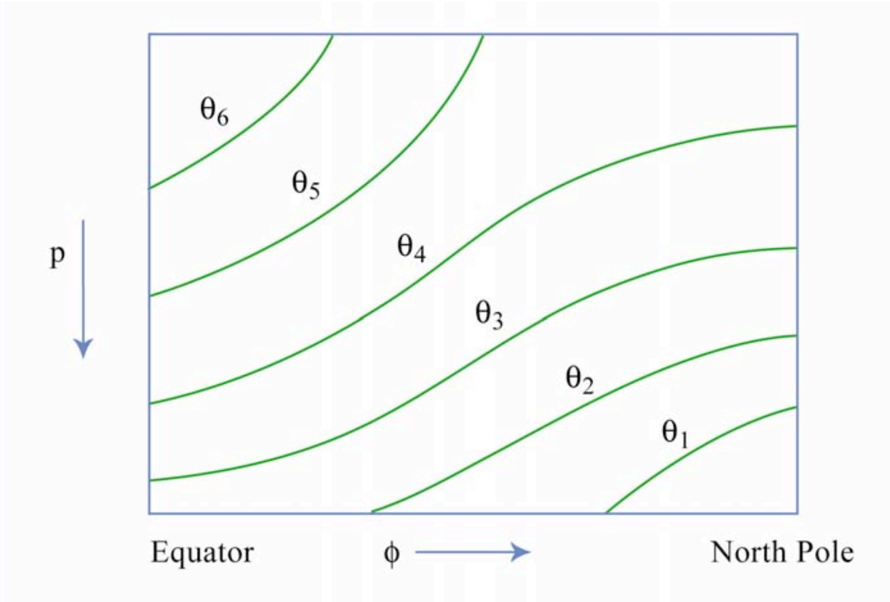

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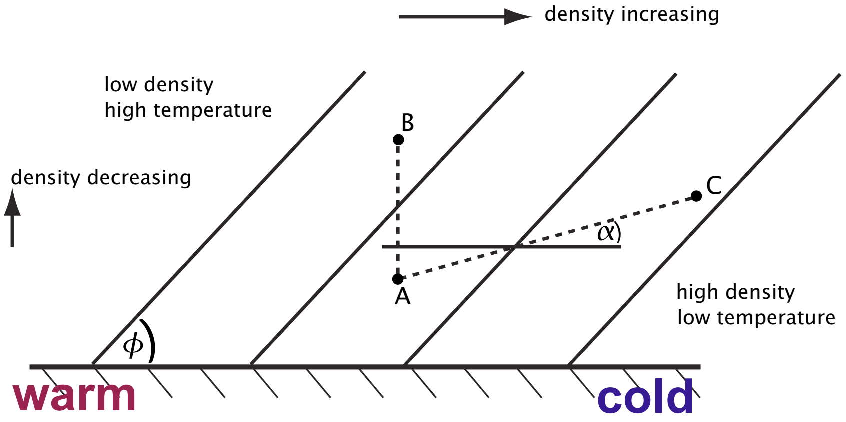

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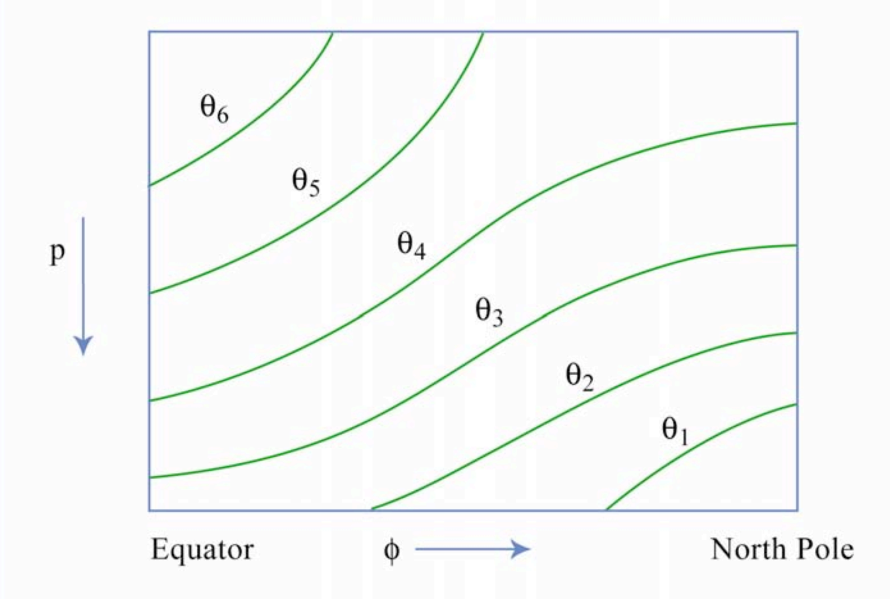

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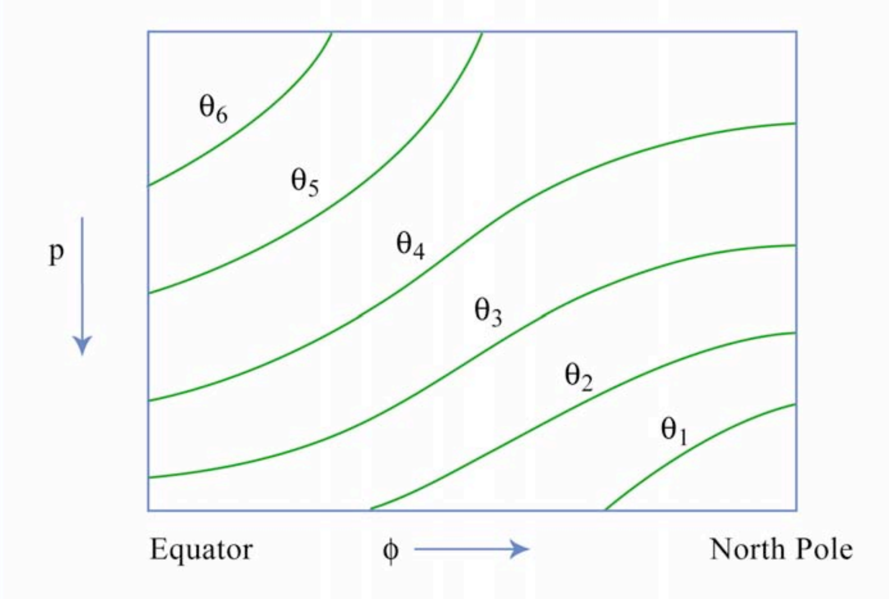

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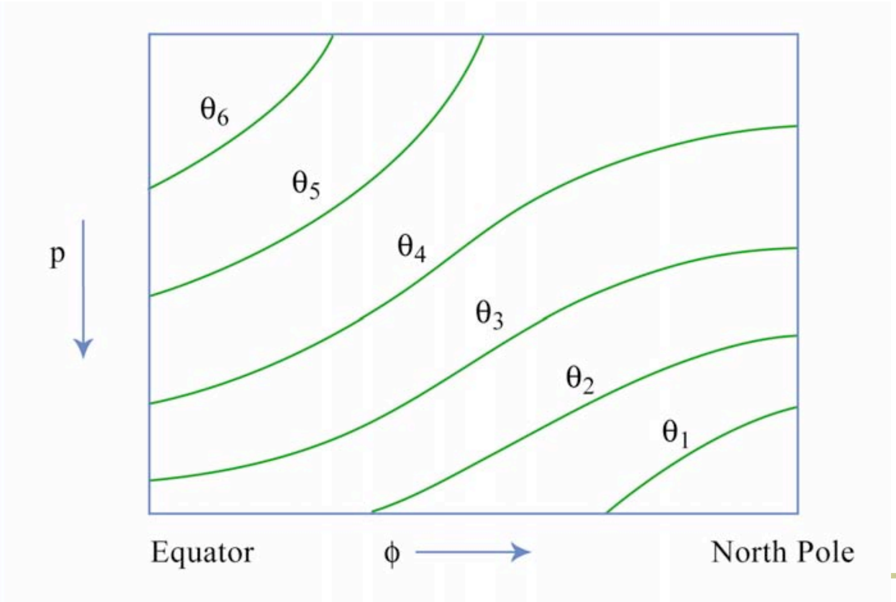

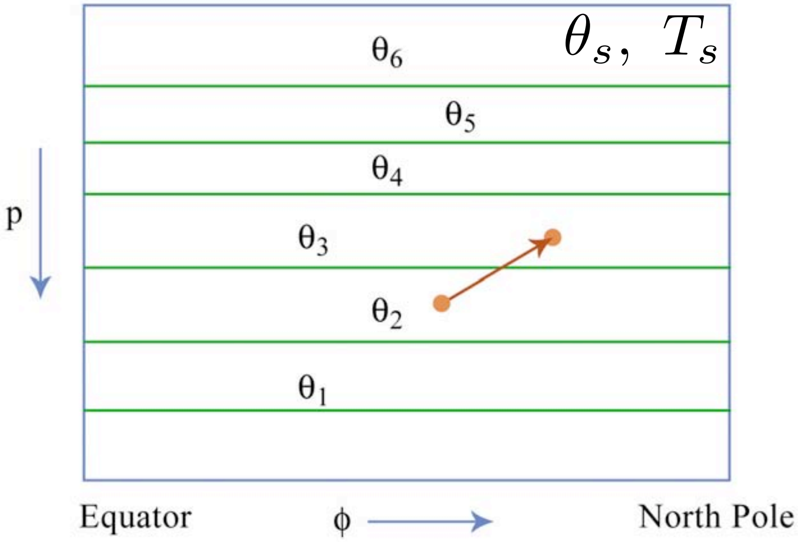

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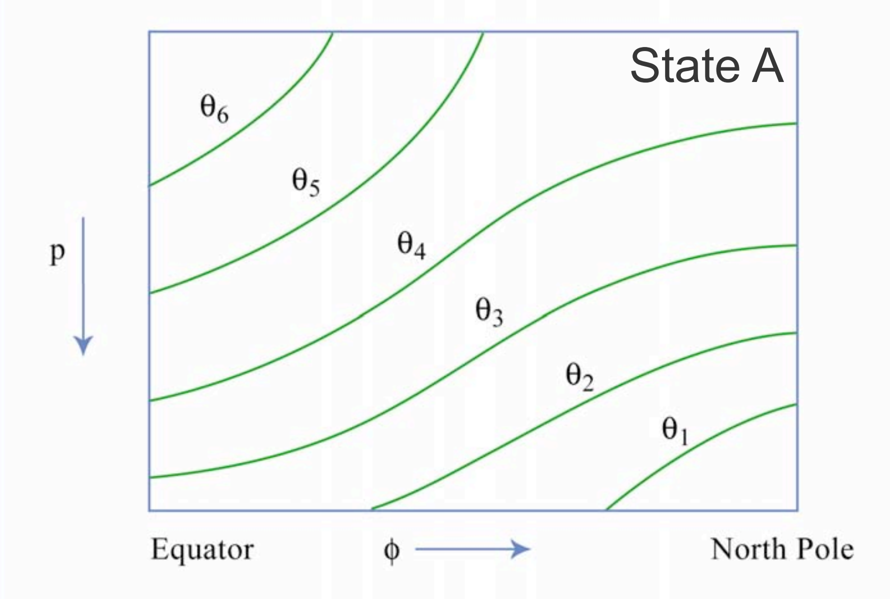

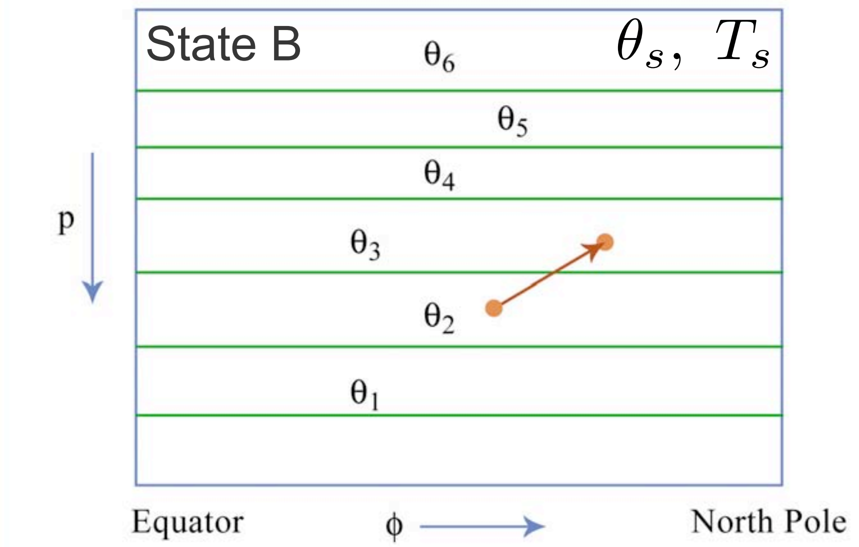

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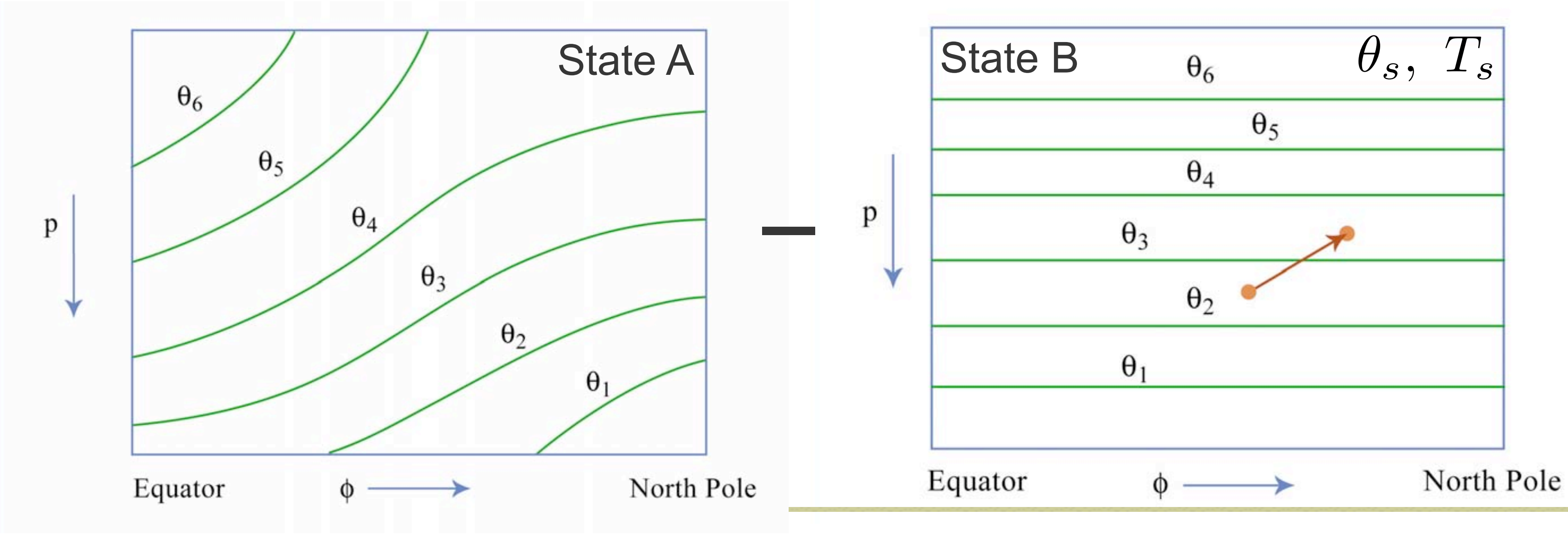

= Available potential energy

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State A − State B **= Available potential energy**

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State A **−** State B **= Available potential energy**

  - Available potential energy (有效位能):

$$P = \frac{1}{2}\int_0^{p_s}\frac{T_s}{\gamma_d-\gamma_s}\left(\frac{T-T_s}{T_s}\right)^2\mathrm{d}p = \frac{c_p}{2g}\int_0^{p_s}\Gamma\left(T-T_s\right)^2\mathrm{d}p$$

$$\begin{aligned}\Gamma &= -\frac{R}{c_p p}\left(\frac{p_s}{p}\right)^{\frac{R}{c_p}}\left(\frac{\partial\theta_s}{\partial p}\right)^{-1}\\ &= \left(\gamma_d/T_s\right)\left(\gamma_d-\gamma_s\right)^{-1}\end{aligned}$$

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  - Kinetic energy (动能):

$$K=\frac{1}{2}(u^2+v^2+w^2)\approx\frac{1}{2}(u^2+v^2)$$

$$\int_0^\infty \rho(I+\Phi)\mathrm{d}z=\frac{1}{g}\int_0^{p_s}(c_vT+RT)\mathrm{d}p=\frac{1}{g}\int_0^{p_s}c_pT\mathrm{d}p$$

State A — State B **= Available potential energy**

  - Available potential energy (有效位能):

$$P=\frac{1}{2}\int_0^{p_s}\frac{T_s}{\gamma_d-\gamma_s}\left(\frac{T-T_s}{T_s}\right)^2\mathrm{d}p=\frac{c_p}{2g}\int_0^{p_s}\Gamma\left(T-T_s\right)^2\mathrm{d}p$$

$$\begin{aligned}\Gamma &= -\frac{R}{c_pp}\left(\frac{p_s}{p}\right)^{\frac{R}{c_p}}\left(\frac{\partial\theta_s}{\partial p}\right)^{-1}\\ &= (\gamma_d/T_s)\left(\gamma_d-\gamma_s\right)^{-1}\end{aligned}$$

From the “approximate” expression
of Lorenz (1955)

Energy cycles

in the baroclinic eddy-mean flow interactions

- Basic forms of energy:
  - Kinetic energy (动能): $K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2)$
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Q - diabatic heating

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- Zonal mean and eddy components:
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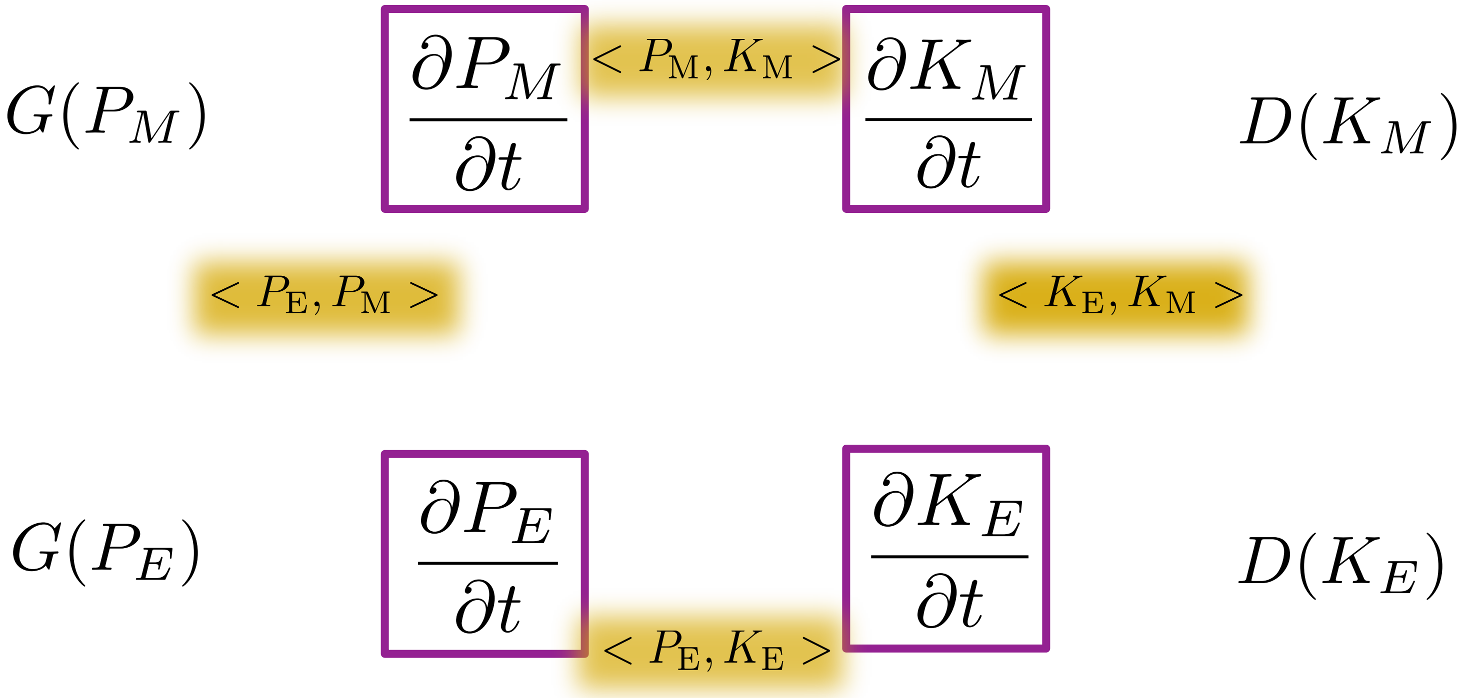
Energy cycles
in the baroclinic eddy-mean flow interactions



$G(P_M)$ $\boxed{\dfrac{\partial P_M}{\partial t}}$ $< P_\mathrm{M}, K_\mathrm{M} >$ $\boxed{\dfrac{\partial K_M}{\partial t}}$ $D(K_M)$

$< P_\mathrm{E}, P_\mathrm{M} >$ $< K_\mathrm{E}, K_\mathrm{M} >$

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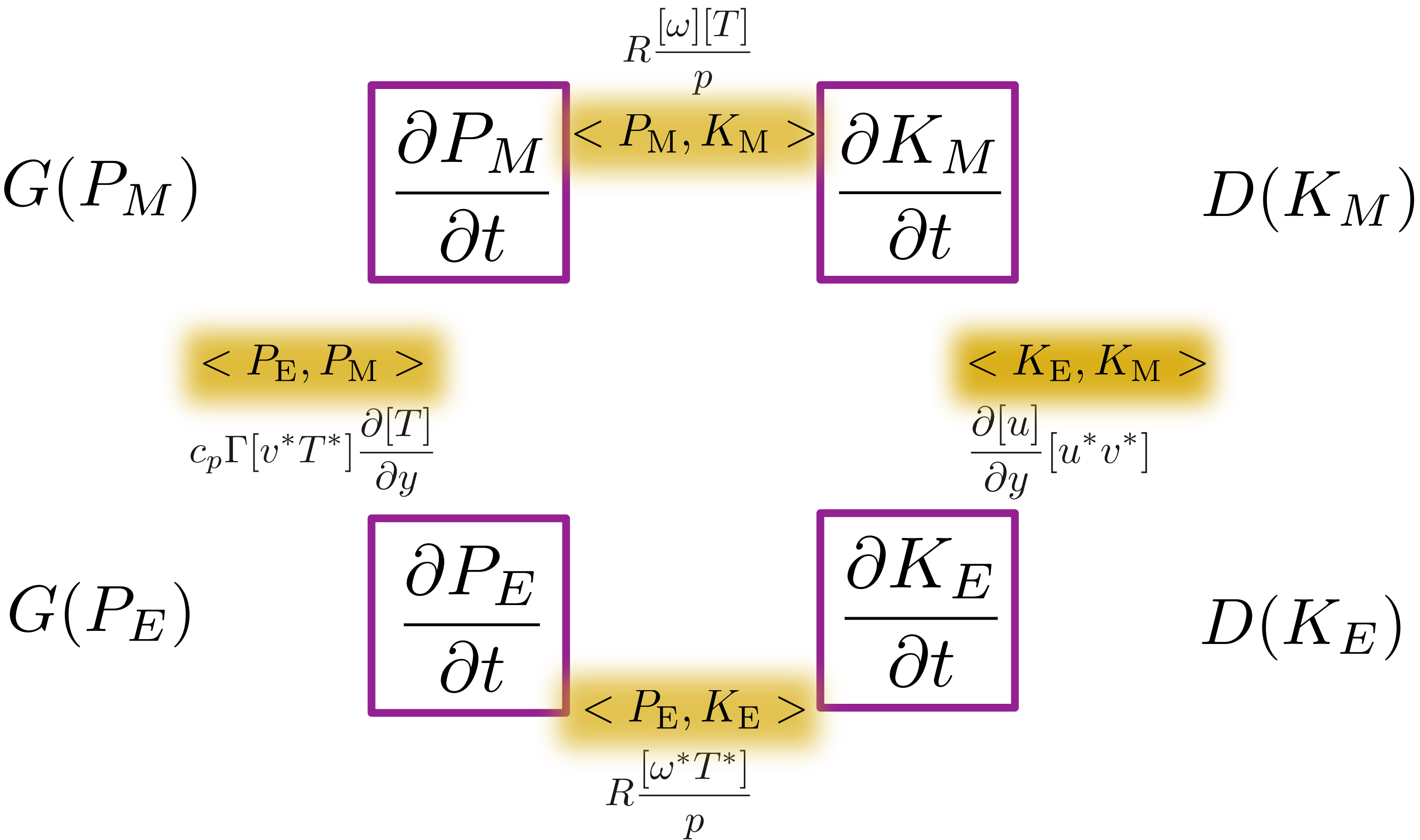

$$R\frac{[\omega][T]}{p}$$

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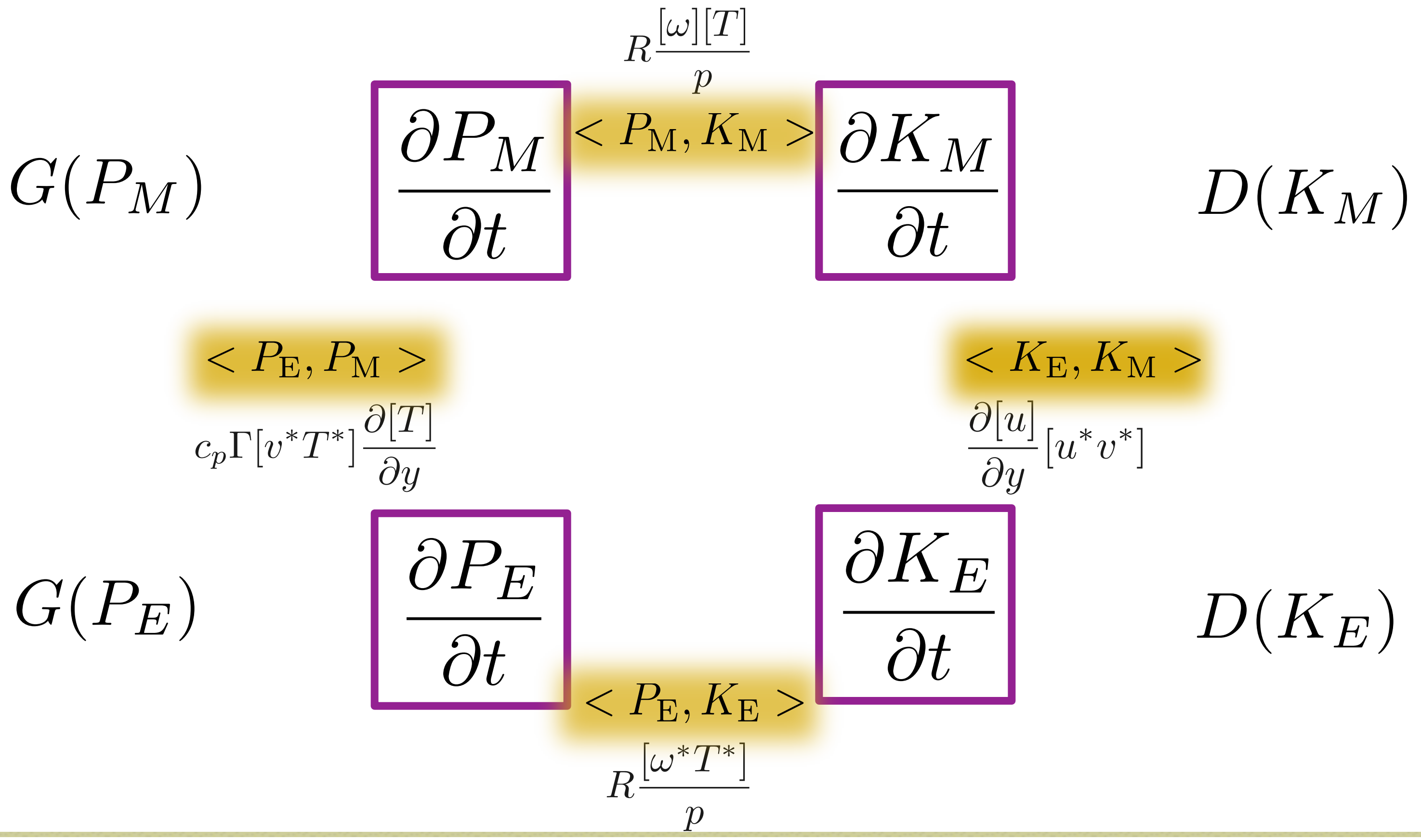
PE → KE



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Energy cycles

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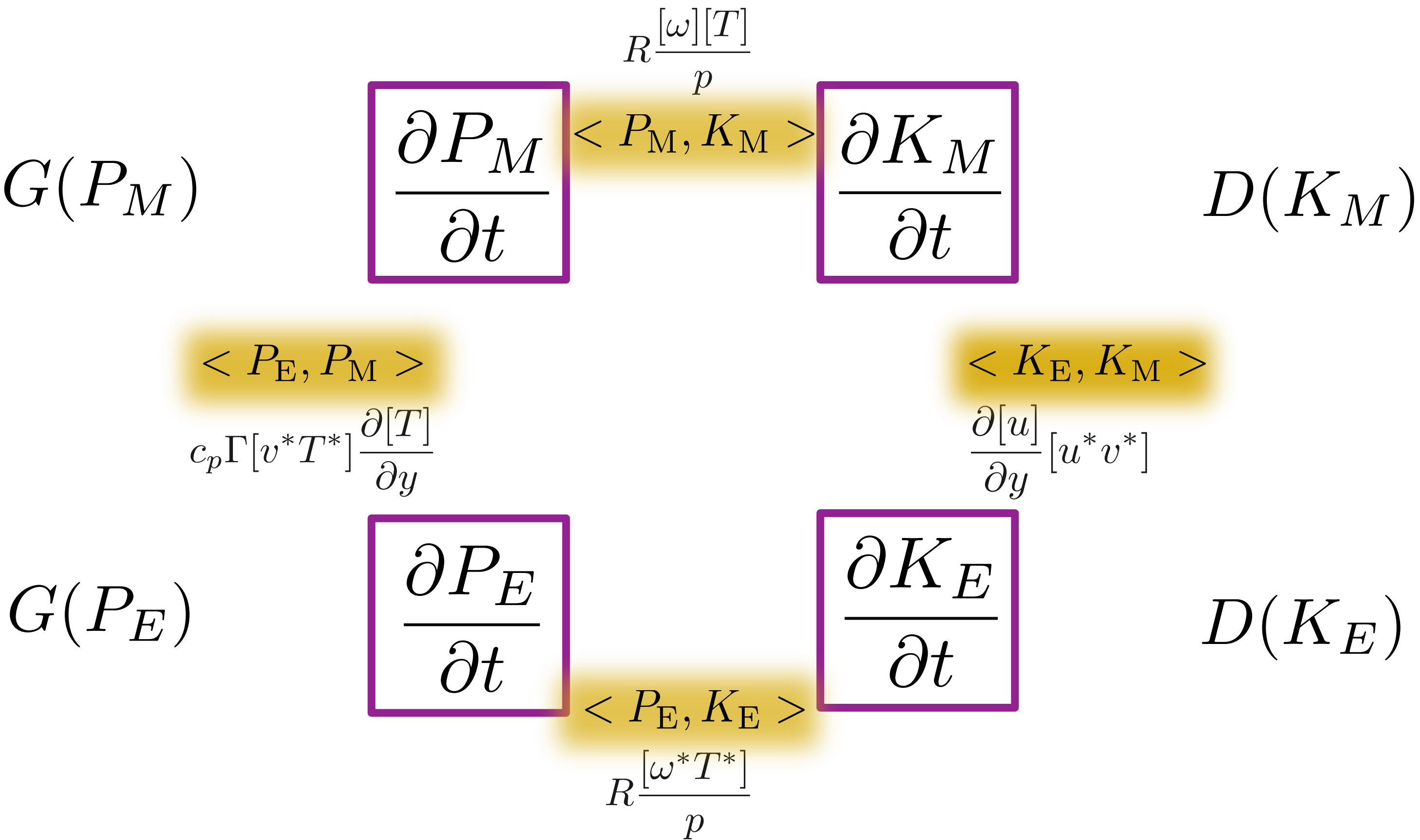
- Energy cycles in eddy life cycle: PE → KE



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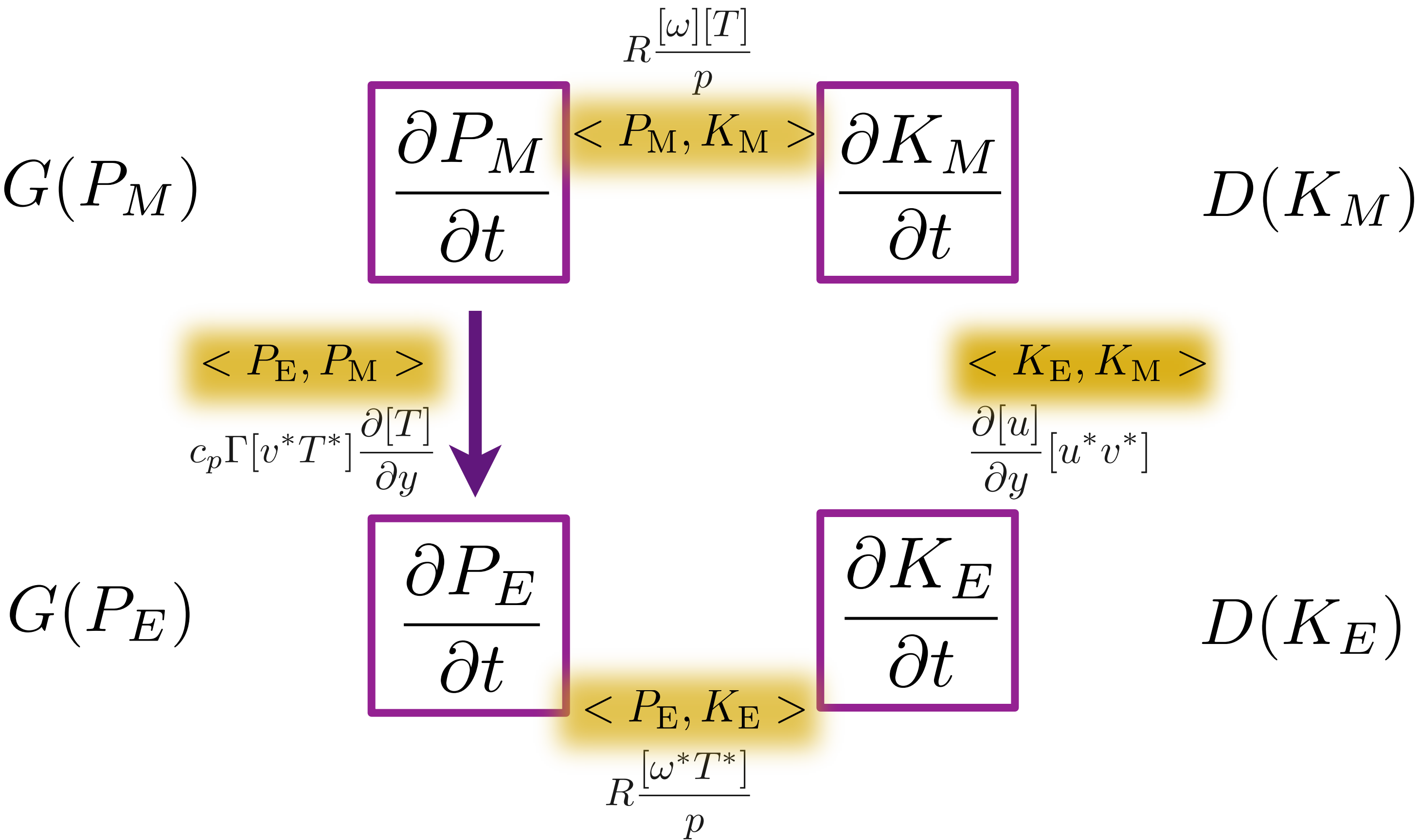

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$$< P_{\rm E}, P_{\rm M} > \qquad < K_{\rm E}, K_{\rm M} >$$

$$c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y} \qquad \frac{\partial[u]}{\partial y}[u^*v^*]$$

$$G(P_E) \quad \boxed{\frac{\partial P_E}{\partial t}} \quad < P_{\rm E}, K_{\rm E} > \quad \boxed{\frac{\partial K_E}{\partial t}} \quad D(K_E)$$

$$R\frac{[\omega^*T^*]}{p}$$

Energy cycles

in the baroclinic eddy-mean flow interactions

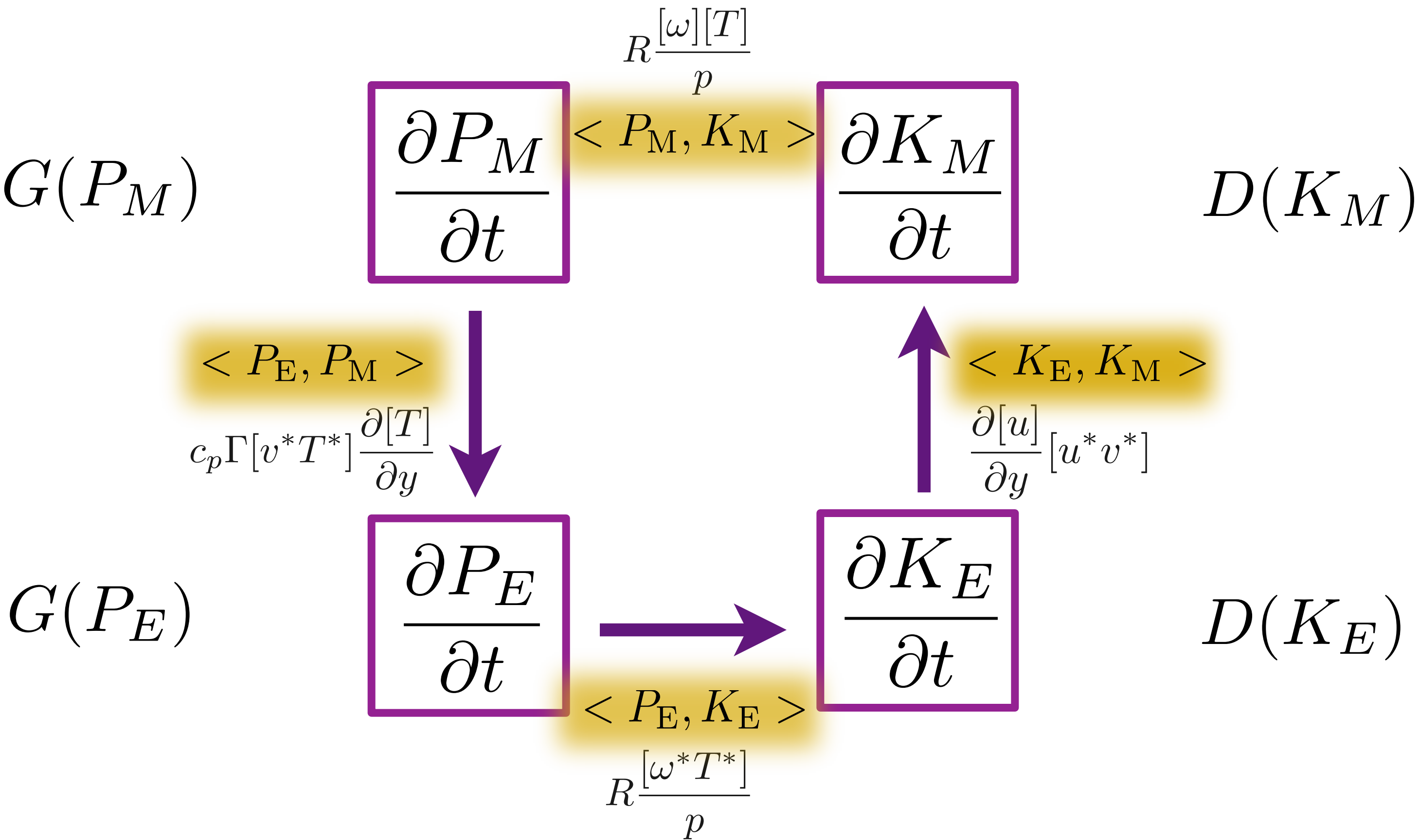
- Energy cycles in eddy life cycle: PE → KE

$$\begin{tikzcd}
G(P_M) & \boxed{\frac{\partial P_M}{\partial t}} \arrow[r, phantom, "{R\frac{[\omega][T]}{p}}" above, "{< P_{\mathrm{M}}, K_{\mathrm{M}} >}" below] \arrow[d, "{< P_{\mathrm{E}}, P_{\mathrm{M}} >}" swap, "{c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}}" swap] & \boxed{\frac{\partial K_M}{\partial t}} \arrow[d, phantom, "{< K_{\mathrm{E}}, K_{\mathrm{M}} >}", "{\frac{\partial[u]}{\partial y}[u^*v^*]}"] & D(K_M) \\
G(P_E) & \boxed{\frac{\partial P_E}{\partial t}} \arrow[r, "{< P_{\mathrm{E}}, K_{\mathrm{E}} >}" swap, "{R\frac{[\omega^*T^*]}{p}}" swap] & \boxed{\frac{\partial K_E}{\partial t}} & D(K_E)
\end{tikzcd}$$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in eddy life cycle: PE → KE



$$\begin{tikzcd}
G(P_M) & \frac{\partial P_M}{\partial t} \arrow[r, phantom, "{< P_{\mathrm{M}}, K_{\mathrm{M}} >}", "{R\frac{[\omega][T]}{p}}"'] \arrow[d, "{< P_{\mathrm{E}}, P_{\mathrm{M}} >}"', "{c_p\Gamma[v^*T^*]\frac{\partial [T]}{\partial y}}"] & \frac{\partial K_M}{\partial t} & D(K_M) \\
G(P_E) & \frac{\partial P_E}{\partial t} \arrow[r, "{< P_{\mathrm{E}}, K_{\mathrm{E}} >}"', "{R\frac{[\omega^*T^*]}{p}}"'] & \frac{\partial K_E}{\partial t} \arrow[u, "{< K_{\mathrm{E}}, K_{\mathrm{M}} >}"', "{\frac{\partial [u]}{\partial y}[u^*v^*]}"'] & D(K_E)
\end{tikzcd}$$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in eddy life cycle: $\boxed{\mathrm{PE}} \rightarrow \boxed{\mathrm{KE}}$

$$\begin{tikzcd}[column sep=large, row sep=large]
G(P_M) & \boxed{\dfrac{\partial P_M}{\partial t}} \arrow[r, phantom, "{R\frac{[\omega][T]}{p}}", "{<P_{\mathrm{M}}, K_{\mathrm{M}}>}"'] \arrow[d, "{<P_{\mathrm{E}}, P_{\mathrm{M}}>}"', "{c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}}"] & \boxed{\dfrac{\partial K_M}{\partial t}} \arrow[r] & D(K_M) \\
G(P_E) & \boxed{\dfrac{\partial P_E}{\partial t}} \arrow[r, "{<P_{\mathrm{E}}, K_{\mathrm{E}}>}"', "{R\frac{[\omega^*T^*]}{p}}"'] & \boxed{\dfrac{\partial K_E}{\partial t}} \arrow[u, "{<K_{\mathrm{E}}, K_{\mathrm{M}}>}", "{\frac{\partial[u]}{\partial y}[u^*v^*]}"] \arrow[r] & D(K_E)
\end{tikzcd}$$

Baroclinic eddy life cycle

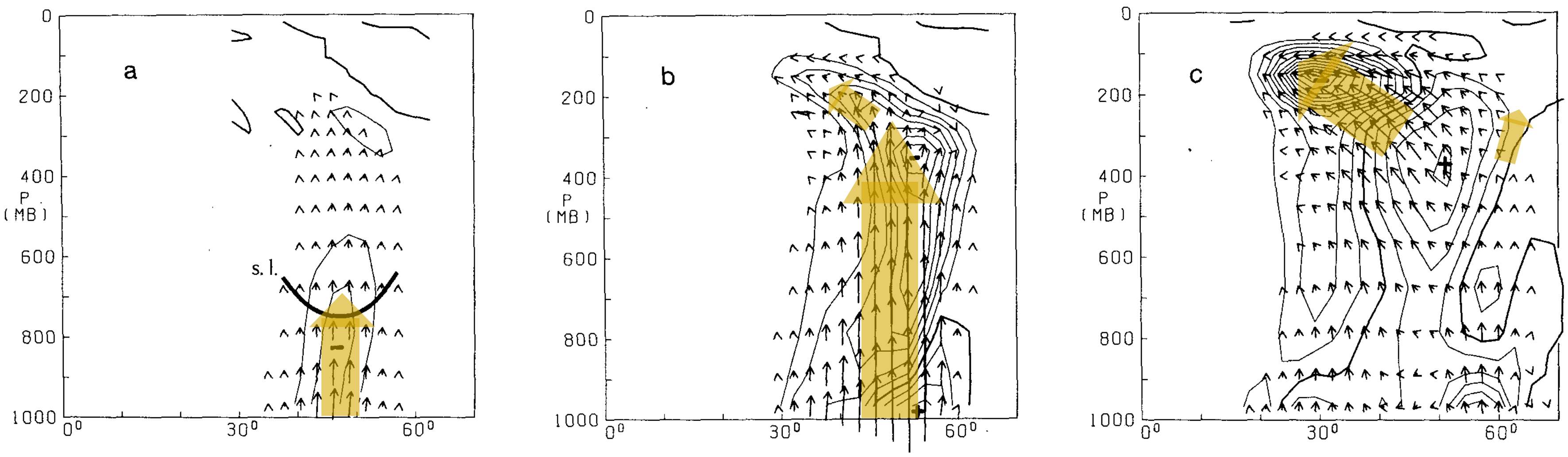
- An E-P flux view

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

Numerical results from
Simmons and Hoskins, 1978, JAS

Eddies: generate at lower level, propagate **upwards** and **away** from the eddy source region



TOTAL E-P FLUX DIVERGENCE
DAY .00

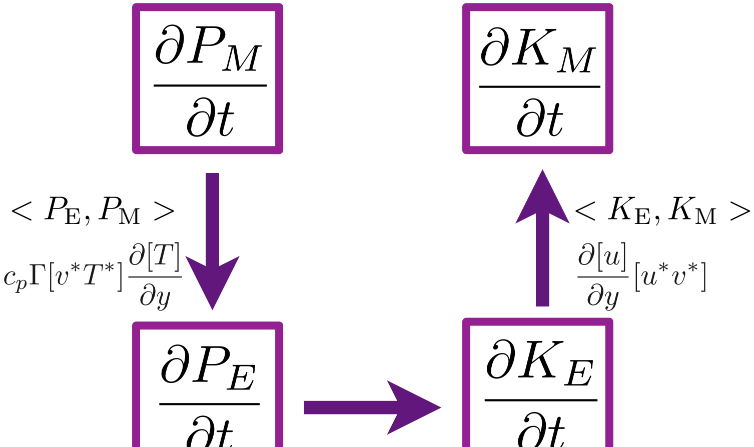
TOTAL E-P FLUX DIVERGENCE
DAY 5.00

TOTAL E-P FLUX DIVERGENCE
DAY 8.00

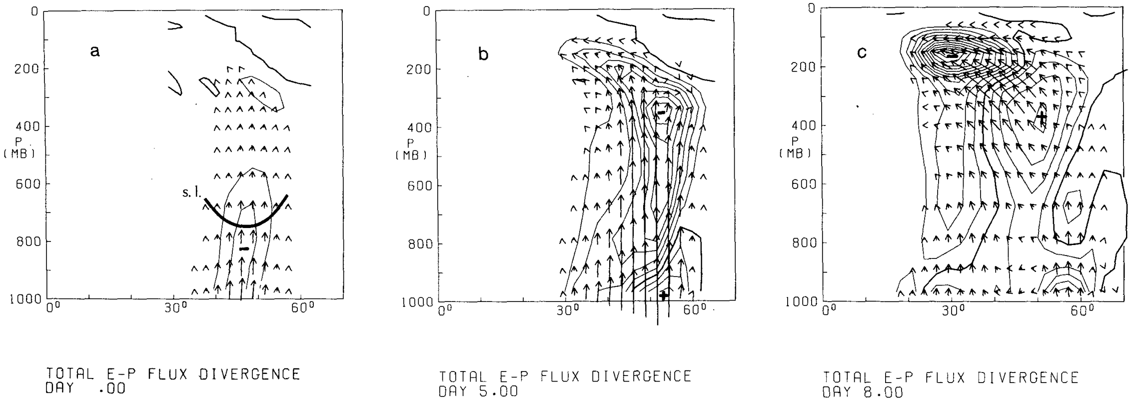
Baroclinic eddies

- baroclinic eddy life cycle

- Westerly jet and energy cycle:



$$\begin{tikzcd}
\boxed{\dfrac{\partial P_M}{\partial t}} \arrow[d, "{< P_\mathrm{E}, P_\mathrm{M} >}"', "{c_p\Gamma[v^*T^*]\frac{\partial [T]}{\partial y}}"'] & \boxed{\dfrac{\partial K_M}{\partial t}} \\
\boxed{\dfrac{\partial P_E}{\partial t}} \arrow[r, "{< P_\mathrm{E}, K_\mathrm{E} >}"', "{R\frac{[\omega^*T^*]}{p}}"'] & \boxed{\dfrac{\partial K_E}{\partial t}} \arrow[u, "{< K_\mathrm{E}, K_\mathrm{M} >}", "{\frac{\partial [u]}{\partial y}[u^*v^*]}"']
\end{tikzcd}$$



TOTAL E-P FLUX DIVERGENCE
DAY .00

TOTAL E-P FLUX DIVERGENCE
DAY 5.00

TOTAL E-P FLUX DIVERGENCE
DAY 8.00

Baroclinic eddies

- baroclinic eddy life cycle

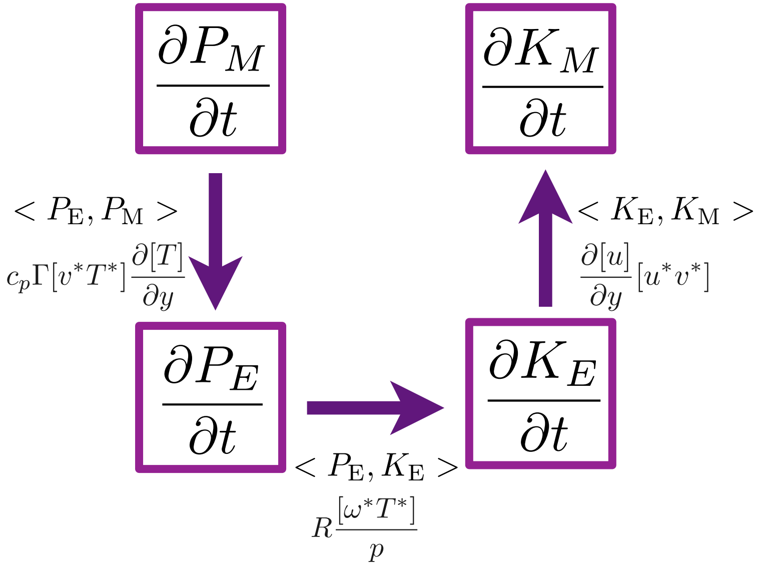
- Westerly jet and energy cycle:

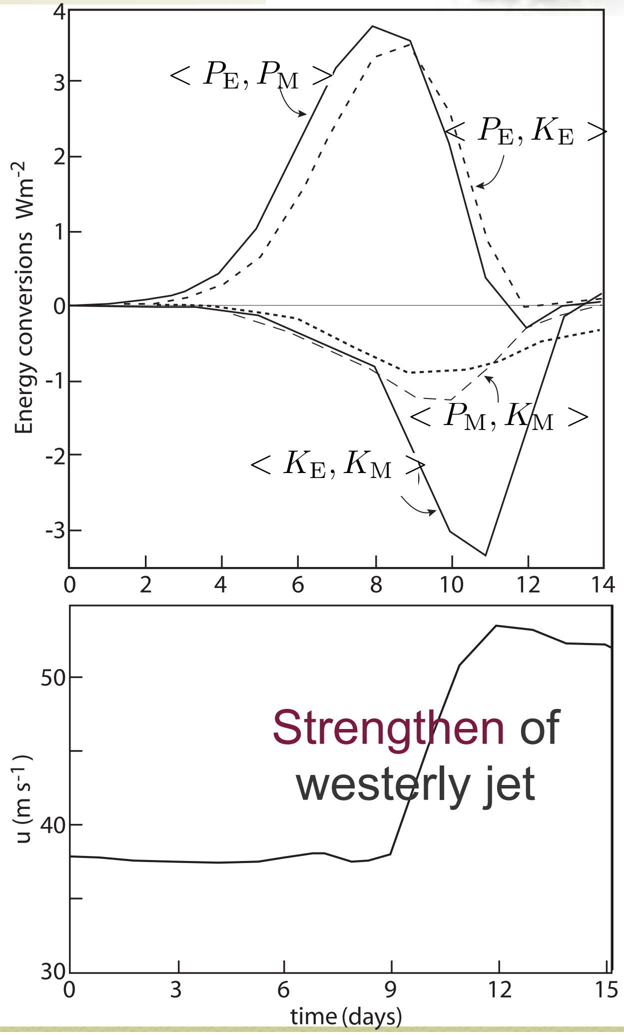
$$\begin{tikzcd}
\boxed{\dfrac{\partial P_M}{\partial t}} \arrow[d, "{< P_{\mathrm{E}}, P_{\mathrm{M}} > \quad c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}}"'] & \boxed{\dfrac{\partial K_M}{\partial t}} \\
\boxed{\dfrac{\partial P_E}{\partial t}} \arrow[r, "{< P_{\mathrm{E}}, K_{\mathrm{E}} > \quad R\frac{[\omega^*T^*]}{p}}"'] & \boxed{\dfrac{\partial K_E}{\partial t}} \arrow[u, "{< K_{\mathrm{E}}, K_{\mathrm{M}} > \quad \frac{\partial[u]}{\partial y}[u^*v^*]}"']
\end{tikzcd}$$

Baroclinic eddies

- baroclinic eddy life cycle

- Westerly jet and energy cycle:



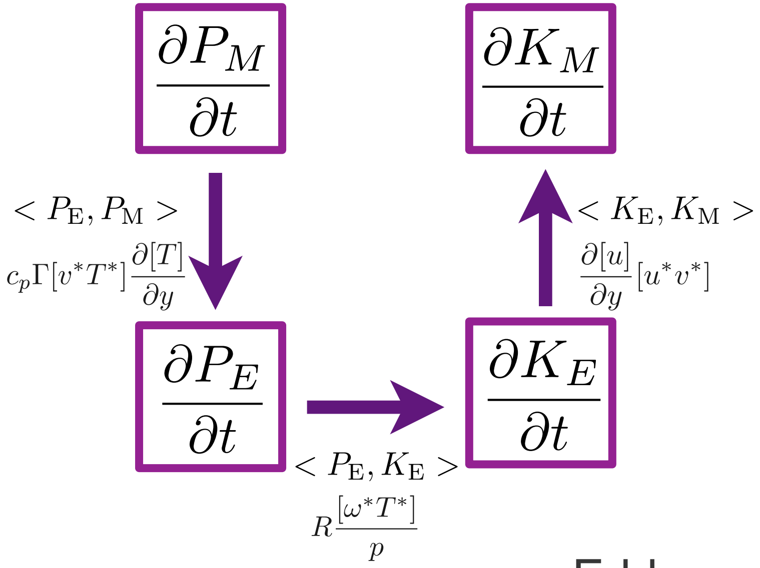


Numerical results from
Simmons and Hoskins, 1978, JAS

Baroclinic eddies

- baroclinic eddy life cycle

- Westerly jet and energy cycle:



$$\frac{\partial P_M}{\partial t}$$

$< P_E, P_M >$

$c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}$

$$\frac{\partial P_E}{\partial t}$$

$< P_E, K_E >$

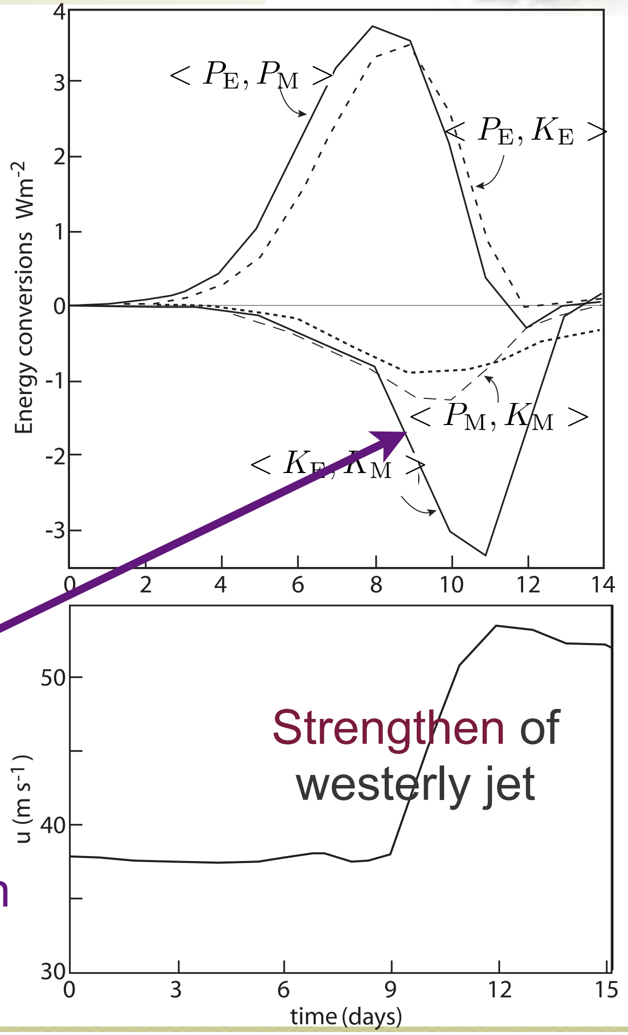
$R\frac{[\omega^*T^*]}{p}$

$$\frac{\partial K_E}{\partial t}$$

$< K_E, K_M >$

$\frac{\partial[u]}{\partial y}[u^*v^*]$

$$\frac{\partial K_M}{\partial t}$$



Strengthen of westerly jet

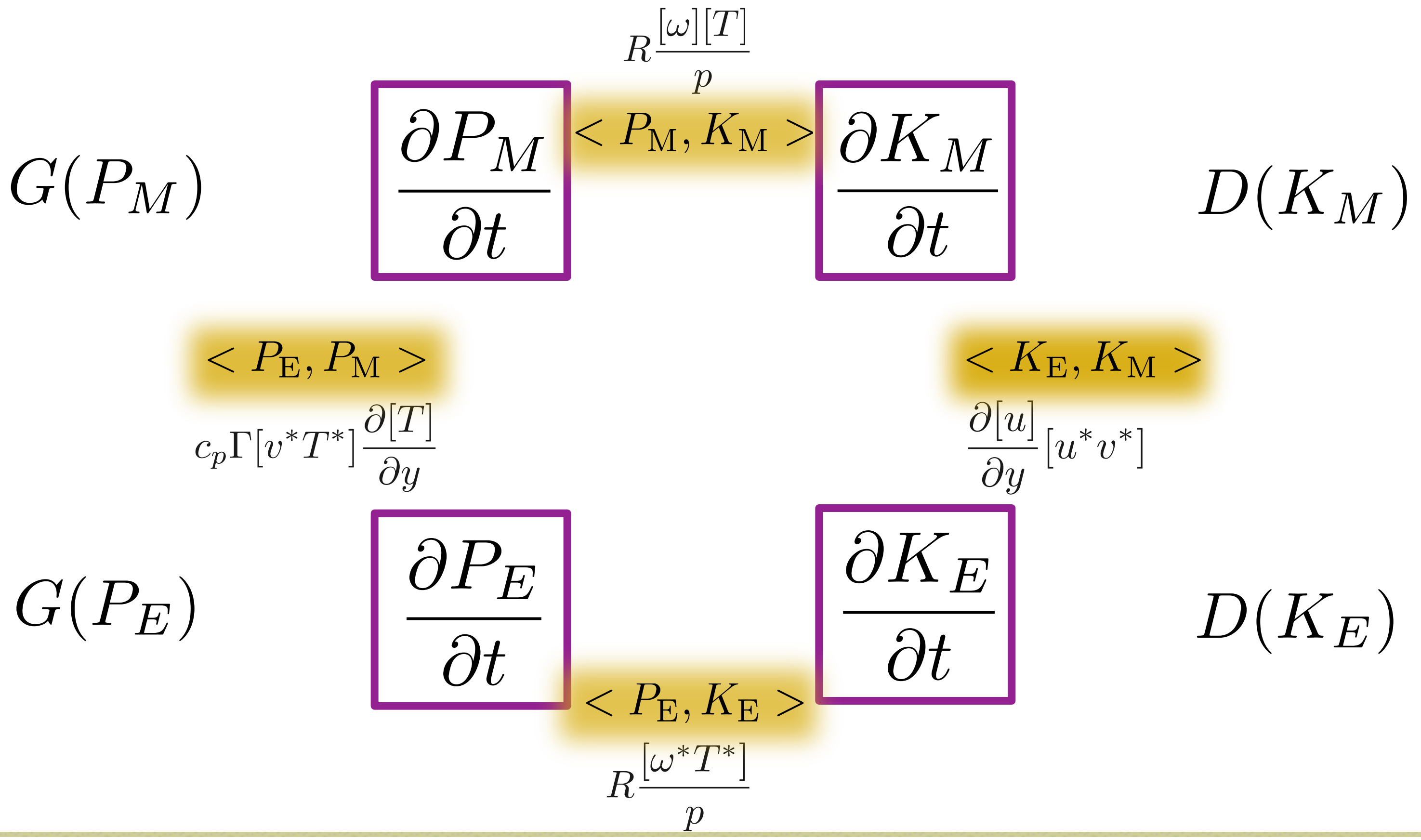
Eddy momentum flux grows, which extracts kinetic energy from the eddies to the zonal mean flow, then the growth of the eddy energy ceases.

Numerical results from
Simmons and Hoskins, 1978, JAS

Energy cycles

in the baroclinic eddy-mean flow interactions

PE → KE



$R\frac{[\omega][T]}{p}$

$< P_{\mathrm{M}}, K_{\mathrm{M}} >$

$G(P_M)$ $\boxed{\frac{\partial P_M}{\partial t}}$ $\boxed{\frac{\partial K_M}{\partial t}}$ $D(K_M)$

$< P_{\mathrm{E}}, P_{\mathrm{M}} >$

$c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}$

$< K_{\mathrm{E}}, K_{\mathrm{M}} >$

$\frac{\partial[u]}{\partial y}[u^*v^*]$

$G(P_E)$ $\boxed{\frac{\partial P_E}{\partial t}}$ $\boxed{\frac{\partial K_E}{\partial t}}$ $D(K_E)$

$< P_{\mathrm{E}}, K_{\mathrm{E}} >$

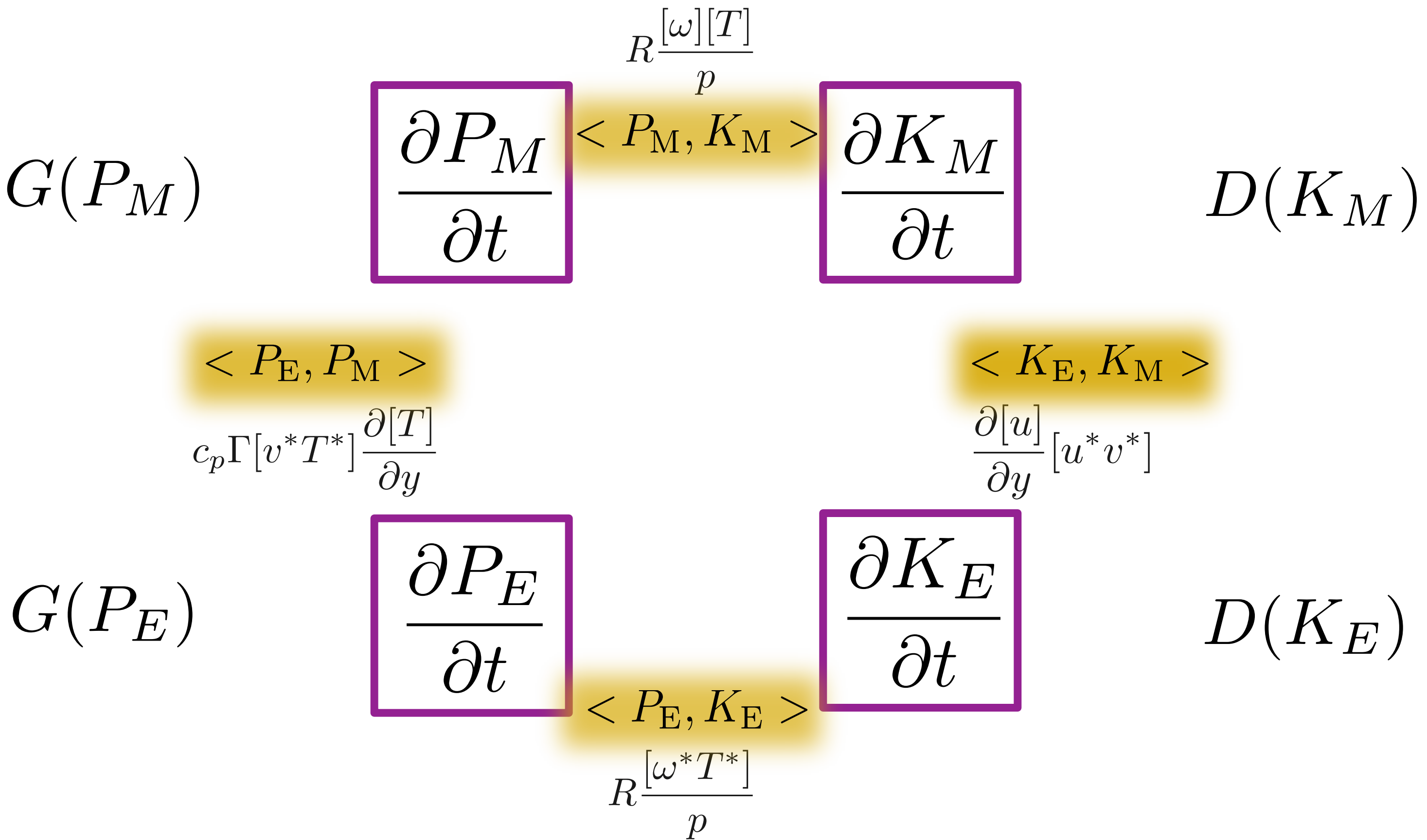
$R\frac{[\omega^*T^*]}{p}$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium:

PE → KE



$G(P_M)$

$$\frac{\partial P_M}{\partial t}$$

$< P_\mathrm{M}, K_\mathrm{M} >$

$R\frac{[\omega][T]}{p}$

$$\frac{\partial K_M}{\partial t}$$

$D(K_M)$

$< P_\mathrm{E}, P_\mathrm{M} >$

$c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}$

$< K_\mathrm{E}, K_\mathrm{M} >$

$\frac{\partial[u]}{\partial y}[u^*v^*]$

$G(P_E)$

$$\frac{\partial P_E}{\partial t}$$

$< P_\mathrm{E}, K_\mathrm{E} >$

$R\frac{[\omega^*T^*]}{p}$

$$\frac{\partial K_E}{\partial t}$$

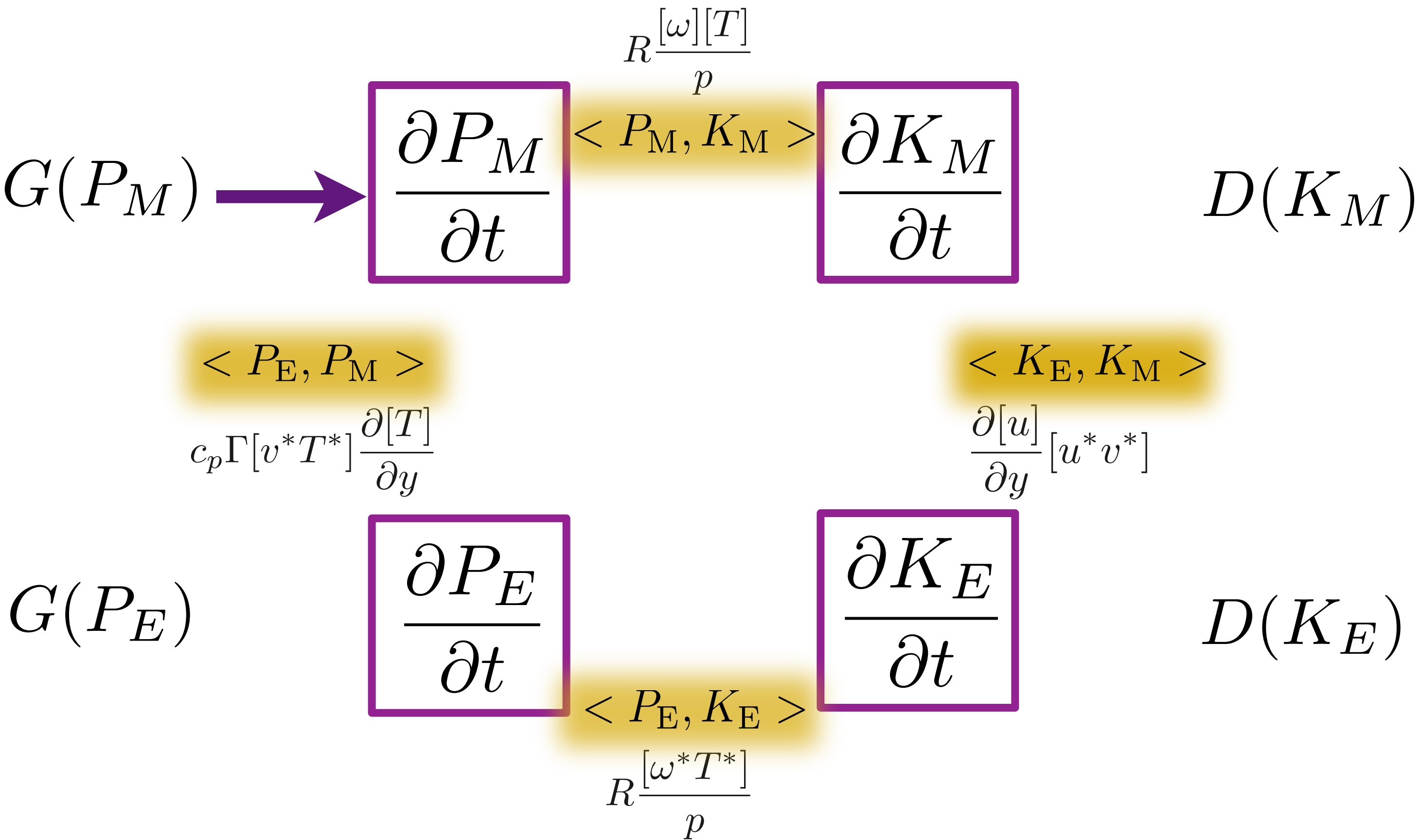
$D(K_E)$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium:

PE → KE



$R\frac{[\omega][T]}{p}$

$< P_{\mathrm{M}}, K_{\mathrm{M}} >$

$G(P_M) \rightarrow \frac{\partial P_M}{\partial t}$

$\frac{\partial K_M}{\partial t}$

$D(K_M)$

$< P_{\mathrm{E}}, P_{\mathrm{M}} >$

$c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}$

$< K_{\mathrm{E}}, K_{\mathrm{M}} >$

$\frac{\partial[u]}{\partial y}[u^*v^*]$

$G(P_E)$

$\frac{\partial P_E}{\partial t}$

$\frac{\partial K_E}{\partial t}$

$D(K_E)$

$< P_{\mathrm{E}}, K_{\mathrm{E}} >$

$R\frac{[\omega^*T^*]}{p}$

Energy cycles
in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium:

PE → KE

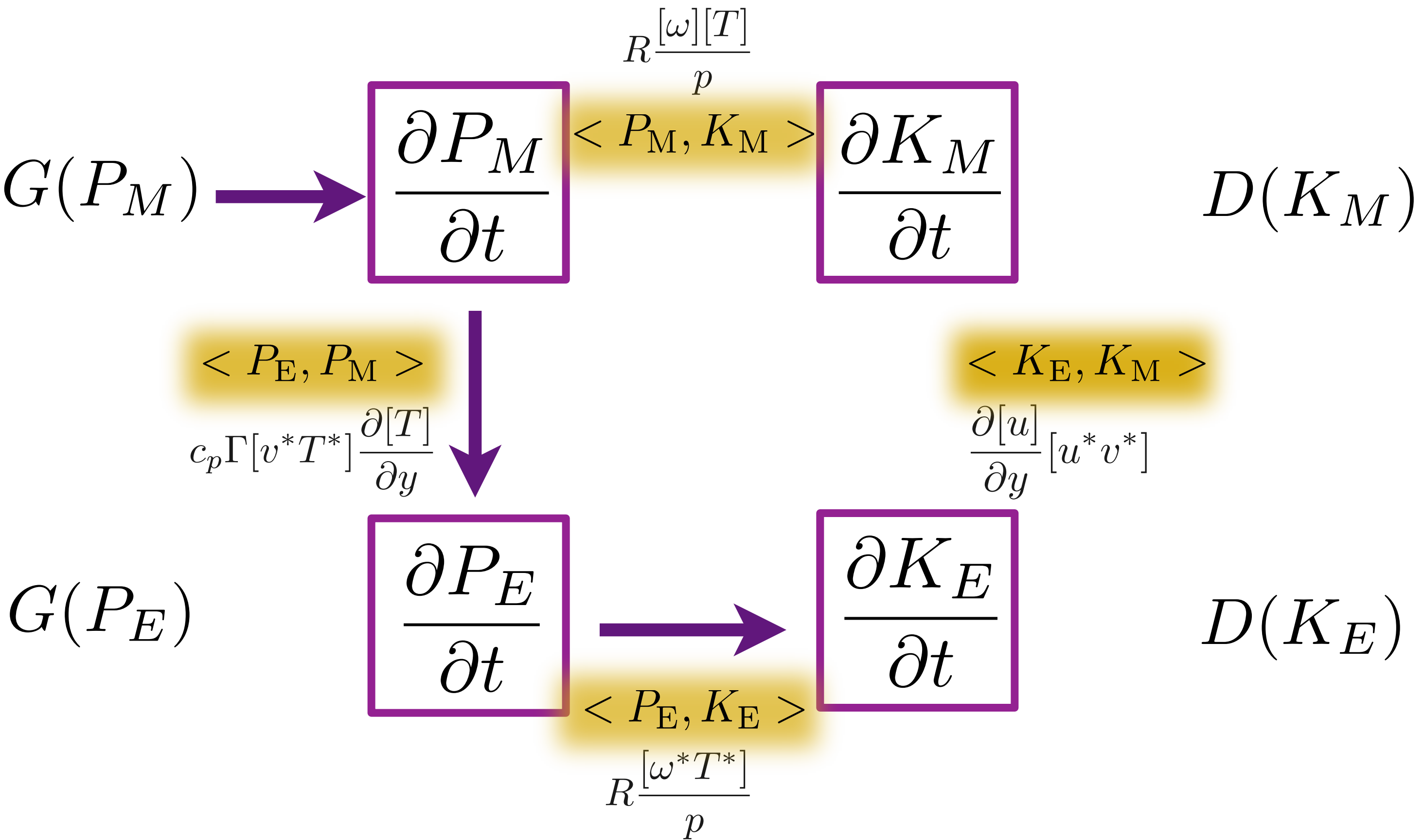
$$\begin{tikzcd}
G(P_M) \arrow[r] & \boxed{\frac{\partial P_M}{\partial t}} \arrow[r, phantom, "{R\frac{[\omega][T]}{p}}", "{< P_\mathrm{M}, K_\mathrm{M} >}"'] \arrow[d, "{< P_\mathrm{E}, P_\mathrm{M} > \quad c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}}"'] & \boxed{\frac{\partial K_M}{\partial t}} \arrow[d, phantom, "{< K_\mathrm{E}, K_\mathrm{M} > \quad \frac{\partial[u]}{\partial y}[u^*v^*]}"] & D(K_M) \\
G(P_E) & \boxed{\frac{\partial P_E}{\partial t}} \arrow[r, phantom, "{< P_\mathrm{E}, K_\mathrm{E} >}", "{R\frac{[\omega^*T^*]}{p}}"'] & \boxed{\frac{\partial K_E}{\partial t}} & D(K_E)
\end{tikzcd}$$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium:

PE → KE



$R\frac{[\omega][T]}{p}$

$G(P_M)$ $\frac{\partial P_M}{\partial t}$ $< P_{\rm M}, K_{\rm M} >$ $\frac{\partial K_M}{\partial t}$ $D(K_M)$

$< P_{\rm E}, P_{\rm M} >$

$c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}$

$< K_{\rm E}, K_{\rm M} >$

$\frac{\partial[u]}{\partial y}[u^*v^*]$

$G(P_E)$ $\frac{\partial P_E}{\partial t}$ $\frac{\partial K_E}{\partial t}$ $D(K_E)$

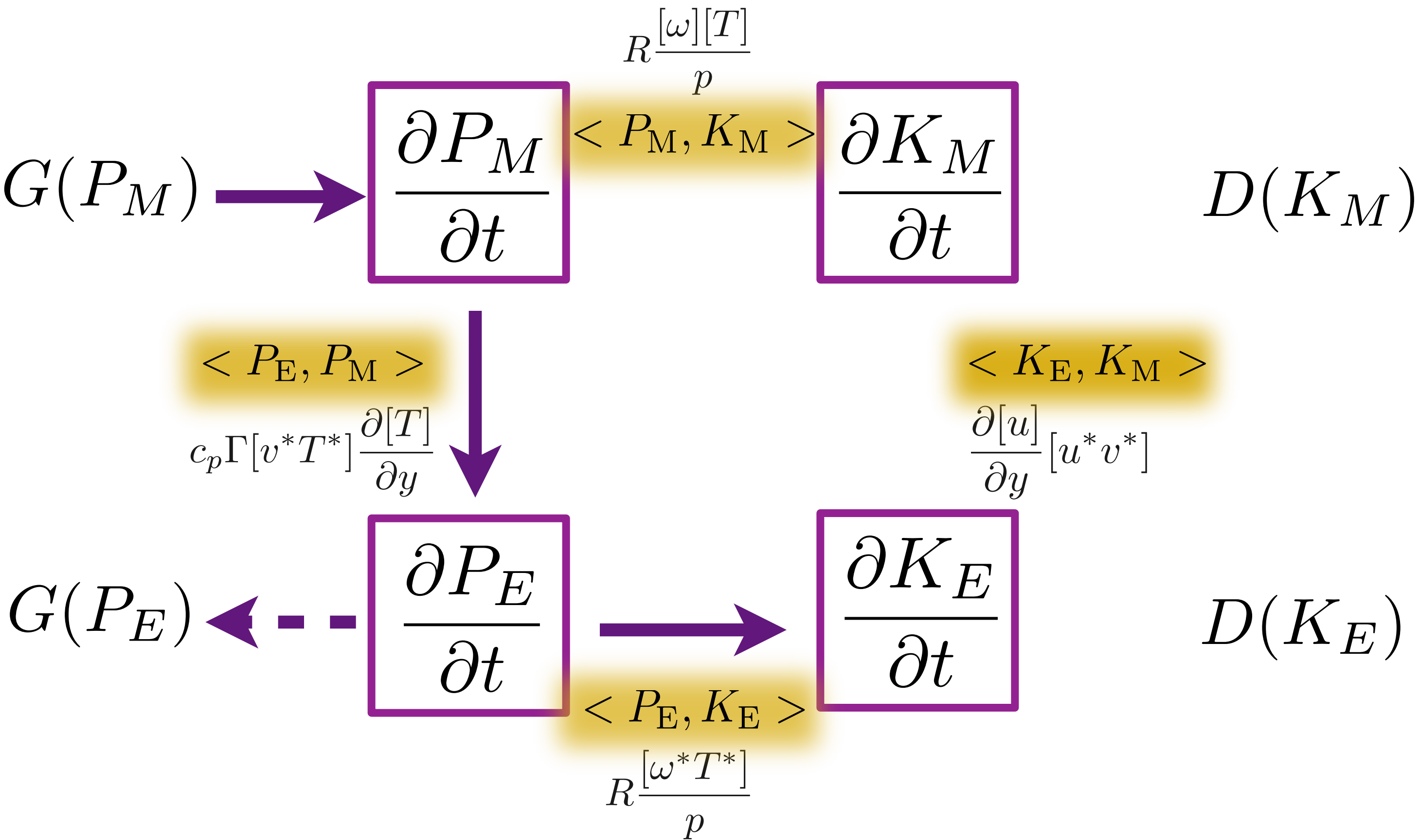
$< P_{\rm E}, K_{\rm E} >$

$R\frac{[\omega^*T^*]}{p}$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium: PE → KE



$R\frac{[\omega][T]}{p}$

$< P_{\mathrm{M}}, K_{\mathrm{M}} >$

$G(P_M)$

$\frac{\partial P_M}{\partial t}$

$\frac{\partial K_M}{\partial t}$

$D(K_M)$

$< P_{\mathrm{E}}, P_{\mathrm{M}} >$

$c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}$

$< K_{\mathrm{E}}, K_{\mathrm{M}} >$

$\frac{\partial[u]}{\partial y}[u^*v^*]$

$G(P_E)$

$\frac{\partial P_E}{\partial t}$

$\frac{\partial K_E}{\partial t}$

$D(K_E)$

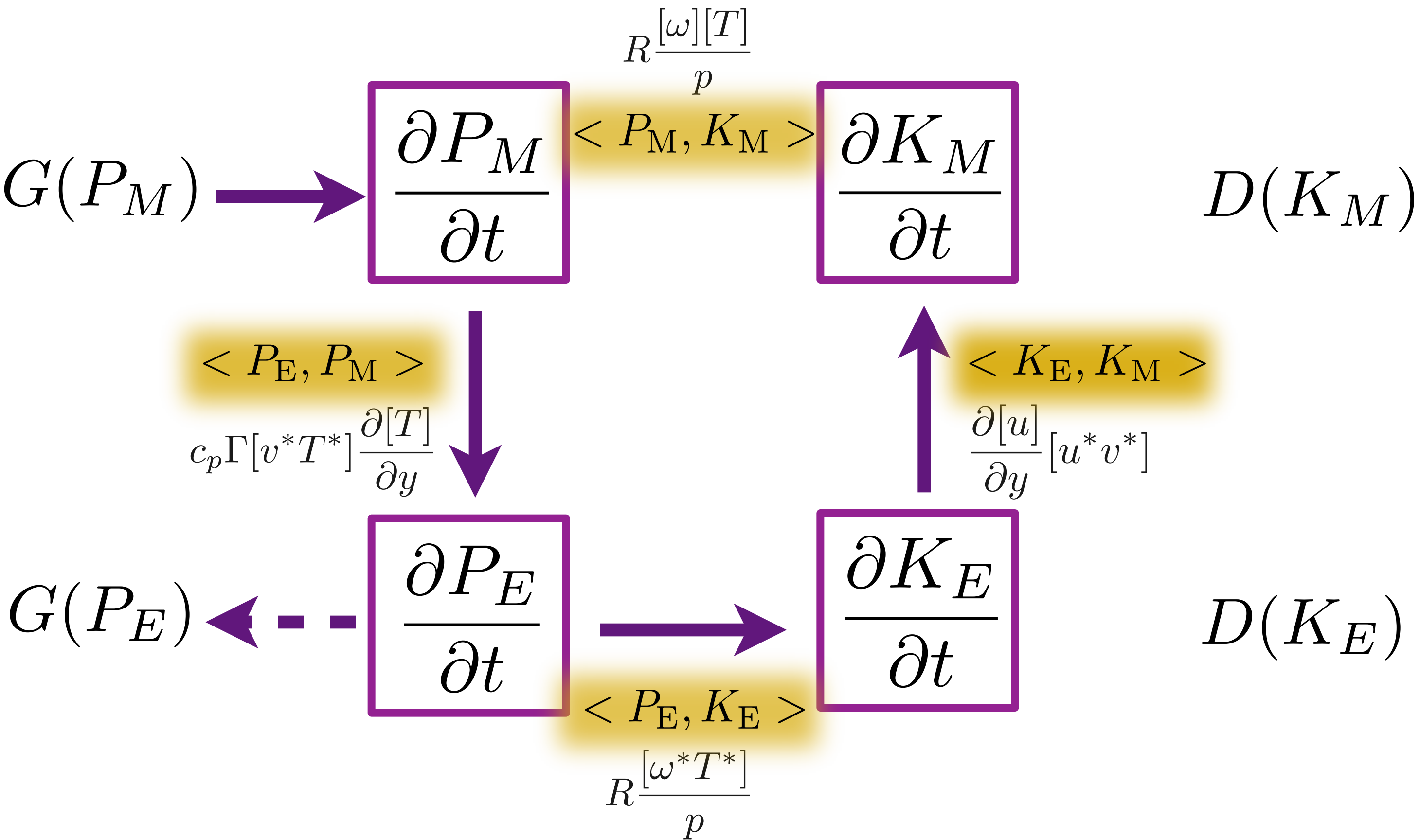
$< P_{\mathrm{E}}, K_{\mathrm{E}} >$

$R\frac{[\omega^*T^*]}{p}$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium: $\boxed{PE} \rightarrow \boxed{KE}$



$G(P_M)$ → $\frac{\partial P_M}{\partial t}$

$R\frac{[\omega][T]}{p}$

$< P_{\rm M}, K_{\rm M} >$

$\frac{\partial K_M}{\partial t}$

$D(K_M)$

$< P_{\rm E}, P_{\rm M} >$

$c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}$

$< K_{\rm E}, K_{\rm M} >$

$\frac{\partial[u]}{\partial y}[u^*v^*]$

$G(P_E)$ ← $\frac{\partial P_E}{\partial t}$

$\frac{\partial K_E}{\partial t}$

$D(K_E)$

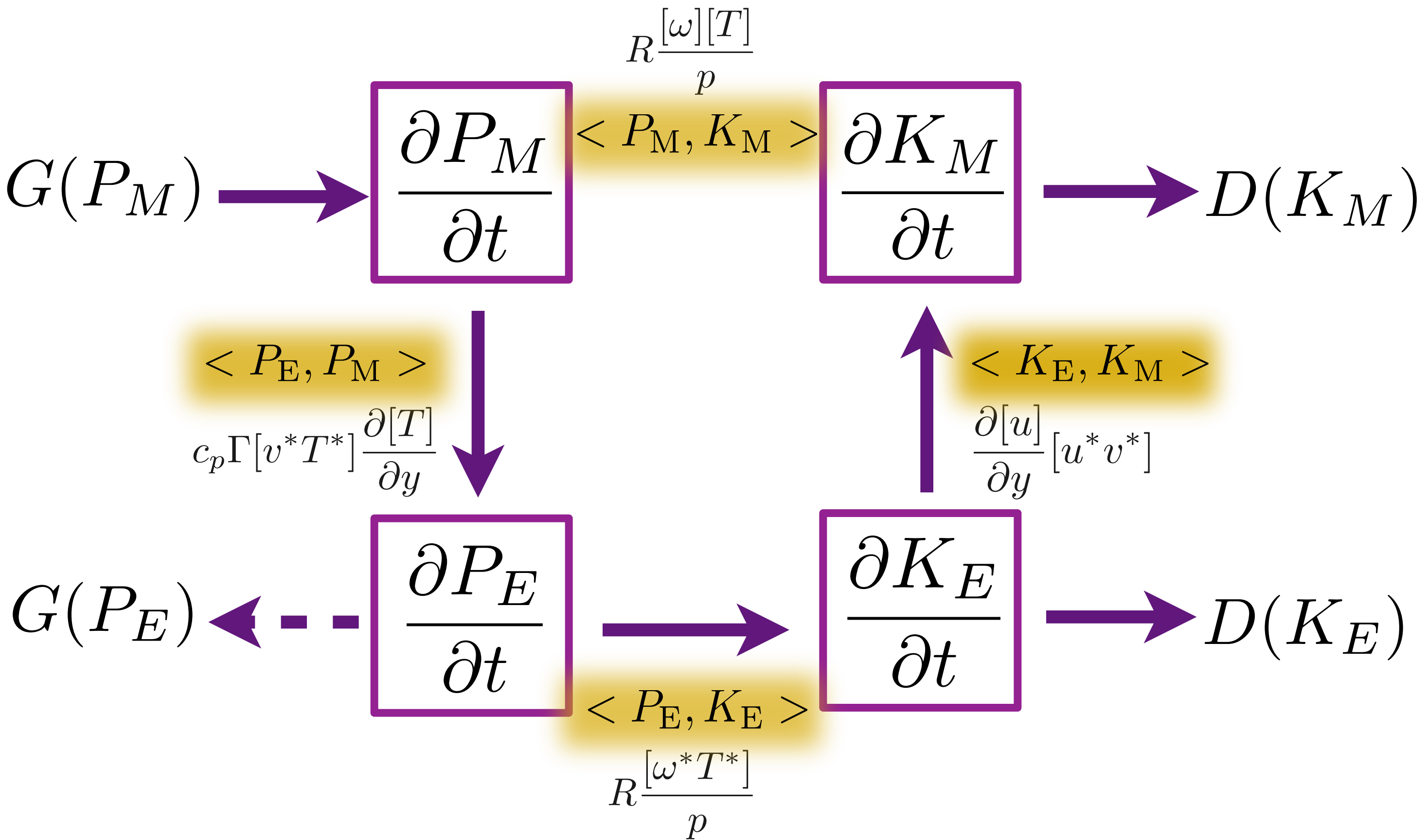
$< P_{\rm E}, K_{\rm E} >$

$R\frac{[\omega^*T^*]}{p}$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium: $\boxed{PE} \rightarrow \boxed{KE}$



$R\frac{[\omega][T]}{p}$

$< P_{\mathrm{M}}, K_{\mathrm{M}} >$

$G(P_M) \rightarrow \frac{\partial P_M}{\partial t}$

$\frac{\partial K_M}{\partial t} \rightarrow D(K_M)$

$< P_{\mathrm{E}}, P_{\mathrm{M}} >$

$c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}$

$< K_{\mathrm{E}}, K_{\mathrm{M}} >$

$\frac{\partial[u]}{\partial y}[u^*v^*]$

$G(P_E) \leftarrow \frac{\partial P_E}{\partial t}$

$\frac{\partial K_E}{\partial t} \rightarrow D(K_E)$

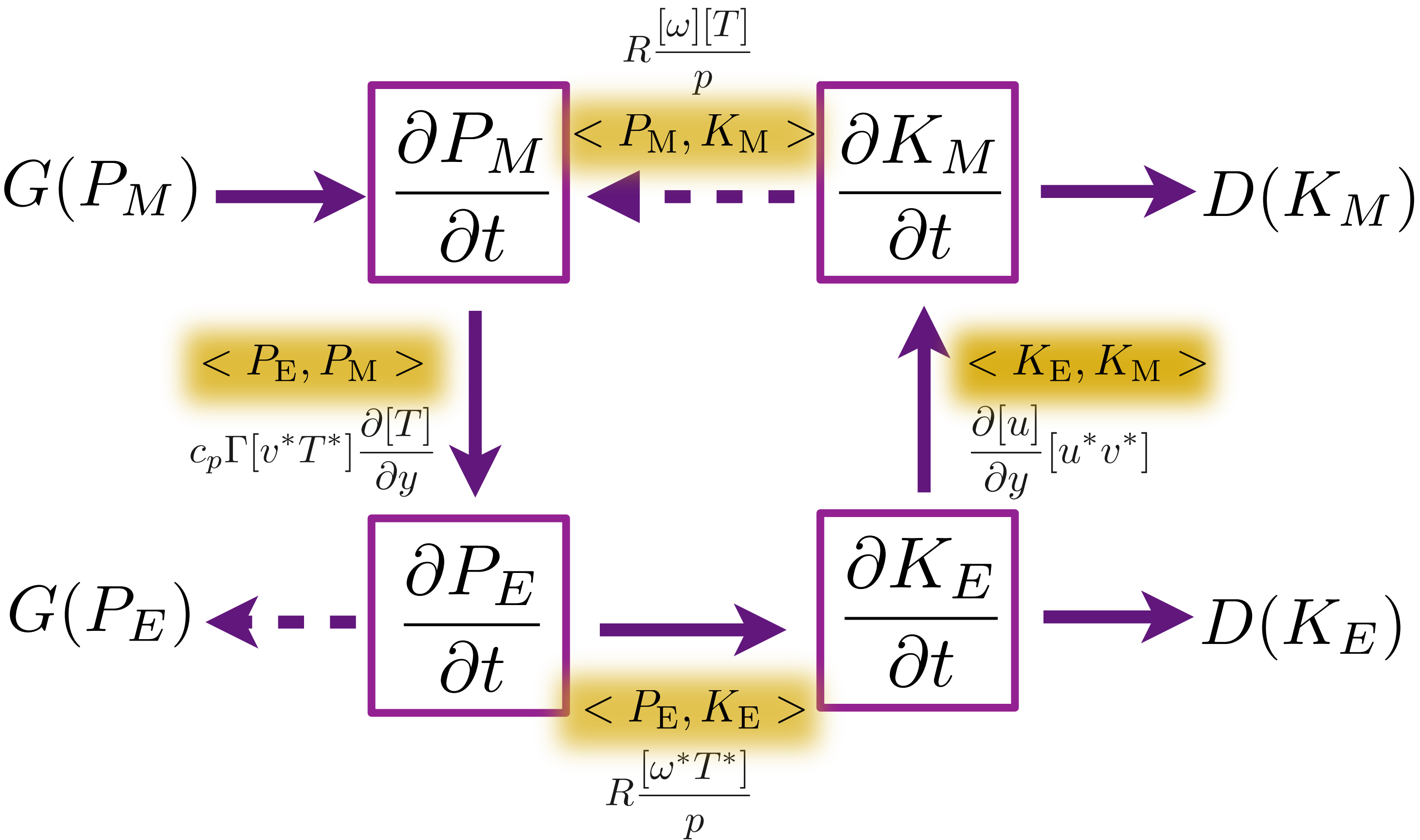
$< P_{\mathrm{E}}, K_{\mathrm{E}} >$

$R\frac{[\omega^*T^*]}{p}$

Energy cycles
in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium:

PE → KE



$R\frac{[\omega][T]}{p}$

$< P_{\mathrm{M}}, K_{\mathrm{M}} >$

$G(P_M) \rightarrow \frac{\partial P_M}{\partial t}$

$\frac{\partial K_M}{\partial t} \rightarrow D(K_M)$

$< P_{\mathrm{E}}, P_{\mathrm{M}} >$

$c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}$

$< K_{\mathrm{E}}, K_{\mathrm{M}} >$

$\frac{\partial[u]}{\partial y}[u^*v^*]$

$G(P_E) \leftarrow \frac{\partial P_E}{\partial t}$

$\frac{\partial K_E}{\partial t} \rightarrow D(K_E)$

$< P_{\mathrm{E}}, K_{\mathrm{E}} >$

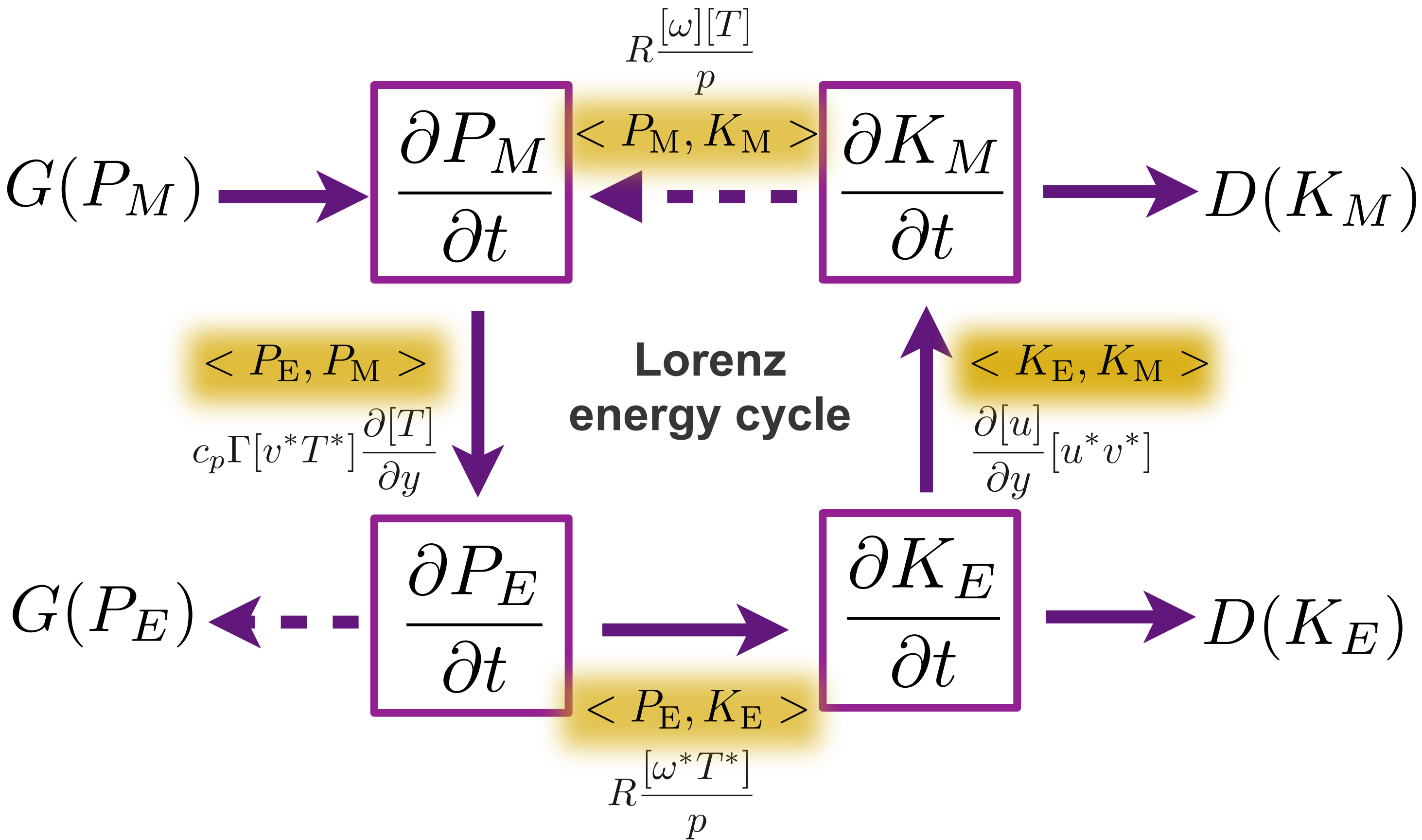
$R\frac{[\omega^*T^*]}{p}$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium:

PE → KE



$R\frac{[\omega][T]}{p}$

$< P_\mathrm{M}, K_\mathrm{M} >$

$G(P_M)$

$\frac{\partial P_M}{\partial t}$

$\frac{\partial K_M}{\partial t}$

$D(K_M)$

$< P_\mathrm{E}, P_\mathrm{M} >$

$c_p\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}$

Lorenz energy cycle

$< K_\mathrm{E}, K_\mathrm{M} >$

$\frac{\partial[u]}{\partial y}[u^*v^*]$

$G(P_E)$

$\frac{\partial P_E}{\partial t}$

$\frac{\partial K_E}{\partial t}$

$D(K_E)$

$< P_\mathrm{E}, K_\mathrm{E} >$

$R\frac{[\omega^*T^*]}{p}$

Energy cycles in Hadley Cell

PE → KE

Energy cycles in Hadley Cell

$$\begin{tikzcd} \boxed{\mathrm{PE}} \arrow[r] & \boxed{\mathrm{KE}} \end{tikzcd}$$

$$R\frac{[\omega][T]}{p}$$

$$\begin{tikzcd} G(P_M) \arrow[r] & \boxed{\frac{\partial P_M}{\partial t}} \arrow[r, "{< P_{\mathrm{M}}, K_{\mathrm{M}} >}"] & \boxed{\frac{\partial K_M}{\partial t}} \arrow[r] & D(K_M) \end{tikzcd}$$

If assume no eddies.

Energy cycles

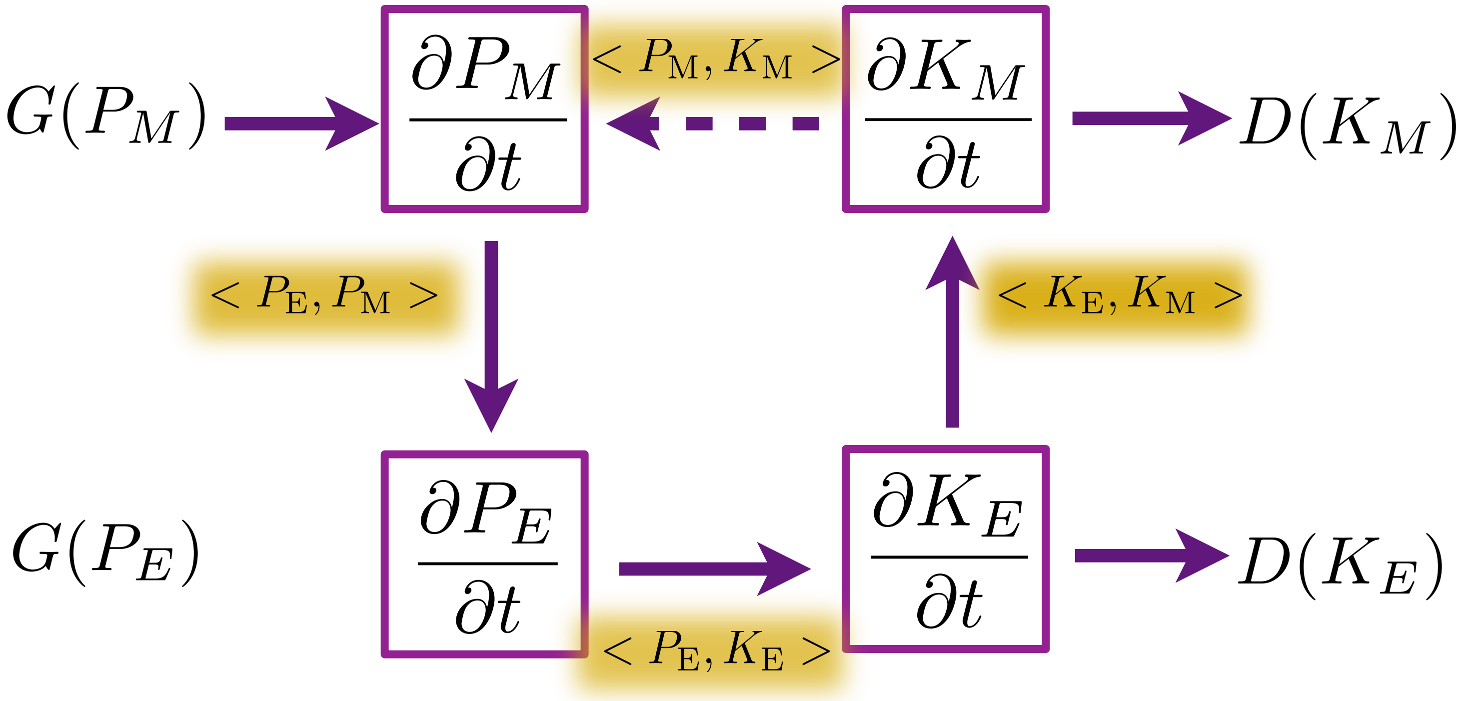
in the baroclinic eddy-mean flow interactions

$$\begin{tikzcd}
G(P_M) \arrow[r] & \dfrac{\partial P_M}{\partial t} \arrow[d, "< P_{\mathrm{E}}, P_{\mathrm{M}} >"'] & \dfrac{\partial K_M}{\partial t} \arrow[l, dashed, "< P_{\mathrm{M}}, K_{\mathrm{M}} >"'] \arrow[r] & D(K_M) \\
G(P_E) & \dfrac{\partial P_E}{\partial t} \arrow[r, "< P_{\mathrm{E}}, K_{\mathrm{E}} >"'] & \dfrac{\partial K_E}{\partial t} \arrow[u, "< K_{\mathrm{E}}, K_{\mathrm{M}} >"'] \arrow[r] & D(K_E)
\end{tikzcd}$$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in real atmosphere:



$G(P_M)$

$\frac{\partial P_M}{\partial t}$

$< P_\mathrm{M}, K_\mathrm{M} >$

$\frac{\partial K_M}{\partial t}$

$D(K_M)$

$< P_\mathrm{E}, P_\mathrm{M} >$

$< K_\mathrm{E}, K_\mathrm{M} >$

$G(P_E)$

$\frac{\partial P_E}{\partial t}$

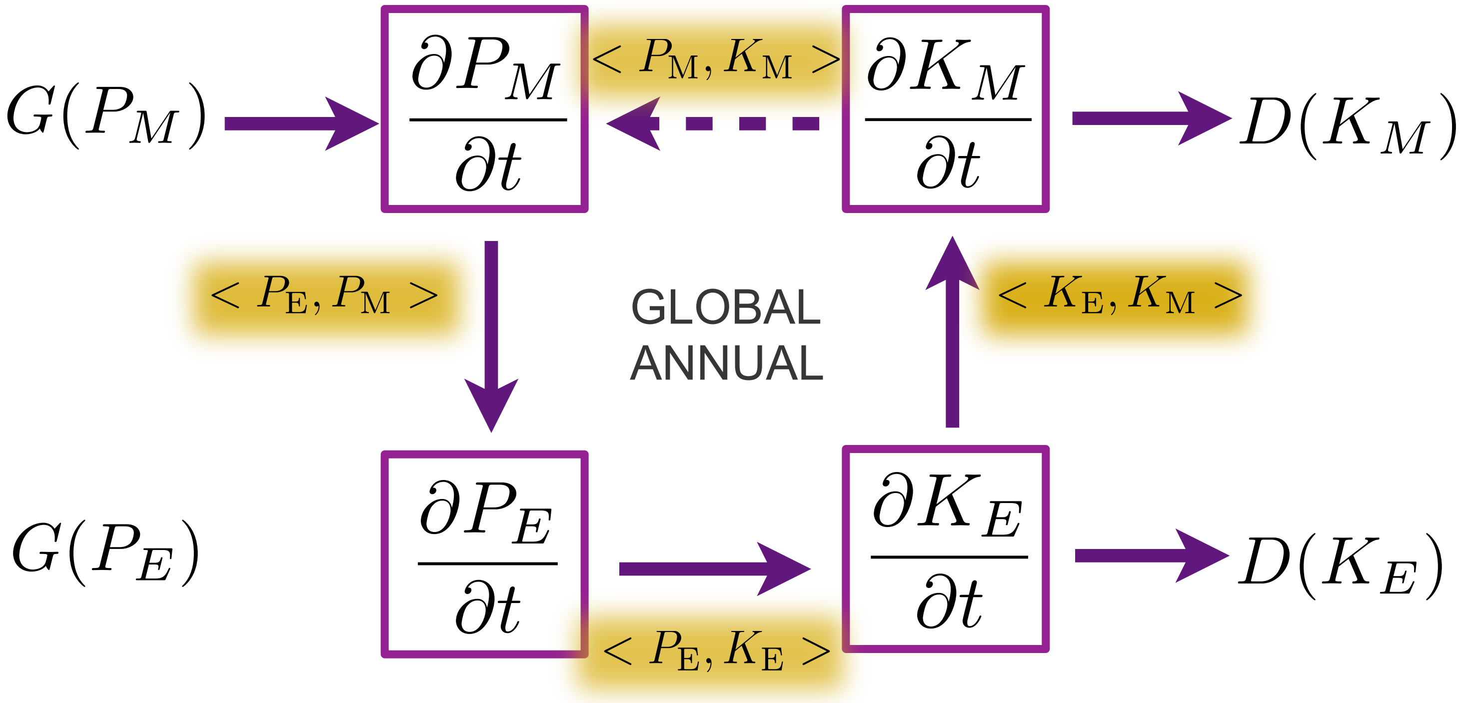
$\frac{\partial K_E}{\partial t}$

$D(K_E)$

$< P_\mathrm{E}, K_\mathrm{E} >$

Energy cycles
in the baroclinic eddy-mean flow interactions

- Energy cycles in real atmosphere:



$G(P_M)$

$\frac{\partial P_M}{\partial t}$

$< P_{\mathrm{M}}, K_{\mathrm{M}} >$

$\frac{\partial K_M}{\partial t}$

$D(K_M)$

$< P_{\mathrm{E}}, P_{\mathrm{M}} >$

GLOBAL
ANNUAL

$< K_{\mathrm{E}}, K_{\mathrm{M}} >$

$G(P_E)$

$\frac{\partial P_E}{\partial t}$

$\frac{\partial K_E}{\partial t}$

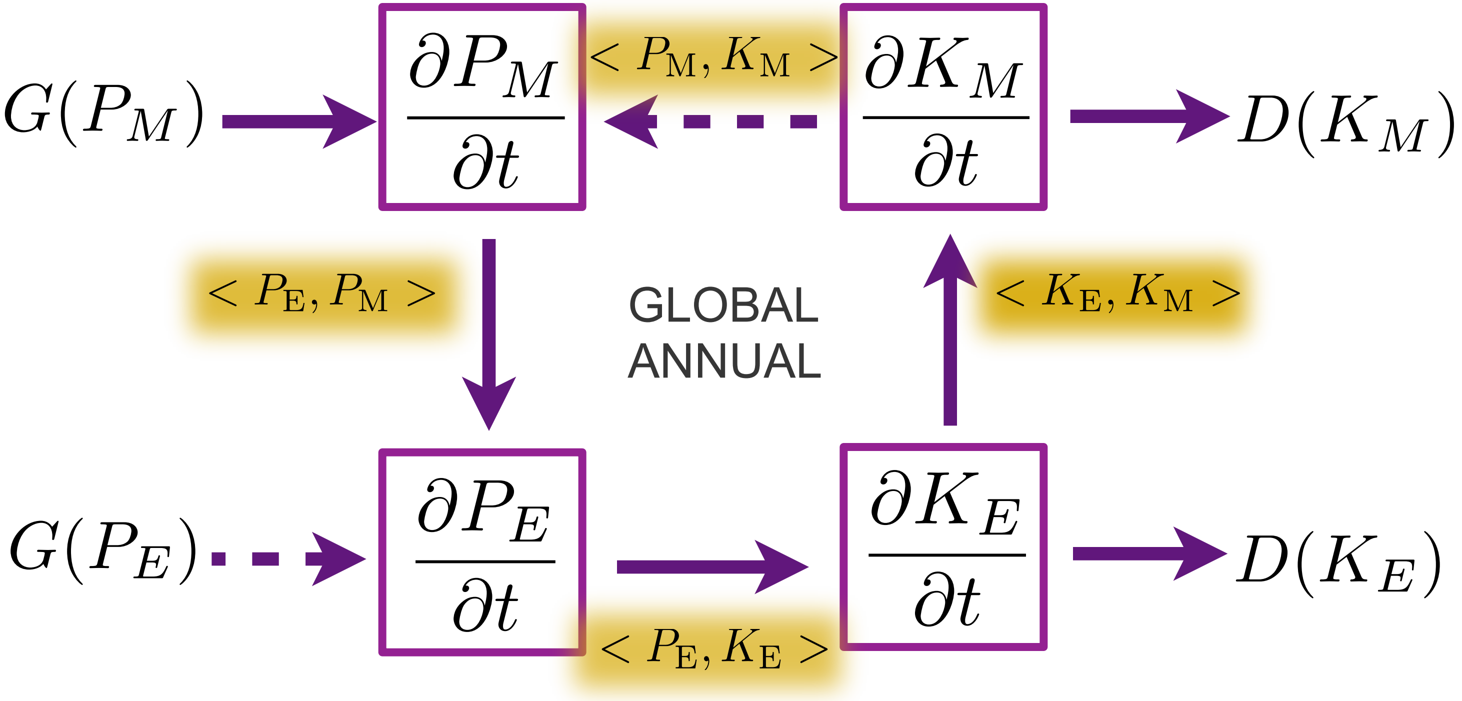
$D(K_E)$

$< P_{\mathrm{E}}, K_{\mathrm{E}} >$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in real atmosphere:



$G(P_M)$ → $\frac{\partial P_M}{\partial t}$ ⇠ $< P_\mathrm{M}, K_\mathrm{M} >$ ⇠ $\frac{\partial K_M}{\partial t}$ → $D(K_M)$

$< P_\mathrm{E}, P_\mathrm{M} >$

GLOBAL
ANNUAL

$< K_\mathrm{E}, K_\mathrm{M} >$

$G(P_E)$ ⇢ $\frac{\partial P_E}{\partial t}$ → $\frac{\partial K_E}{\partial t}$ → $D(K_E)$

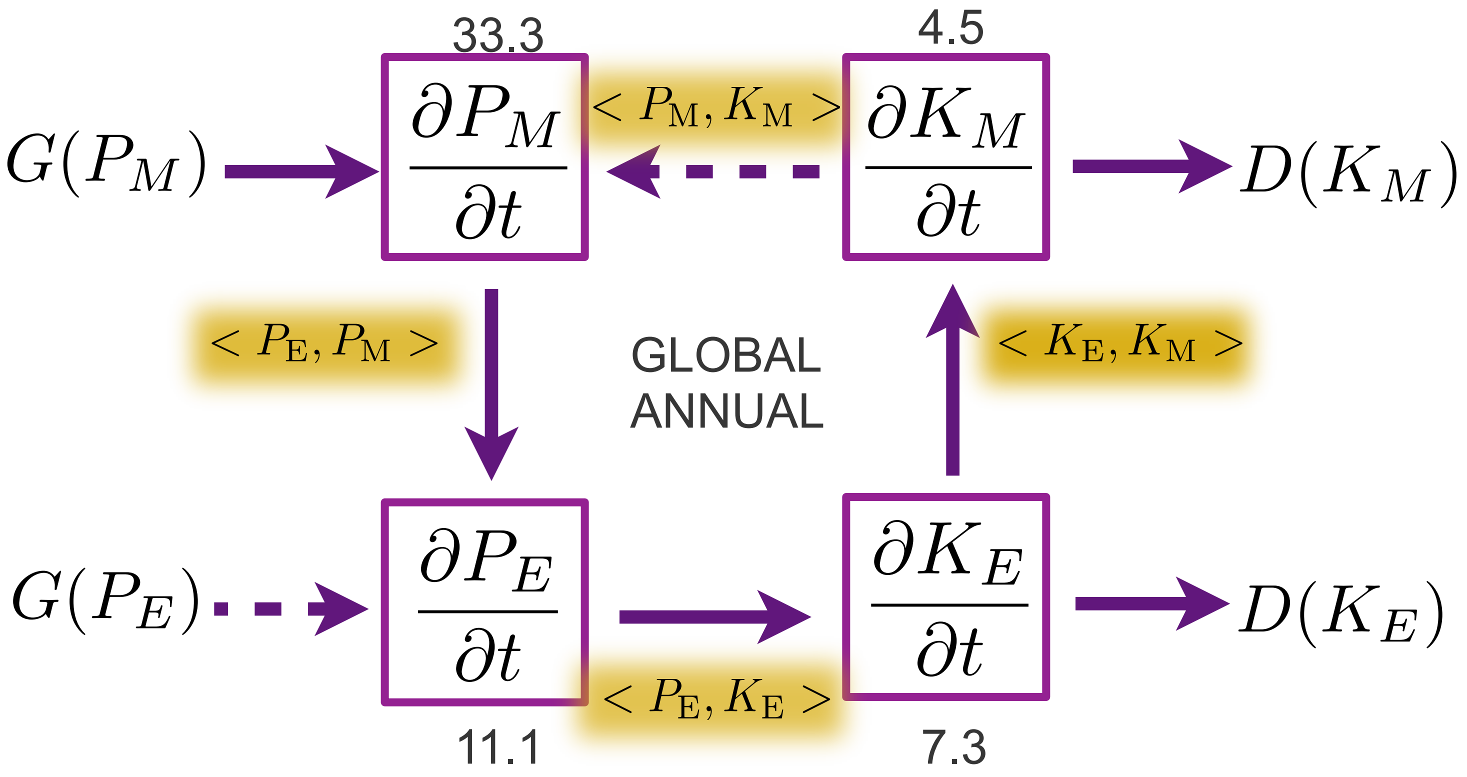
$< P_\mathrm{E}, K_\mathrm{E} >$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in real atmosphere:

energy: $10^5 Jm^{-2}$



33.3

4.5

$G(P_M)$

$\frac{\partial P_M}{\partial t}$

$< P_{\mathrm{M}}, K_{\mathrm{M}} >$

$\frac{\partial K_M}{\partial t}$

$D(K_M)$

$< P_{\mathrm{E}}, P_{\mathrm{M}} >$

GLOBAL
ANNUAL

$< K_{\mathrm{E}}, K_{\mathrm{M}} >$

$G(P_E)$

$\frac{\partial P_E}{\partial t}$

$\frac{\partial K_E}{\partial t}$

$D(K_E)$

$< P_{\mathrm{E}}, K_{\mathrm{E}} >$

11.1

7.3

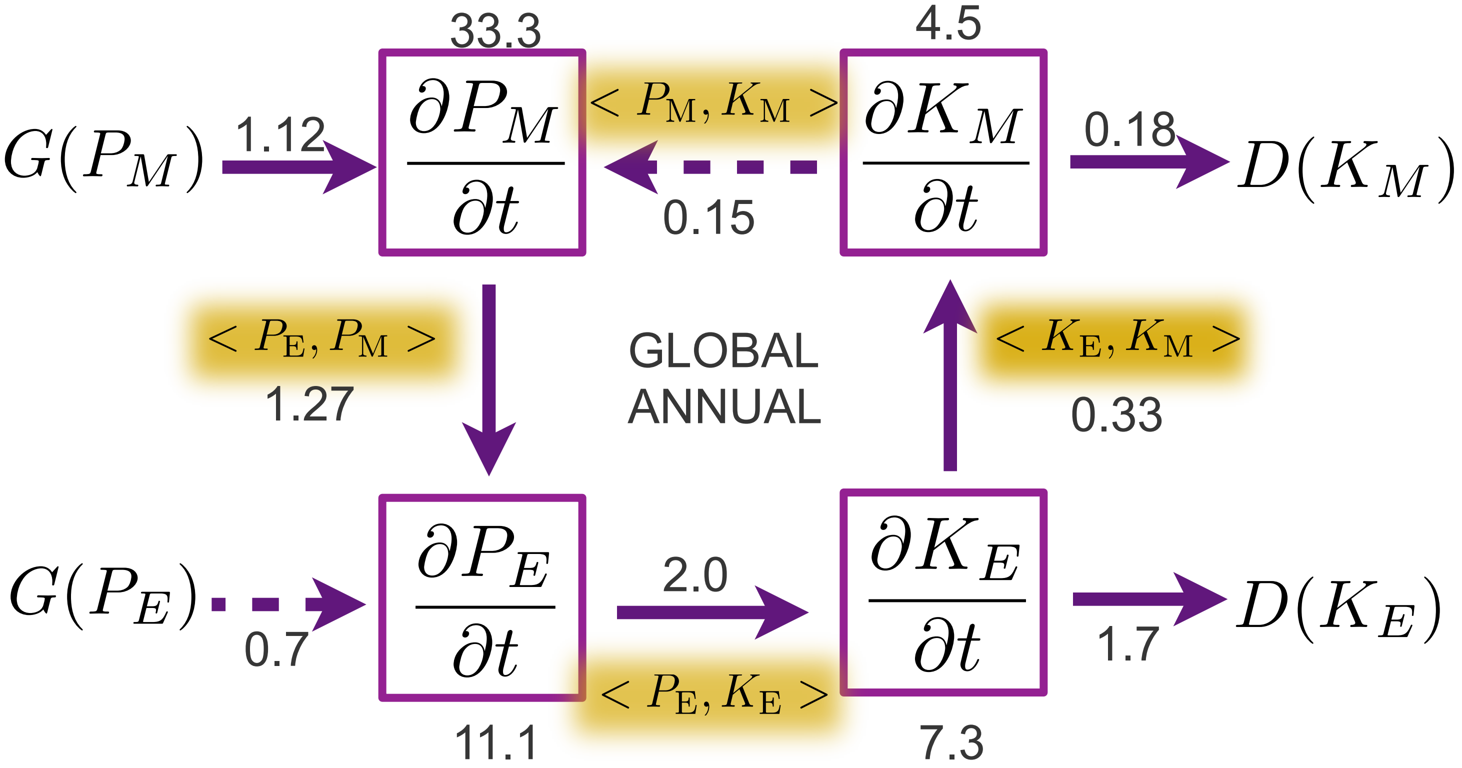
Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in real atmosphere:

energy: $10^5 Jm^{-2}$

conversion: Wm^{-2}



33.3

$\frac{\partial P_M}{\partial t}$

4.5

$\frac{\partial K_M}{\partial t}$

$G(P_M)$ 1.12

$< P_{\mathrm{M}}, K_{\mathrm{M}} >$ 0.15

0.18 $D(K_M)$

$< P_{\mathrm{E}}, P_{\mathrm{M}} >$ 1.27

GLOBAL ANNUAL

$< K_{\mathrm{E}}, K_{\mathrm{M}} >$ 0.33

$G(P_E)$ 0.7

$\frac{\partial P_E}{\partial t}$

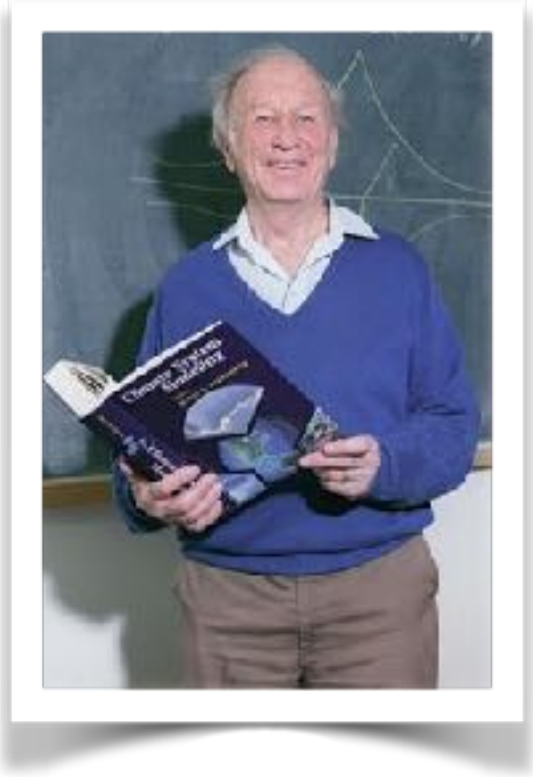
2.0 $< P_{\mathrm{E}}, K_{\mathrm{E}} >$

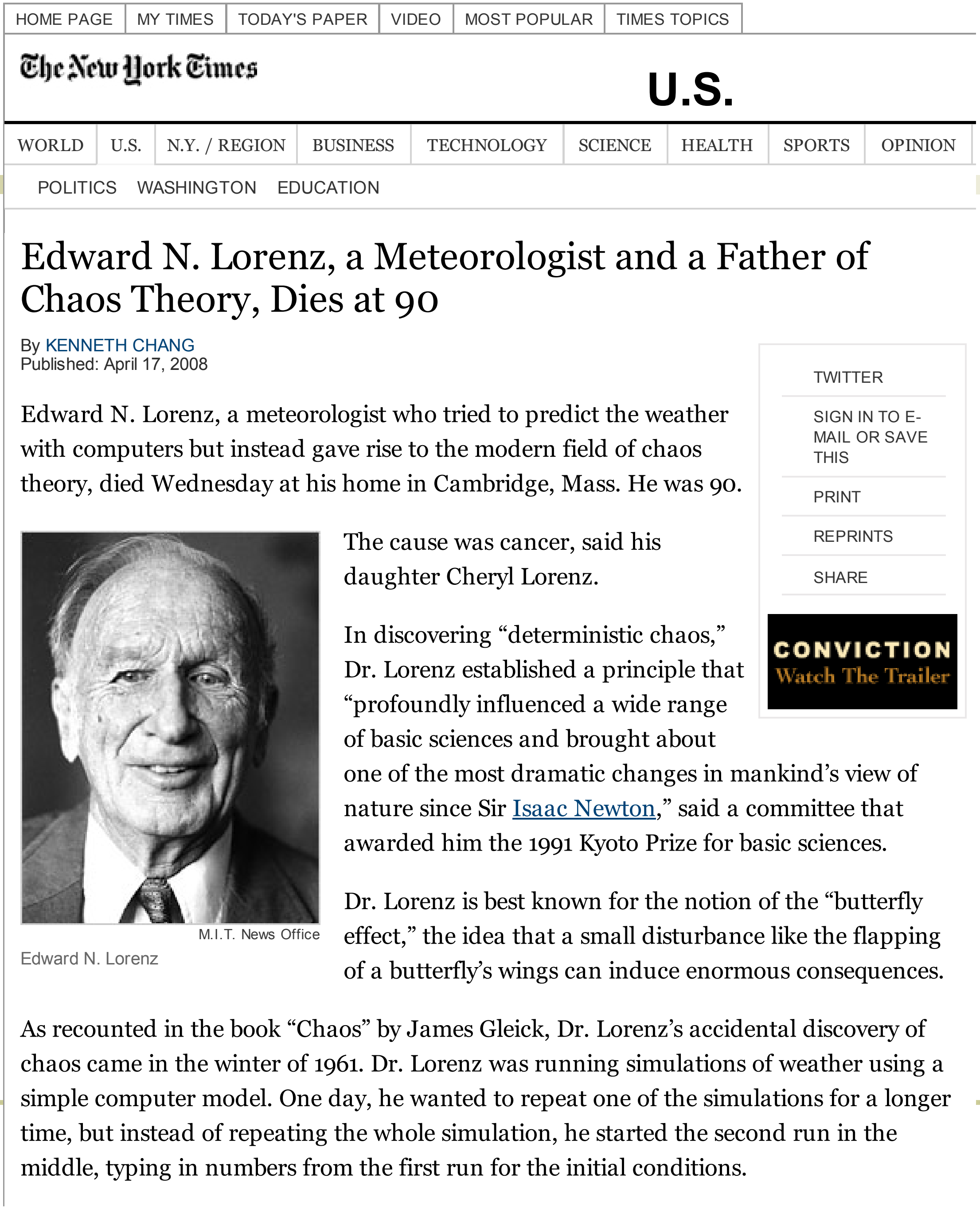
$\frac{\partial K_E}{\partial t}$

1.7 $D(K_E)$

11.1

7.3





Edward N. Lorenz, a Meteorologist and a Father of Chaos Theory, Dies at 90

By KENNETH CHANG
Published: April 17, 2008

Edward N. Lorenz, a meteorologist who tried to predict the weather with computers but instead gave rise to the modern field of chaos theory, died Wednesday at his home in Cambridge, Mass. He was 90.

M.I.T. News Office

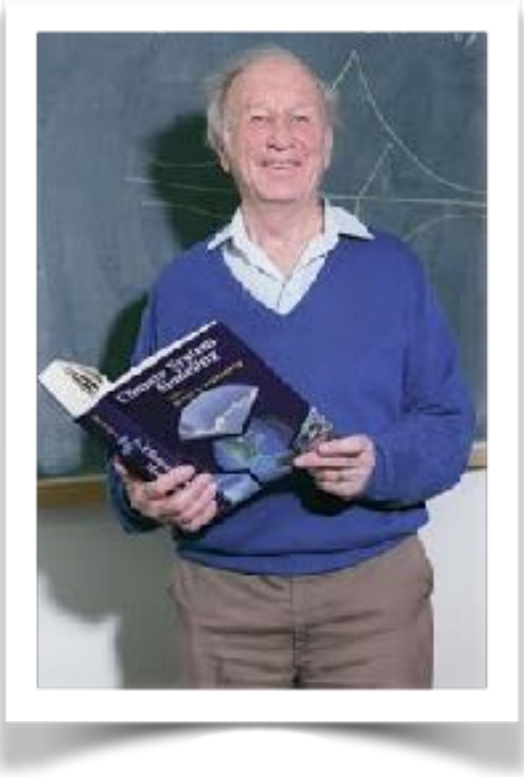
Edward N. Lorenz

The cause was cancer, said his daughter Cheryl Lorenz.

In discovering “deterministic chaos,” Dr. Lorenz established a principle that “profoundly influenced a wide range of basic sciences and brought about one of the most dramatic changes in mankind’s view of nature since Sir Isaac Newton,” said a committee that awarded him the 1991 Kyoto Prize for basic sciences.

Dr. Lorenz is best known for the notion of the “butterfly effect,” the idea that a small disturbance like the flapping of a butterfly’s wings can induce enormous consequences.

As recounted in the book “Chaos” by James Gleick, Dr. Lorenz’s accidental discovery of chaos came in the winter of 1961. Dr. Lorenz was running simulations of weather using a simple computer model. One day, he wanted to repeat one of the simulations for a longer time, but instead of repeating the whole simulation, he started the second run in the middle, typing in numbers from the first run for the initial conditions.



Summary & Discussion

- Observations
- The Ferrel Cell
- Baroclinic eddies
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction, E-P flux
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- The energy cycle

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1. The role of moisture;

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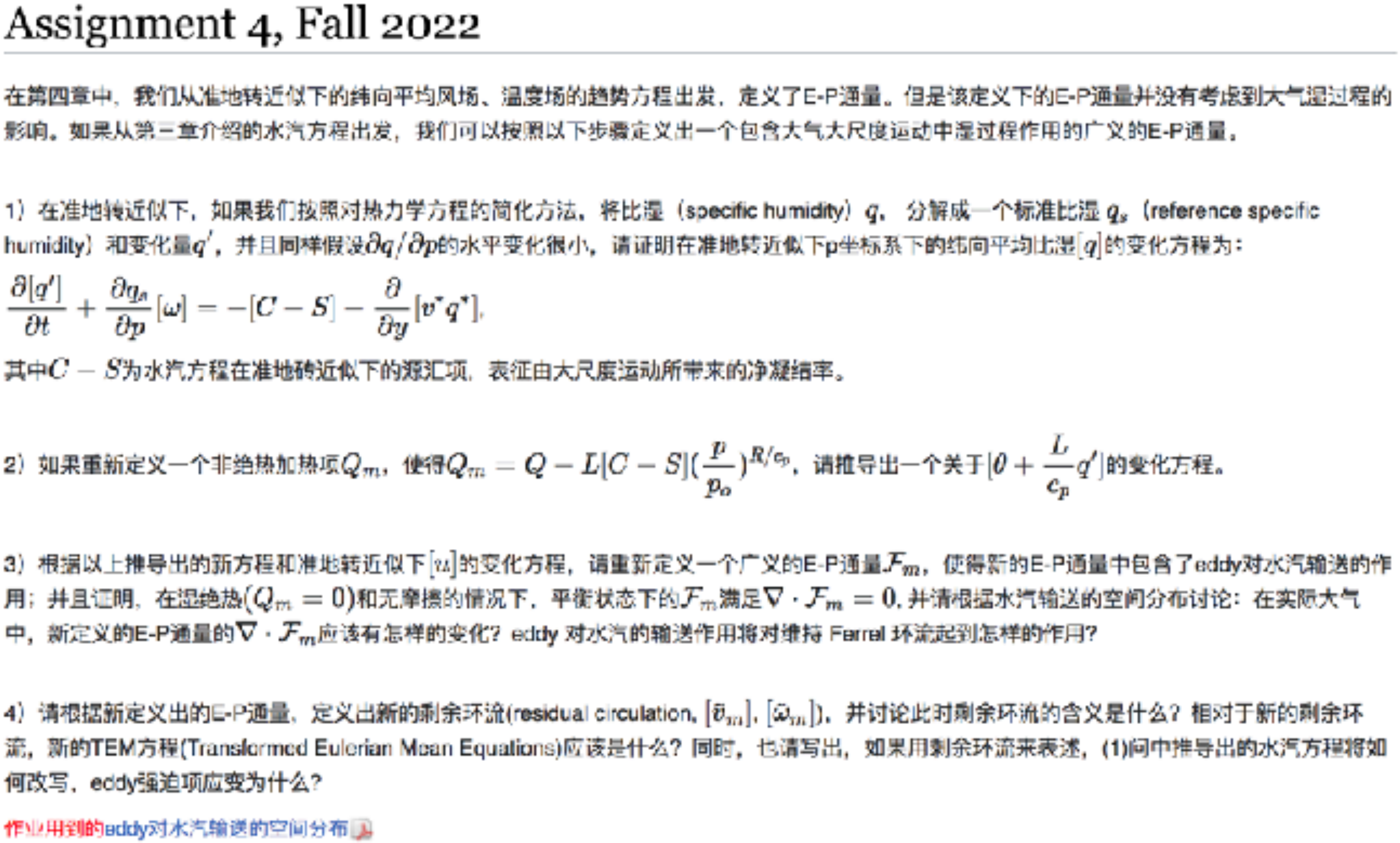
1. The role of moisture;

2. Quantify (parameterize) the relation between eddies and mean flow;

3. Zonal variations.

Due: 2022.12.12 (周一)



Assignment 4, Fall 2022

在第四章中，我们从准地转近似下的纬向平均风场、温度场的趋势方程出发，定义了E-P通量。但是该定义下的E-P通量并没有考虑到大气湿过程的影响。如果从第三章介绍的水汽方程出发，我们可以按照以下步骤定义出一个包含大气大尺度运动中湿过程作用的广义的E-P通量。

1）在准地转近似下，如果我们按照对热力学方程的简化方法，将比湿（specific humidity）q，分解成一个标准比湿 q_s（reference specific humidity）和变化量q'，并且同样假设$\partial q/\partial p$的水平变化很小，请证明在准地转近似下p坐标系下的纬向平均比湿$[q]$的变化方程为：

$$\frac{\partial[q']}{\partial t}+\frac{\partial q_s}{\partial p}[\omega]=-[C-S]-\frac{\partial}{\partial y}[v^*q^*],$$

其中$C-S$为水汽方程在准地砖近似下的源汇项，表征由大尺度运动所带来的净凝结率。

2）如果重新定义一个非绝热加热项Q_m，使得$Q_m=Q-L[C-S](\frac{p}{p_o})^{R/c_p}$，请推导出一个关于$[\theta+\frac{L}{c_p}q']$的变化方程。

3）根据以上推导出的新方程和准地转近似下$[u]$的变化方程，请重新定义一个广义的E-P通量$\mathcal{F}_m$，使得新的E-P通量中包含了eddy对水汽输送的作用；并且证明，在湿绝热$(Q_m=0)$和无摩擦的情况下，平衡状态下的$\mathcal{F}_m$满足$\nabla\cdot\mathcal{F}_m=0$，并请根据水汽输送的空间分布讨论：在实际大气中，新定义的E-P通量的$\nabla\cdot\mathcal{F}_m$应该有怎样的变化？eddy 对水汽的输送作用将对维持 Ferrel 环流起到怎样的作用？

4）请根据新定义出的E-P通量，定义出新的剩余环流(residual circulation, $[\bar{v}_m]$, $[\bar{\omega}_m]$)，并讨论此时剩余环流的含义是什么？相对于新的剩余环流，新的TEM方程(Transformed Eulerian Mean Equations)应该是什么？同时，也请写出，如果用剩余环流来表述，(1)问中推导出的水汽方程将如何改写，eddy强迫项应变为什么？

作业用到的eddy对水汽输送的空间分布

授课教师：张洋