Phase Transition of Hypergraph Matchings

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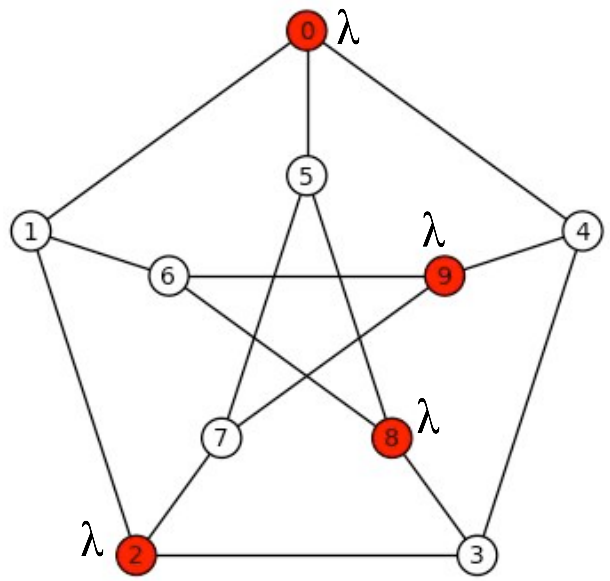
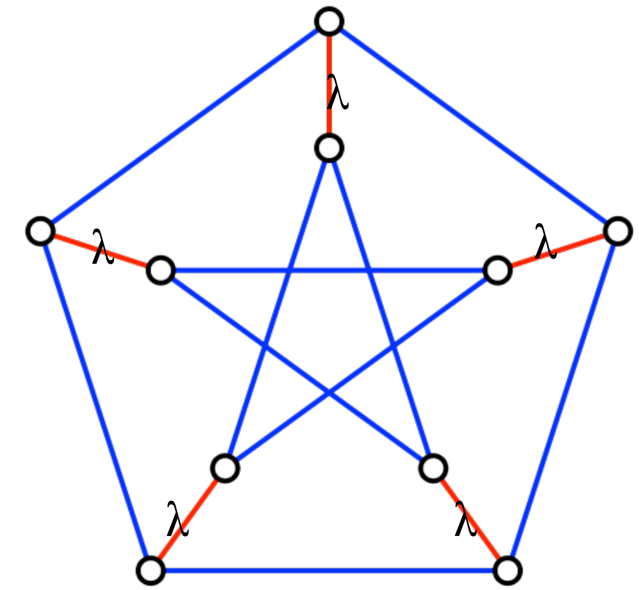
<table>
<tr><th></th><th>hardcore model</th><th>monomer-dimer model</th></tr>
<tr><td>undirected graph
$G = (V, E)$
activity λ</td><td>
</td><td>
</td></tr>
<tr><td>configurations:</td><td>independent sets I</td><td>matchings M</td></tr>
<tr><td>weight:</td><td>$w(I) = \lambda^{|I|}$</td><td>$w(M) = \lambda^{|M|}$</td></tr>
<tr><td>partition function:</td><td>$Z = \sum_{I:\text{independent sets in } G} w(I)$</td><td>$Z = \sum_{M:\text{matchings in } G} w(M)$</td></tr>
<tr><td>Gibbs distribution:</td><td>$\mu(I) = w(I) / Z$</td><td>$\mu(M) = w(M) / Z$</td></tr>
</table>

approximate counting: FPTAS/FPRAS for Z

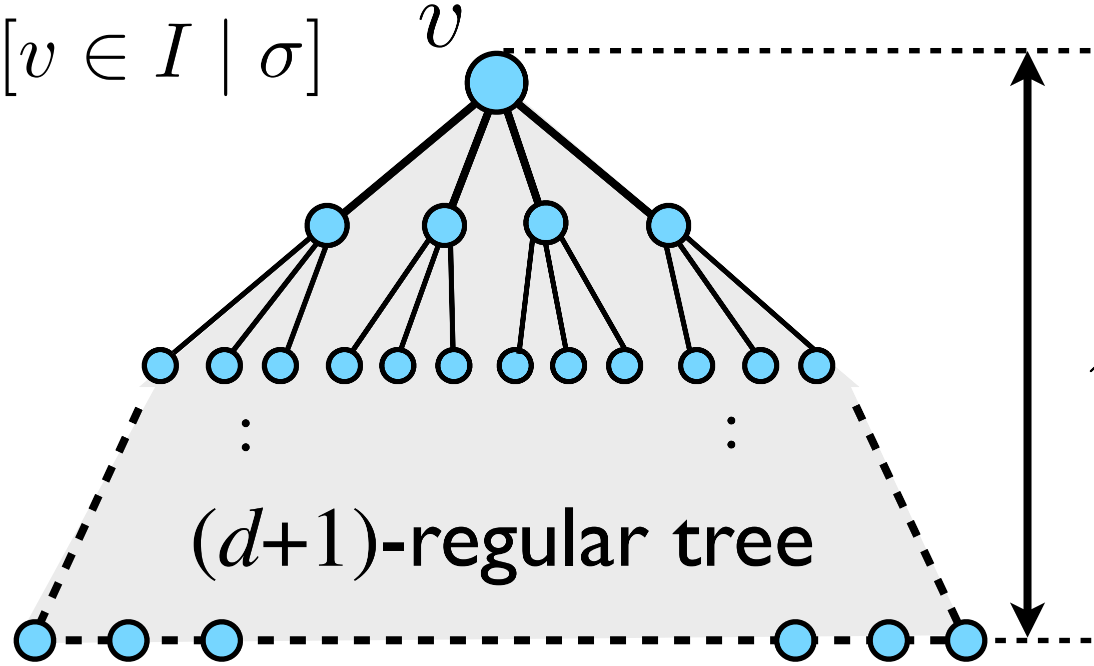
sampling: sampling from μ within TV-distance ε in time $\text{poly}(n, \log 1/\varepsilon)$

Decay of Correlation

(Weak Spatial Mixing, WSM)

hardcore model:

$I \sim \mu$



boundary condition σ : fixing leaves at level l to be *occupied*/*unoccupied* by I

WSM: $\Pr[v \in I \mid \sigma]$ does not depend on σ when $l \to \infty$

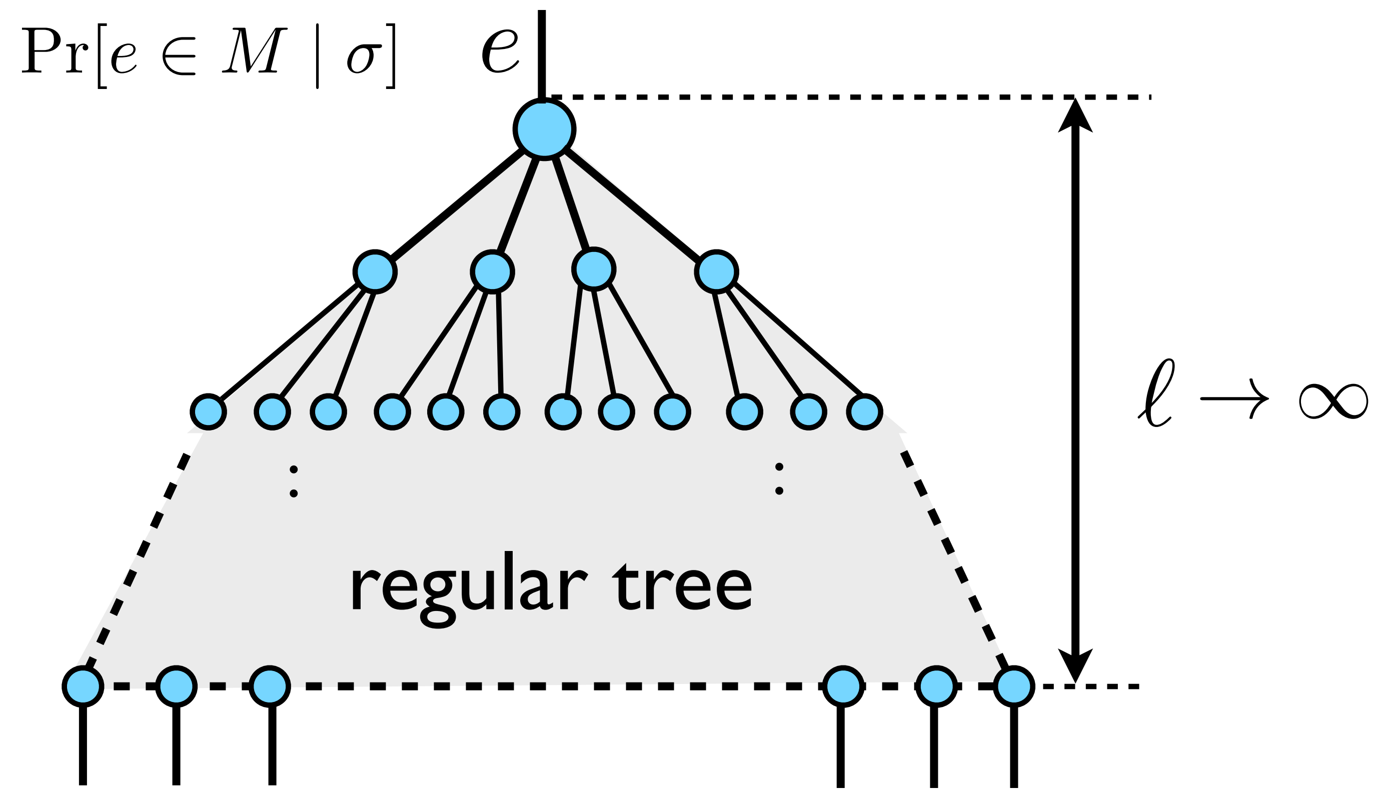
uniqueness threshold: $\lambda_c = \frac{d^d}{(d-1)^{(d+1)}}$

- $\lambda \le \lambda_c \Leftrightarrow$ WSM holds on $(d+1)$-regular tree $\Leftrightarrow$ Gibbs measure is unique
- [Weitz '06]: $\lambda < \lambda_c \Rightarrow$ FPTAS for graphs with max-degree $\le d+1$
- [Galanis, Štefankovič, Vigoda '12; Sly, Sun '12]: $\lambda > \lambda_c \Rightarrow$ inapproximable unless NP=RP

Decay of Correlation
(Weak Spatial Mixing, WSM)

monomer-dimer model:

$M \sim \mu$

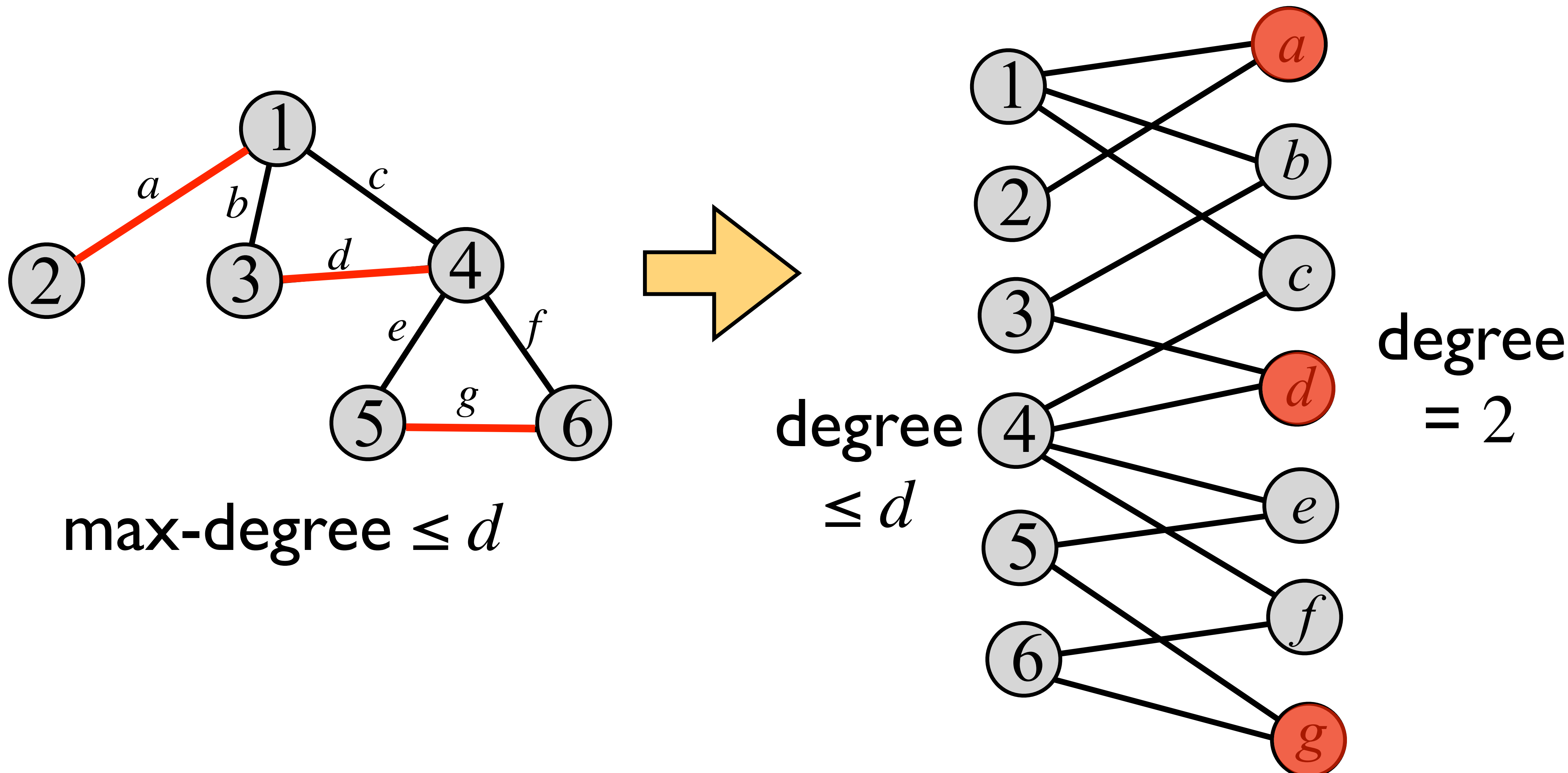


boundary condition σ : fixing leaf-edges at level l to be *occupied*/*unoccupied* by M

WSM: $\Pr[e \in M \mid \sigma]$ does not depend on σ when $l \to \infty$

- WSM always holds ⇔ Gibbs measure is always unique
- [Jerrum, Sinclair '89]: FPRAS for all graphs
- [Bayati, Gamarnik, Katz, Nair, Tetali '08]: FPTAS for graphs with bounded max-degree

CSP (Constraint Satisfaction Problem)

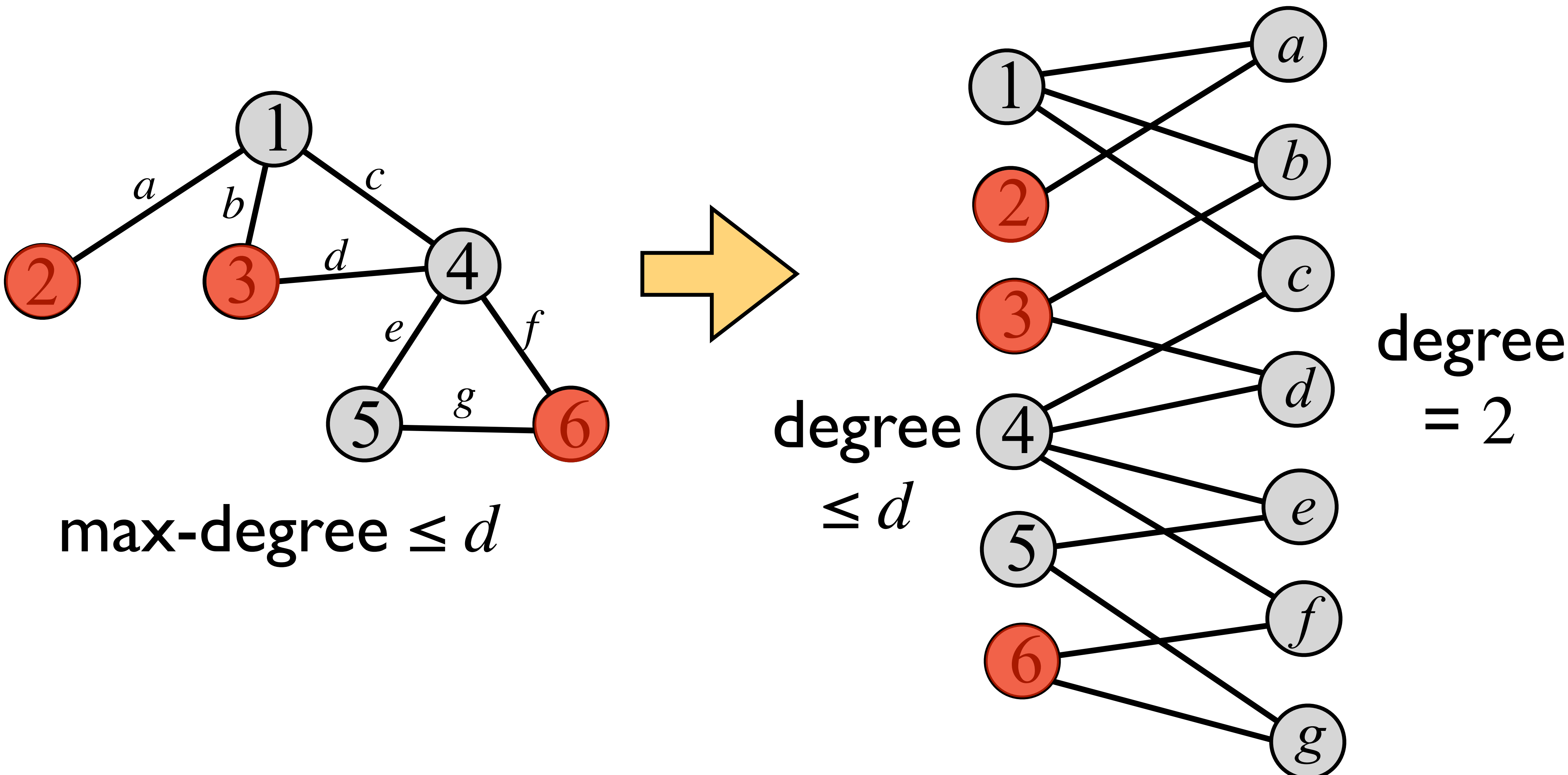


matchings:

matching constraint (at-most-1)

variables $x_i \in \{0,1\}$

CSP (Constraint Satisfaction Problem)



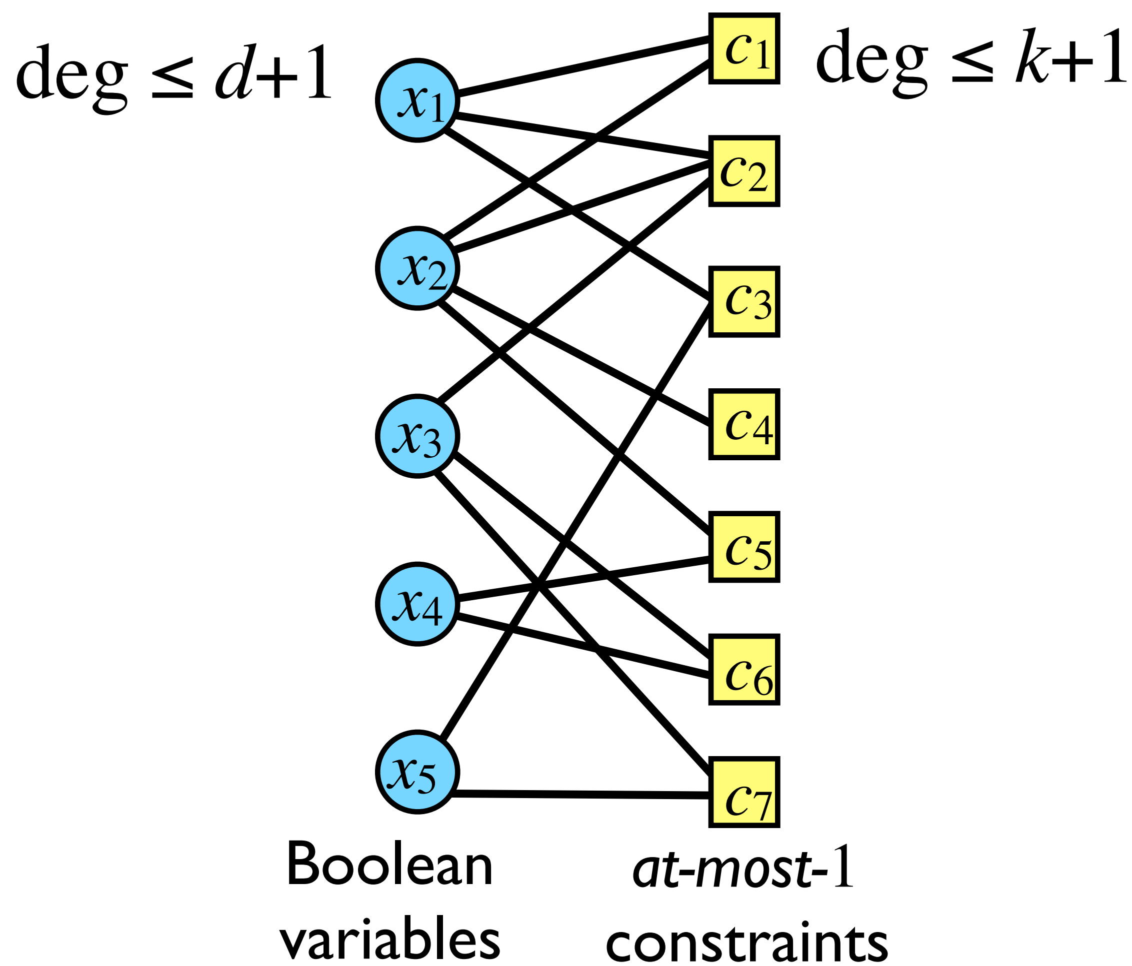
matchings: matching constraint (at-most-1); variables $x_i \in \{0,1\}$

independent sets: variables $x_i \in \{0,1\}$; matching constraint (at-most-1)

partition function:

$$Z = \sum_{\substack{\vec{x}\in\{0,1\}^n \text{ satisfying} \\ \text{all constraints}}} \lambda^{\|\vec{x}\|_1}$$

CSP (Constraint Satisfaction Problem)



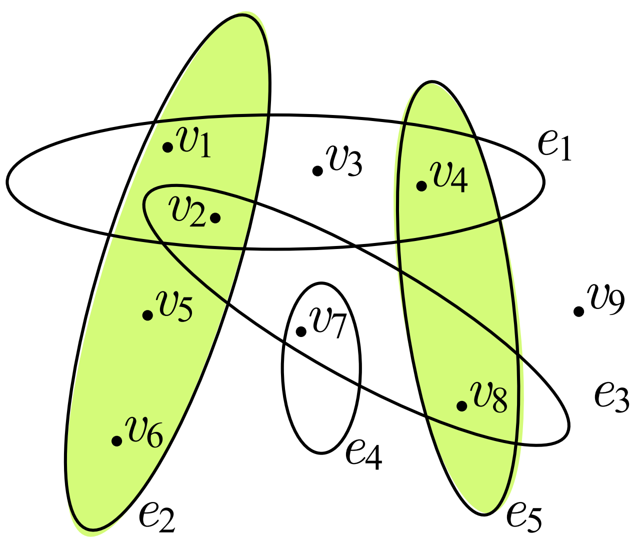
partition function: $$Z = \sum_{\substack{\vec{x}\in\{0,1\}^n \text{ satisfying} \\ \text{all constraints}}} \lambda^{\|\vec{x}\|_1}$$

Hypergraph matching

hypergraph $\mathcal{H}=(V,E)$ vertex set V

hyperedge $e\in E, \quad e\subset V$

a *matching* is an subset $M\subset E$ of *disjoint* hyperedges

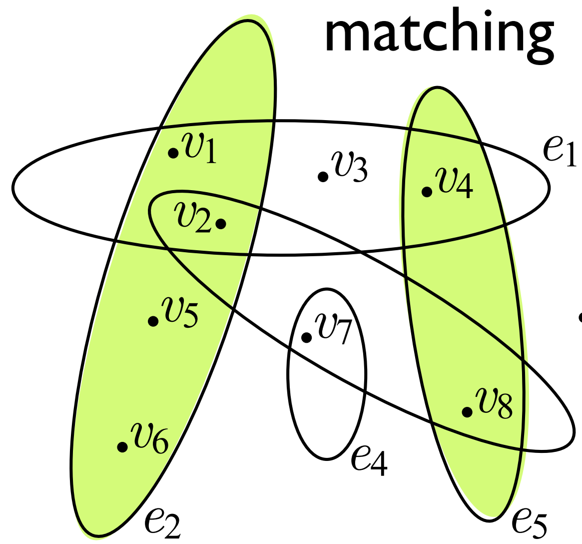


partition functions: $$Z_\lambda(\mathcal{H})=\sum_{M:\text{ matching of }\mathcal{H}}\lambda^{|M|}$$

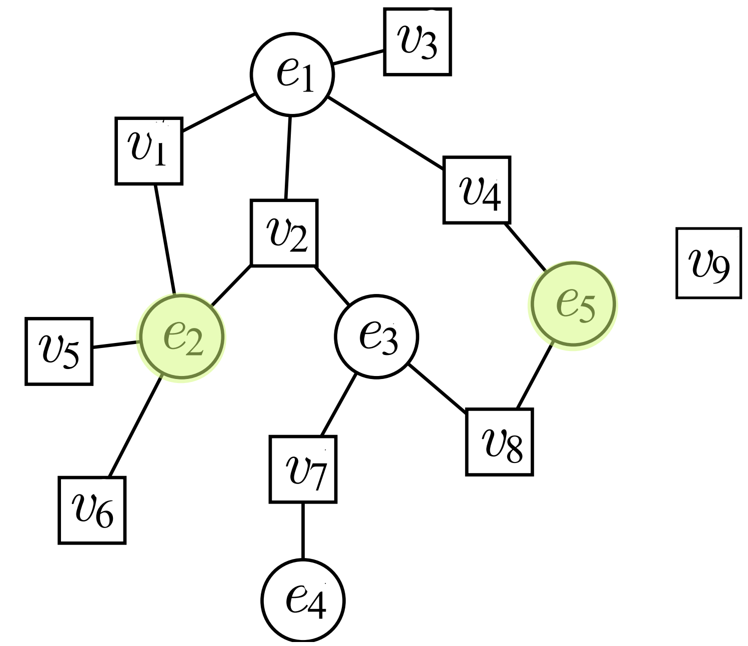
Gibbs distribution: $$\mu(M)=\frac{\lambda^{|M|}}{Z_\lambda(\mathcal{H})}$$

matchings in hypergraphs of max-degree $\le k+1$ and max-edge-size $\le d+1$

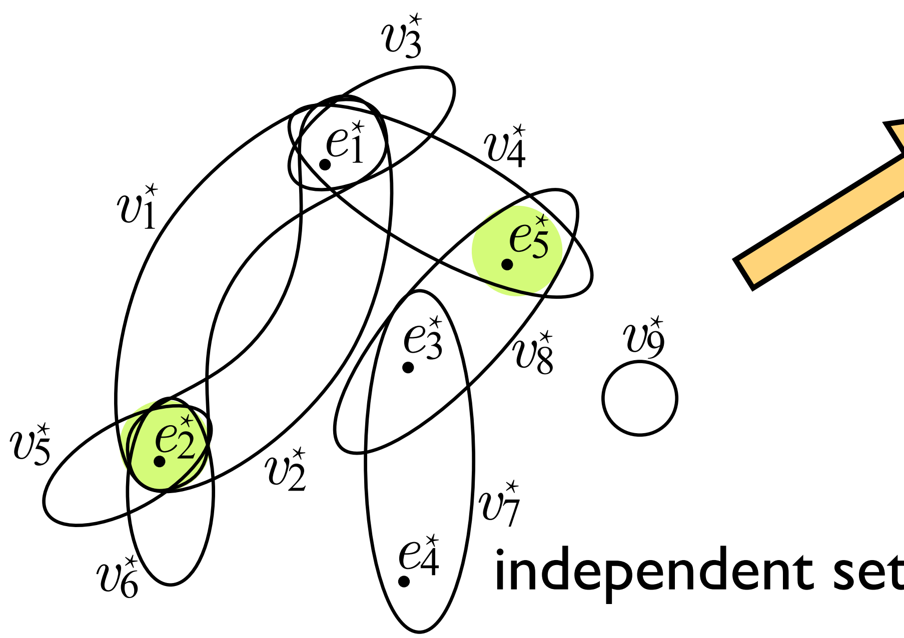
primal:



incidence graph



dual:



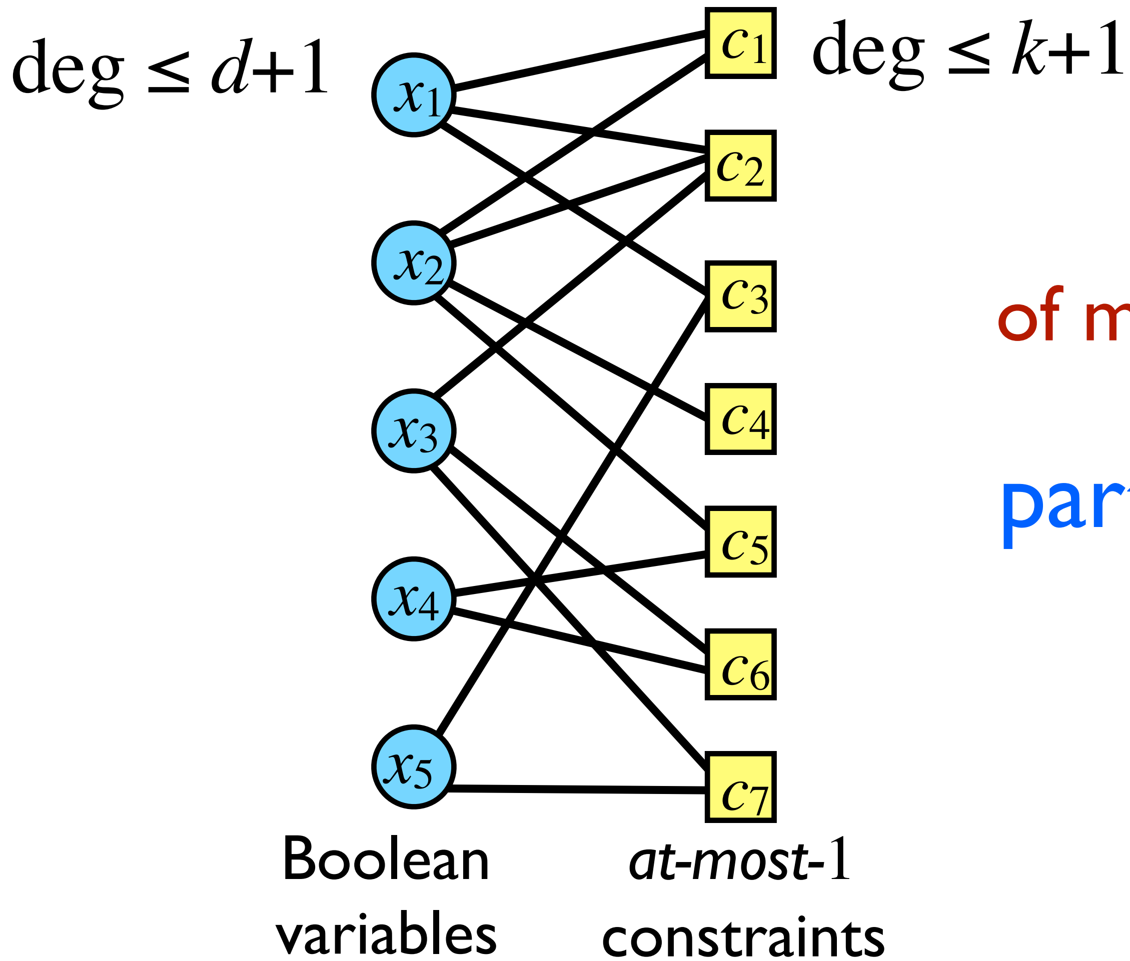
CSP defined by
matching(*packing*) constraint

independent sets in hypergraphs of max-degree $\le d+1$ and max-edge-size $\le k+1$

independent sets: a subset of non-adjacent vertices

(to be distinguished with: vertex subsets containing no hyperedge as subset)

Known results



independent sets of hypergraphs
of max-degree ≤ $d+1$ and max-edge-size ≤ $k+1$

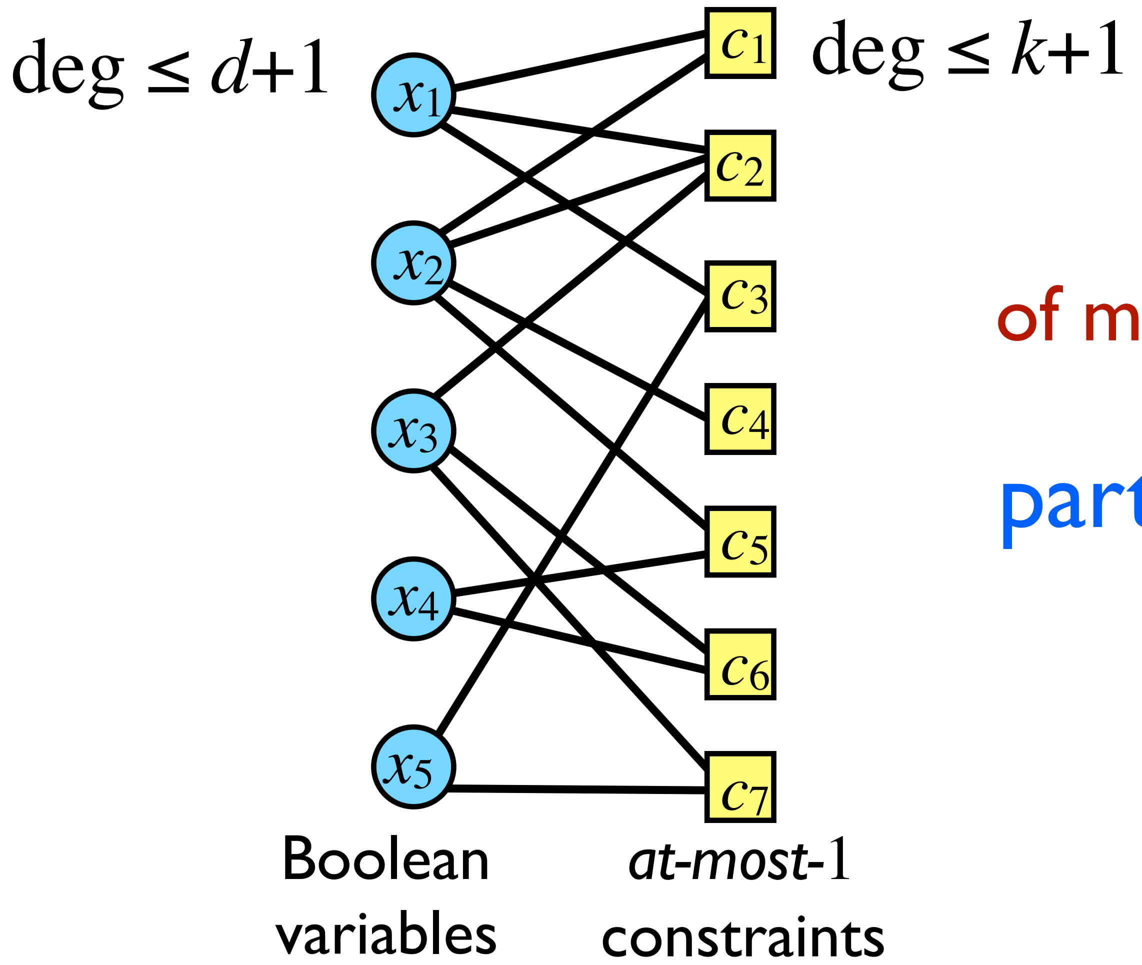
partition function:

$$Z = \sum_{\substack{\vec{x}\in\{0,1\}^n \text{ satisfying} \\ \text{all constraints}}} \lambda^{\|\vec{x}\|_1}$$

Classification of approximability in terms of λ, d, k ?

- $k=1$: hardcore model
- $d=1$: monomer-dimer model
- for $\lambda=1$:
  - [Dudek, Karpinski, Rucinski, Szymanska 2014]: FPTAS when $d=2$, $k\leq 2$
  - [Liu and Lu 2015] FPTAS when $d=2$, $k\leq 3$

Our Results



independent sets of hypergraphs
of max-degree $\le d+1$ and max-edge-size $\le k+1$

partition function:

$$Z = \sum_{\substack{\vec{x}\in\{0,1\}^n \text{ satisfying} \\ \text{all constraints}}} \lambda^{\|\vec{x}\|_1}$$

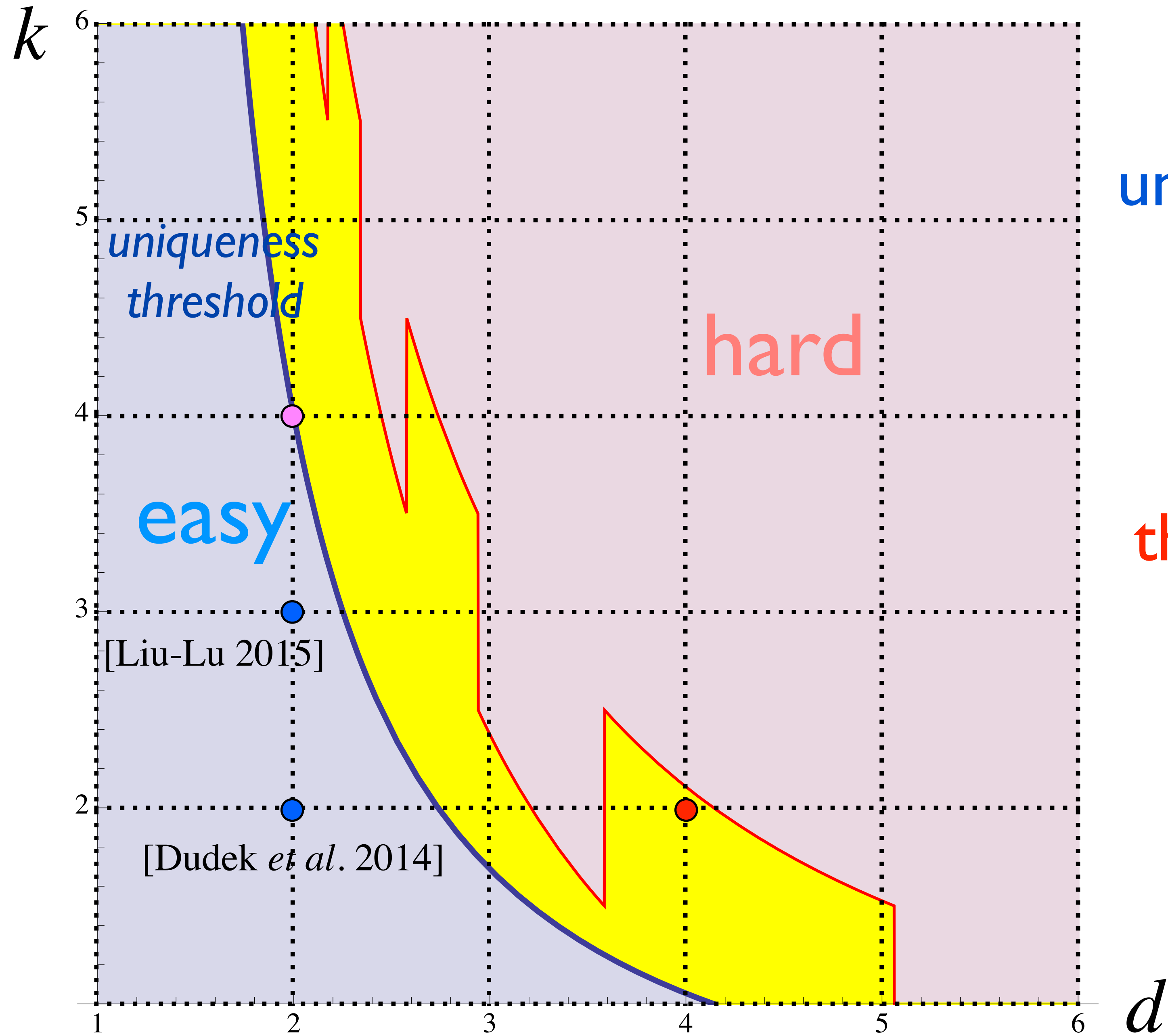
- uniqueness threshold for $(k+1)$-uniform $(d+1)$-regular infinite hypertree:

$$\lambda_c(k,d) = \frac{d^d}{k(d-1)^{d+1}}$$

- $\lambda<\lambda_c$: FPTAS
- $\lambda > \frac{2k+1+(-1)^k}{k+1}\lambda_c \approx 2\lambda_c$: inapproximable unless NP=RP

$\lambda = 1$: matchings of hypergraphs of max-degree $(k+1)$ and max-edge-size $(d+1)$

independent sets of hypergraphs of max-degree $(d+1)$ and max-edge-size $(k+1)$



uniqueness threshold:

$$\lambda_c = \frac{d^d}{k(d-1)^{(d+1)}}$$

threshold for hardness:

$$\frac{2k+1+(-1)^k}{k+1}\lambda_c \approx 2\lambda_c$$

(4,2): independent sets of 3-uniform hypergraphs of max-degree 5,
the only open case for counting Boolean CSP with max-degree 5.

(2,4): matchings of 3-uniform hypergraphs of max-degree 5,
exact at the critical threshold: $\frac{d^d}{k(d-1)^{(d+1)}} = \frac{2^2}{4 \cdot 1^5} = 1$

Spatial Mixing (Decay of Correlation)

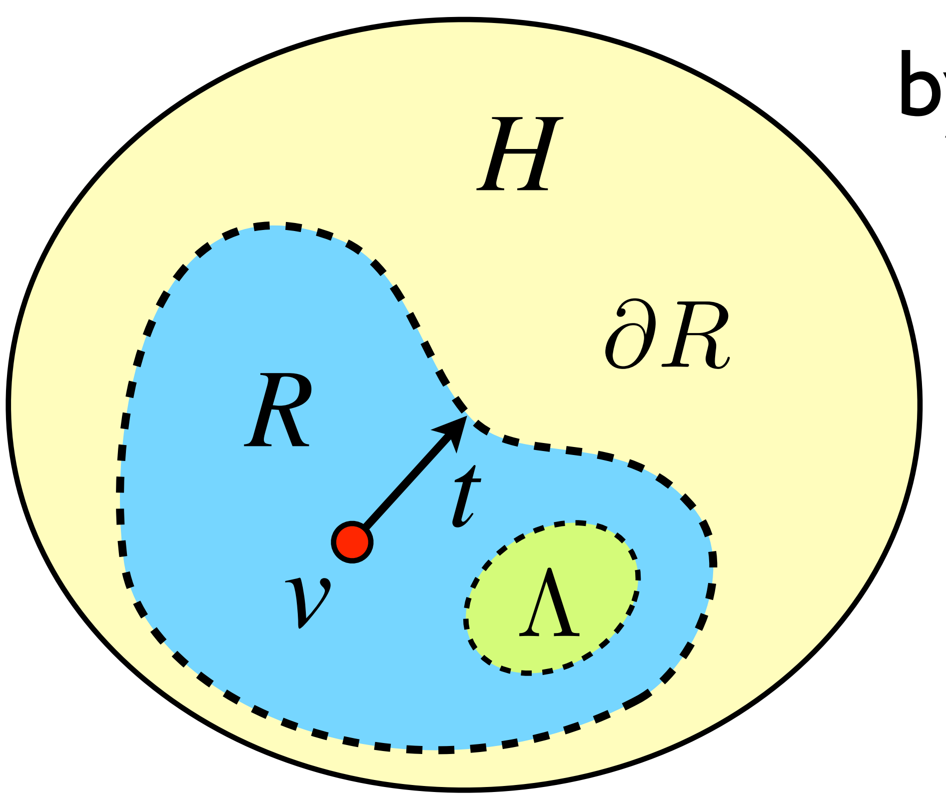
weak spatial mixing (WSM):

$$\Pr[v \text{ is occupied} \mid \sigma_{\partial R}] \approx \Pr[v \text{ is occupied} \mid \tau_{\partial R}]$$

error < exp (-t)

strong spatial mixing (SSM):

$$\Pr[v \text{ is occupied} \mid \sigma_{\partial R}, \sigma_\Lambda] \approx \Pr[v \text{ is occupied} \mid \tau_{\partial R}, \sigma_\Lambda]$$



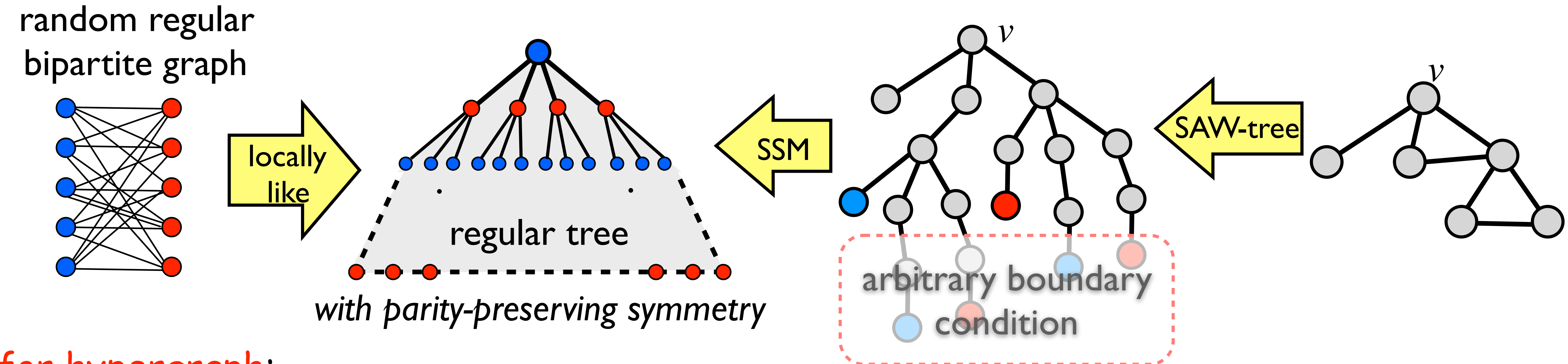
by self-reduction:

$$\Pr[v \text{ is occupied} \mid \sigma_\Lambda]$$

is approximable with additive error ε in time poly(n, 1/ε)

FPTAS for partition function Z

Hardcore model:



for hypergraph:

Similar...

- on infinite regular tree: Gibbs measure is unique ⟺ *semi-translation invariant* (invariant under *parity-preserving* automorphisms) Gibbs measure is unique

Yes.

- **algorithm**: Gibbs measure is unique on regular tree ⟺(generic) WSM on regular tree ⟺ SSM on trees

  self-avoiding walk (SAW) tree ⟹ { SSM on graphs; FPTAS for graphs }

No.

- **hardness**: a sequence of finite graphs G_n (random regular bipartite graph) is *locally like* the infinite regular tree
  - a sequence of *labeled* G_n *locally converges* to the infinite regular tree with *parity labeling*

Theorem: $\lambda \le \lambda_c(k,d) = \dfrac{d^d}{k(d-1)^{d+1}}$

⇒ WSM holds for $(k+1)$-uniform $(d+1)$-regular hypertree

Theorem: on infinite uniform regular hypertree

WSM ⇒ SSM

Theorem:

on infinite (k,d)-hypertree ⇒ for $(\le k, \le d)$-hypergraphs

SSM ⇒ SSM with the same rate

SSM with exponential rate ⇒ FPTAS

all statements are for hypergraph independent sets

Tree Recursion

let $R_T = \frac{\Pr[v \text{ is occupied } \mid \sigma]}{\Pr[v \text{ is unoccupied } \mid \sigma]}$

tree recursion:

$$R_T = \lambda \prod_{i=1}^{d} \frac{1}{1 + \sum_{j=1}^{k_i} R_{T_{ij}}}$$

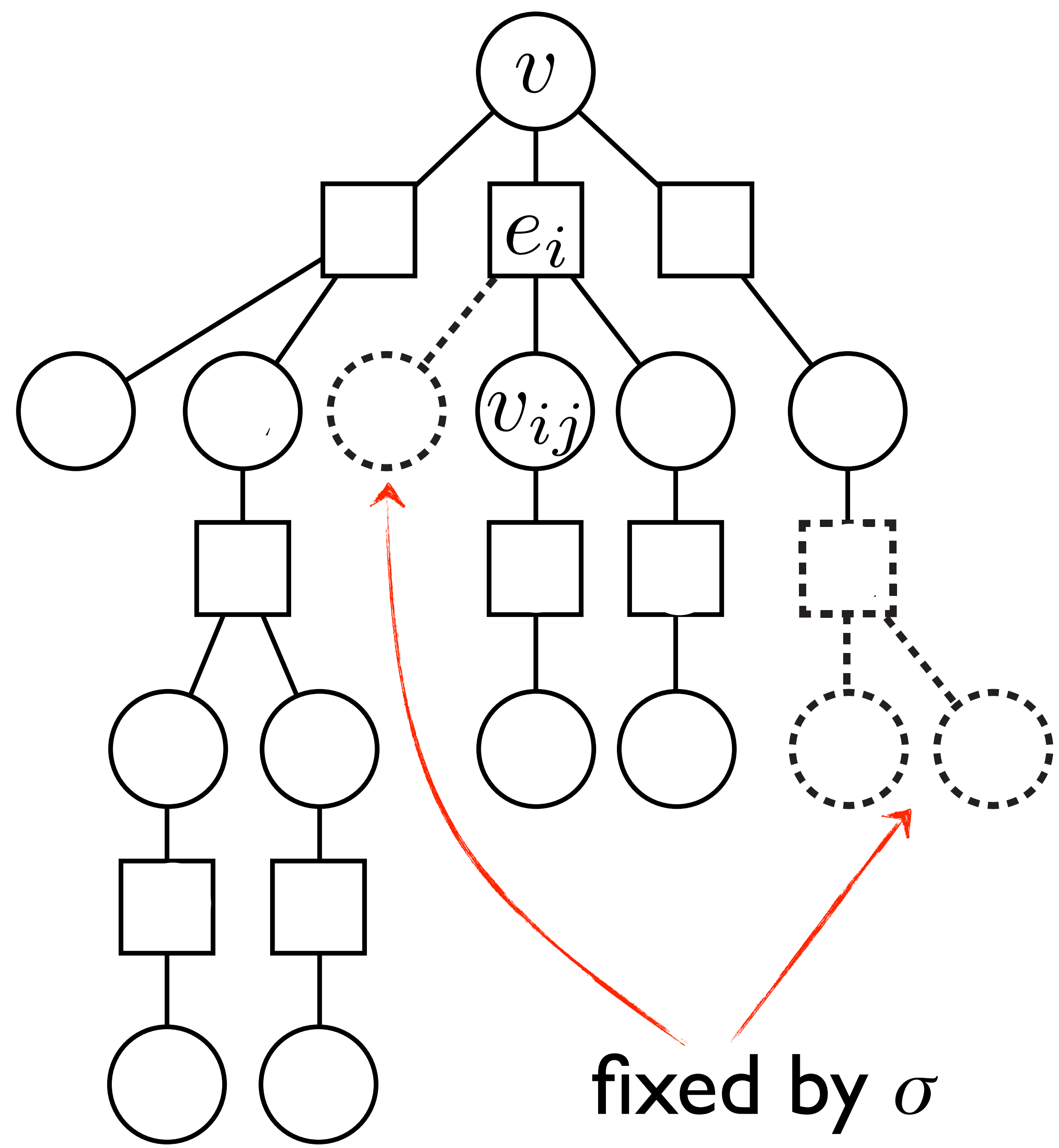
monomer-dimer model:

$$R_T = \frac{\lambda}{1 + \sum_{j=1}^{k} R_{T_j}}$$

hardcore model:

$$R_T = \lambda \prod_{i=1}^{d} \frac{1}{1 + R_{T_i}}$$

independent sets of hypertree T:



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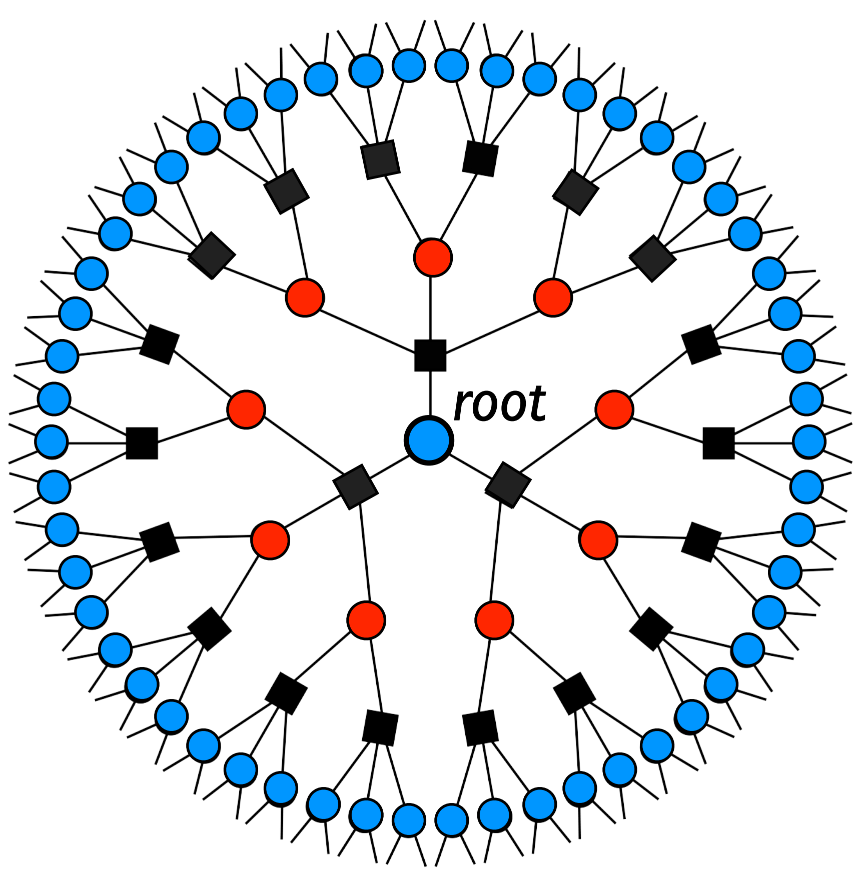
⇨ WSM holds for $(k+1)$-uniform $(d+1)$-regular hypertree



monotonicity of the recursion

⇨ the 2 extremal boundaries at level-l are all occupied / all unoccupied

the recursion becomes $R_\ell = \lambda \prod_{i=1}^{d} \frac{1}{1 + kR_{\ell-1}}$

whose convergence is the same as hardcore model: $R'_\ell = \lambda' \prod_{i=1}^{d} \frac{1}{1 + R'_{\ell-1}}$

with activity $\lambda' = k\lambda$

Theorem: $\lambda \le \lambda_c(k,d) = \frac{d^d}{k(d-1)^{d+1}}$

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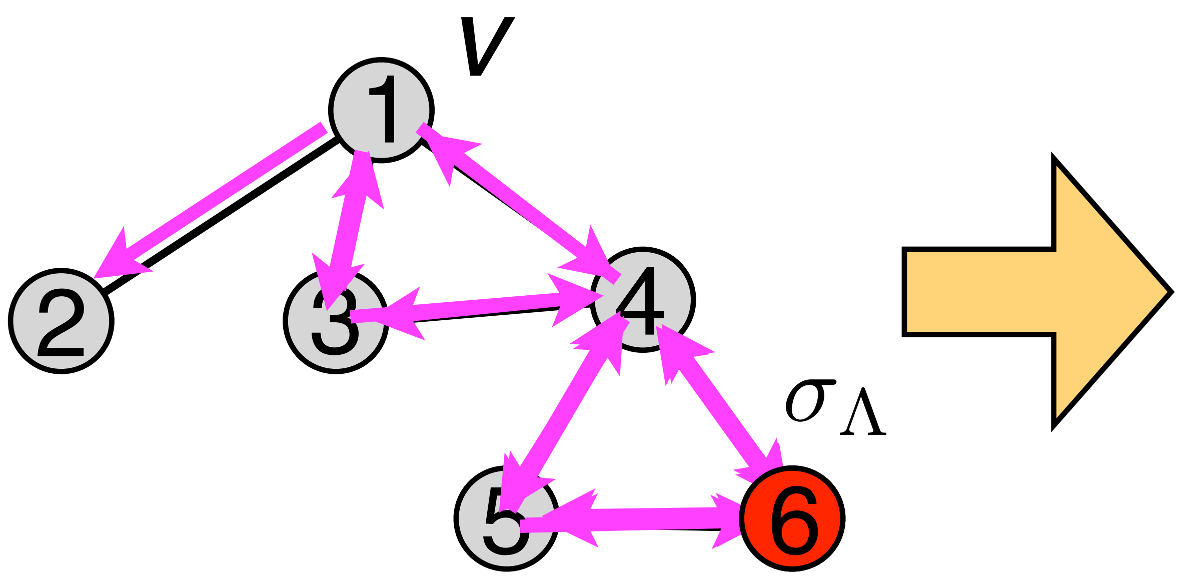
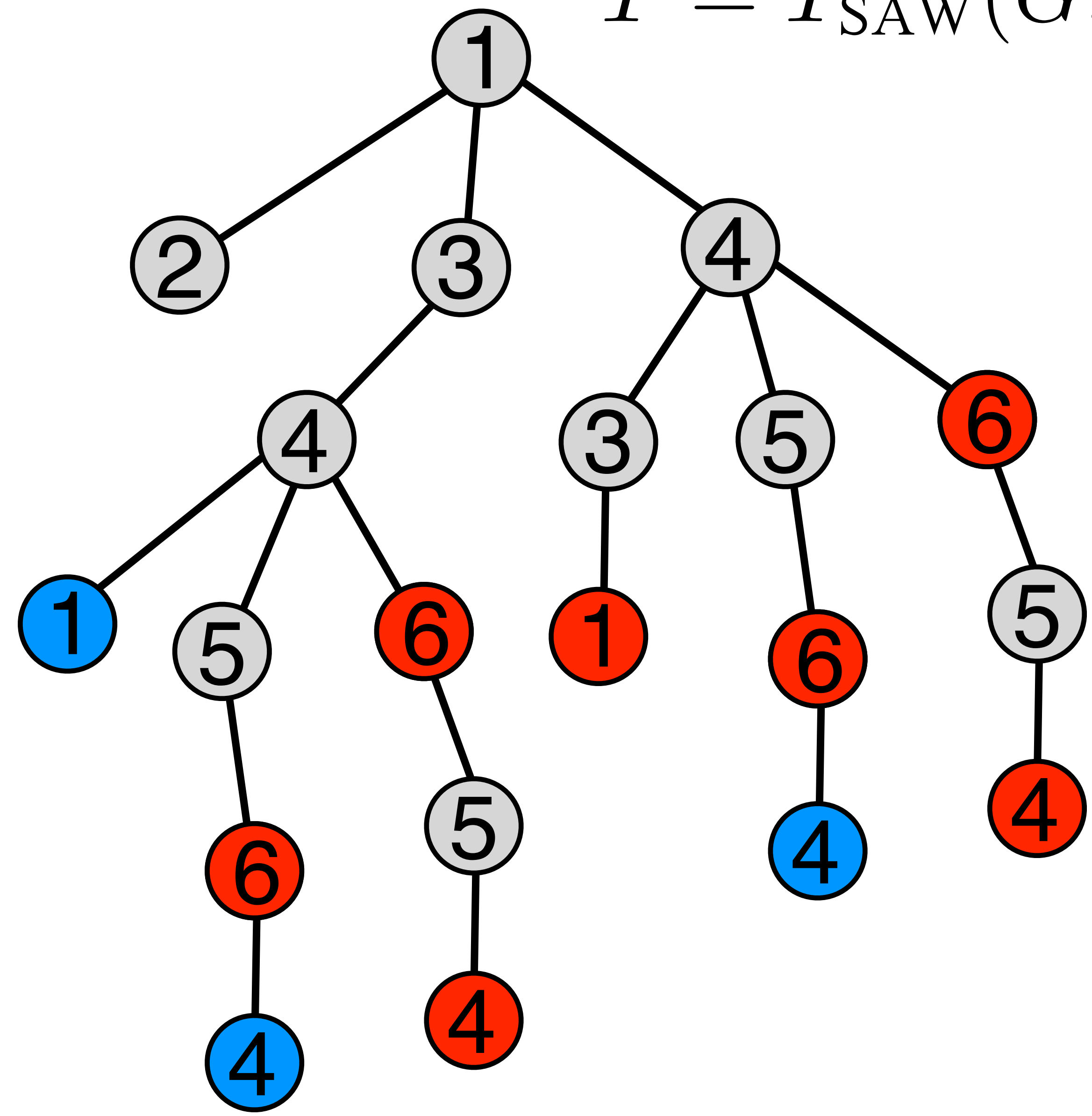
Self-Avoiding Walk Tree

(Weitz 2006)

$G=(V,E)$



$T = T_{\text{SAW}}(G, v)$



for hardcore:

$$\mathbb{P}_G[v \text{ is occupied } \mid \sigma_\Lambda]$$
$$= \mathbb{P}_T[v \text{ is occupied } \mid \sigma_\Lambda]$$

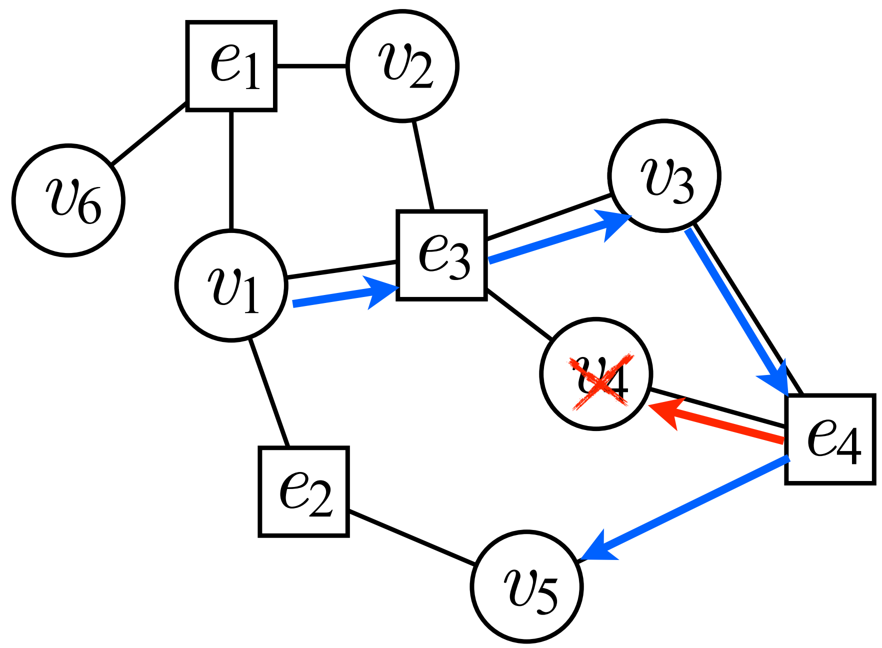
if cycle closing > cycle starting

if cycle closing < cycle starting

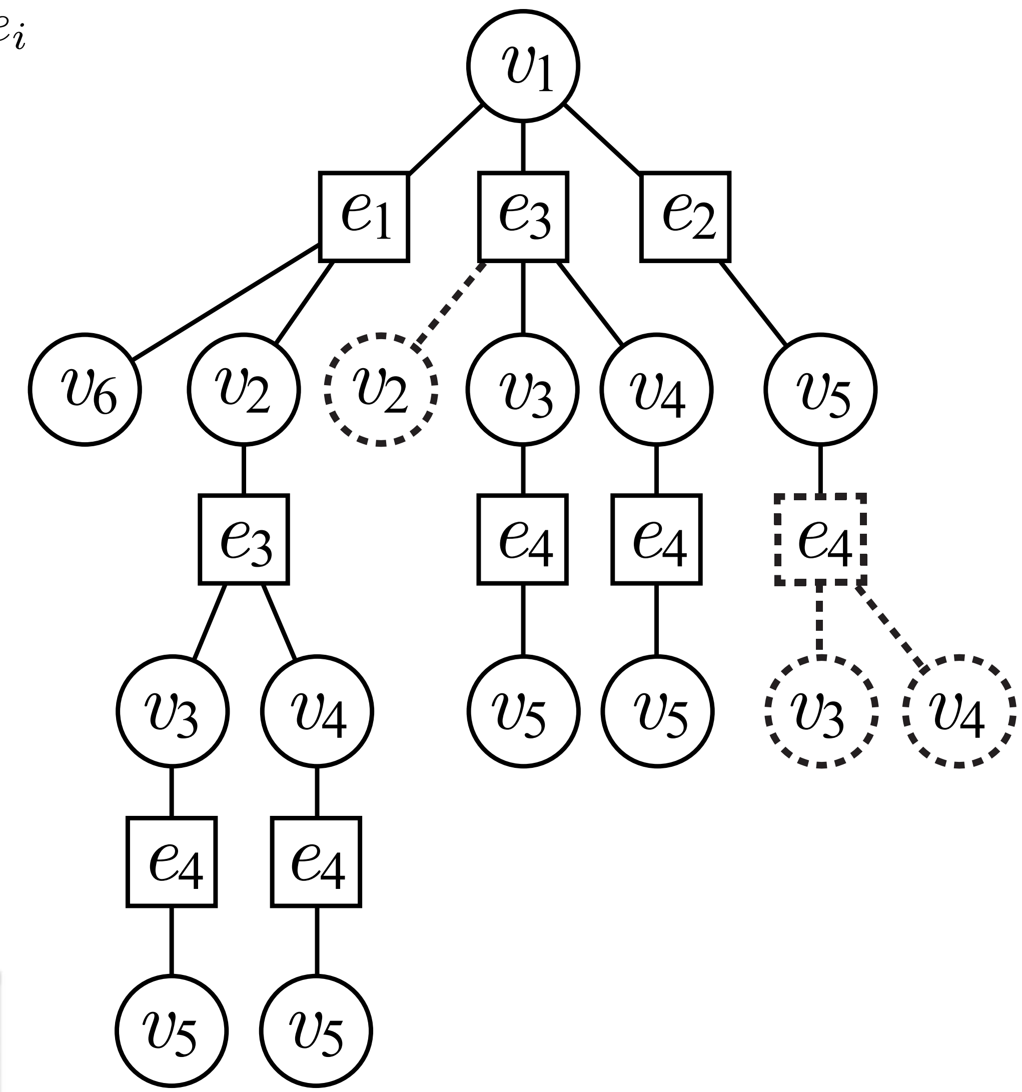
Hypergraph SAW Tree

self-avoiding walk(SAW): $(v_0, e_1, v_1, \ldots, e_\ell, v_\ell)$

is a simple path in incidence graph and $v_i \notin \bigcup_{j<i} e_i$

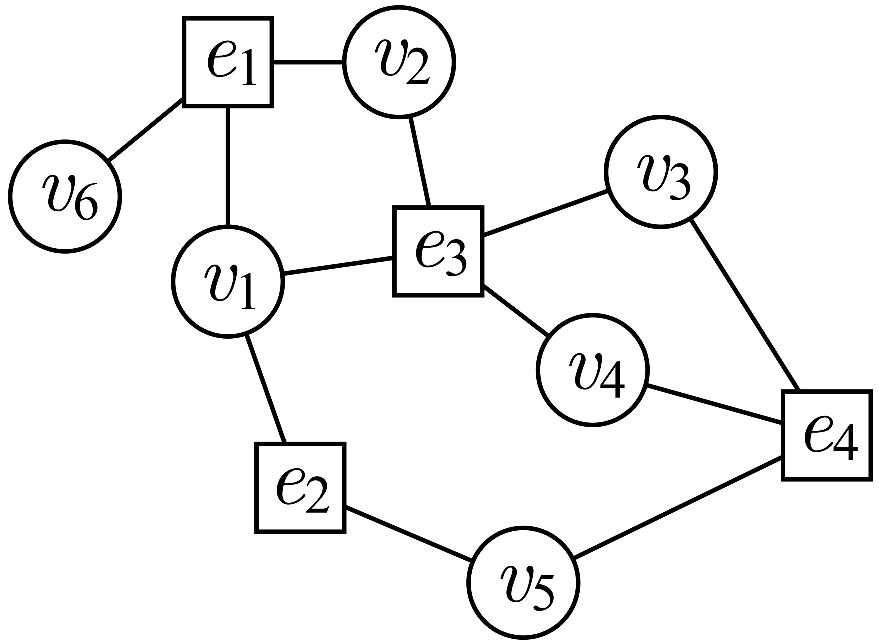


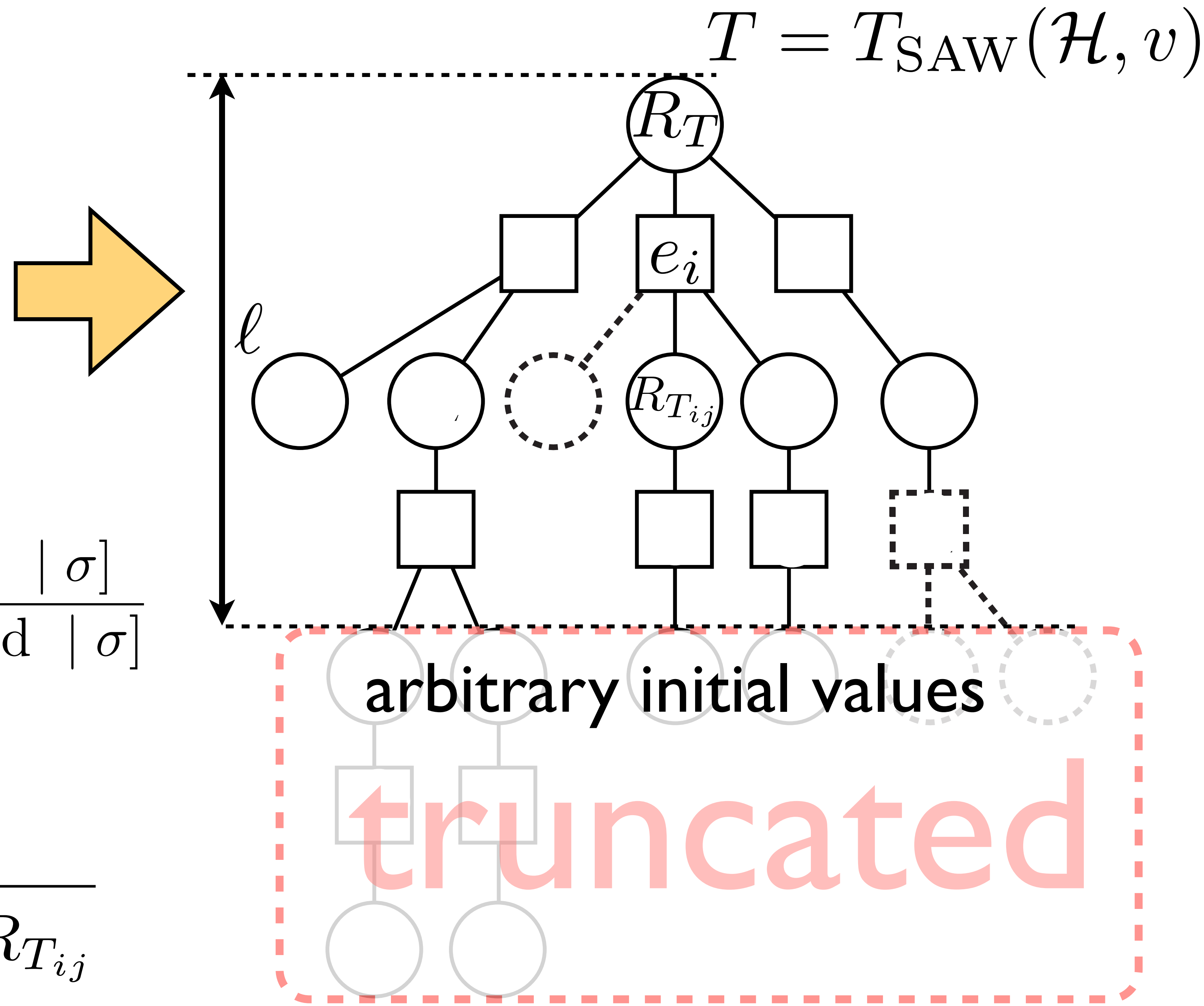
$$T = T_{\text{SAW}}(\mathcal{H}, v)$$



$$\mathbb{P}_{\mathcal{H}}[v \text{ is occupied } \mid \sigma]$$
$$= \mathbb{P}_{T}[v \text{ is occupied } \mid \sigma]$$

mark any cycle-closing vertex *unoccupied* if:
cycle-closing edge locally < cycle-starting edge
and *occupied* if otherwise





let $R_T = \dfrac{\Pr[v \text{ is occupied } \mid \sigma]}{\Pr[v \text{ is unoccupied } \mid \sigma]}$

tree recursion:

$$R_T = \lambda \prod_{i=1}^{d} \frac{1}{1 + \sum_{j=1}^{k_i} R_{T_{ij}}}$$

Theorem:

on infinite $(k+1,d+1)$-hypertree → for $(\leq k+1, \leq d+1)$-hypergraphs

SSM → SSM with the same rate

SSM with exponential rate → FPTAS

Theorem: $\lambda \le \lambda_c(k,d) = \frac{d^d}{k(d-1)^{d+1}}$

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Theorem: on infinite uniform regular hypertree

WSM ⇨ SSM

T : the infinite uniform regular hypertree

R_ℓ^+ : the max value of R_T conditioning on a boundary at level-l

R_ℓ^- : the min value of R_T conditioning on a boundary at level-l

$$R_\ell^{\pm} = \frac{\lambda}{(1 + kR_{\ell-1}^{\mp})^d}$$

$\vec{\lambda}$: the vector assigning each vertex a non-uniform activity $\leq\lambda$

$R_\ell^+(\vec{\lambda}), R_\ell^-(\vec{\lambda})$ are similarly defined

$$\frac{R_\ell^+(\vec{\lambda})}{R_\ell^-(\vec{\lambda})} \leq \frac{R_\ell^+}{R_\ell^-}$$

proved by induction on l with hypotheses:

$$\frac{R_\ell^+(\vec{\lambda})}{R_\ell^-(\vec{\lambda})} \leq \frac{R_\ell^+}{R_\ell^-} \quad \text{and} \quad \frac{1 + kR_\ell^+(\vec{\lambda})}{1 + kR_\ell^-(\vec{\lambda})} \leq \frac{1 + kR_\ell^+}{1 + kR_\ell^-}$$

sandwiching property: $R_\ell^- \leq R_{\ell-1}^- \leq R_{\ell-1}^+ \leq R_\ell^+$

with some extra efforts to deal with hyperedges

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SSM ⇒ SSM with the same rate

SSM with exponential rate ⇒ FPTAS

- $\lambda < \lambda_c = \frac{d^d}{k(d-1)^{d+1}}$ ⇒ FPTAS
- $\lambda = \lambda_c$ ⇒ SSM with sub-poly rate

Inapproximability

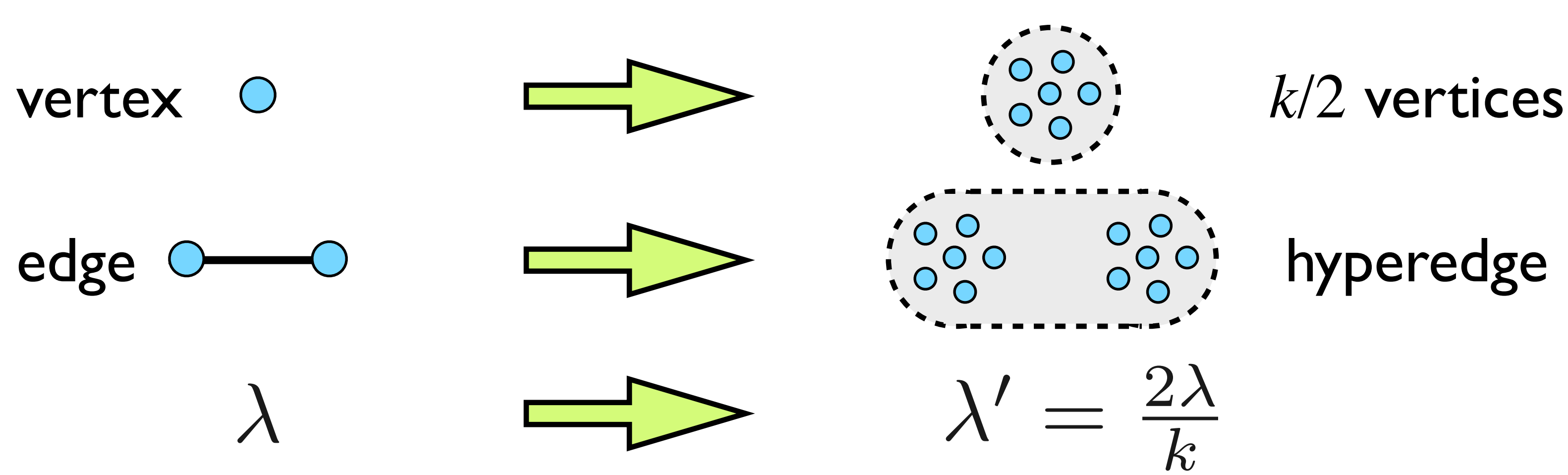
Theorem: let $\lambda_c = \frac{d^d}{k(d-1)^{(d+1)}}$

$\lambda > \frac{2k+1+(-1)^k}{k+1}\lambda_c \approx 2\lambda_c$ ⟹ no FPRAS unless NP=RP

reduction from hardcore model:

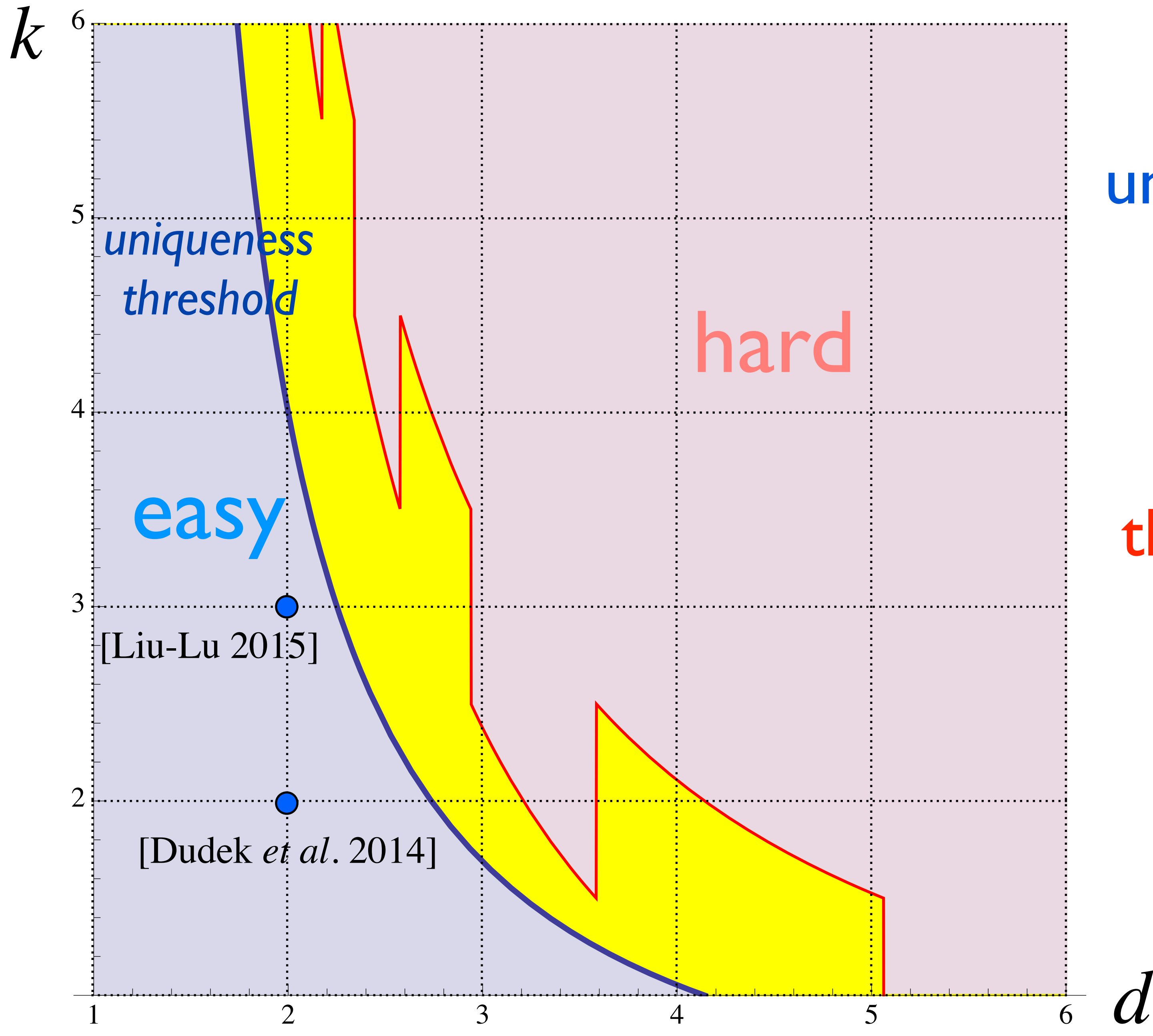
[folklore; Bordewich, Dyer, Karpinski 2008]

hardcore instance: hypergraph instance:



$\lambda = 1$: matchings of hypergraphs of max-degree $(k+1)$ and max-edge-size $(d+1)$

independent sets of hypergraphs of max-degree $(d+1)$ and max-edge-size $(k+1)$



uniqueness threshold:

$$\lambda_c = \frac{d^d}{k(d-1)^{(d+1)}}$$

threshold for hardness:

$$\frac{2k+1+(-1)^k}{k+1}\lambda_c \approx 2\lambda_c$$

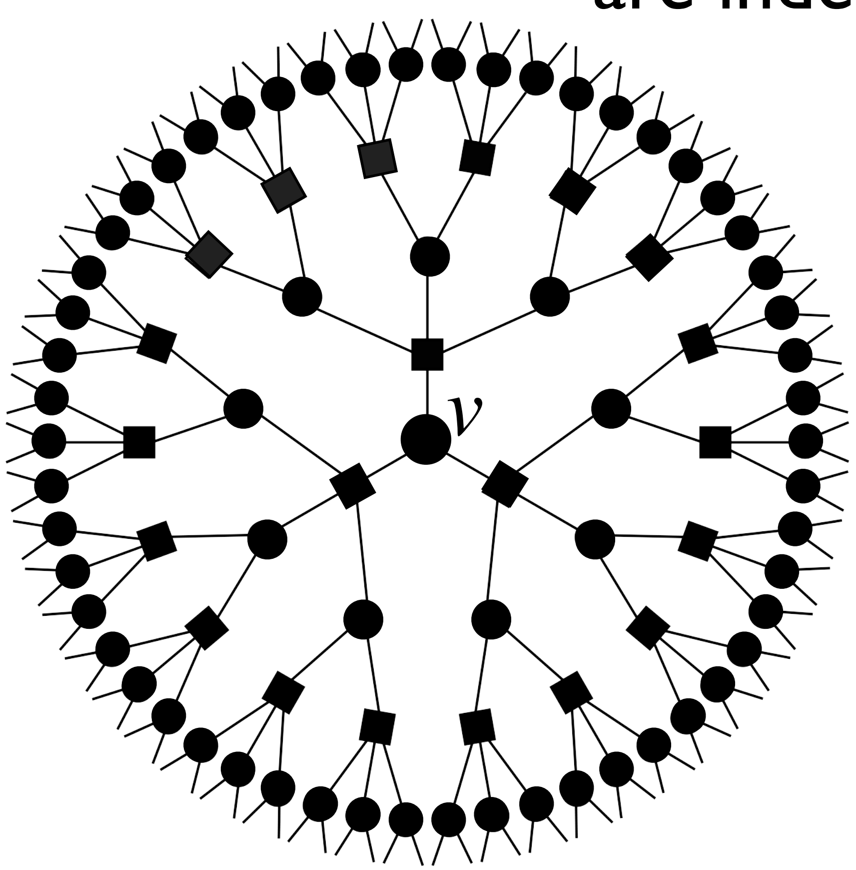
Gibbs Measures

T: the infinite $(k+1)$-uniform $(d+1)$-regular hypertree

μ is a measure over independent sets of T

μ is Gibbs: conditioning on any unoccupied finite boundary, the distribution over the truncated tree is the finite Gibbs distribution (DLR compatibility conditions)

μ is simple: conditioning on the root being unoccupied, the subtrees are independent of each other



$$\mu[\,v \text{ is occupied}\,] = \frac{\lambda}{1+\lambda} \cdot \mu[\,\text{all the neighbors of } v \text{ are unoccupied}\,]$$

(μ is Gibbs)

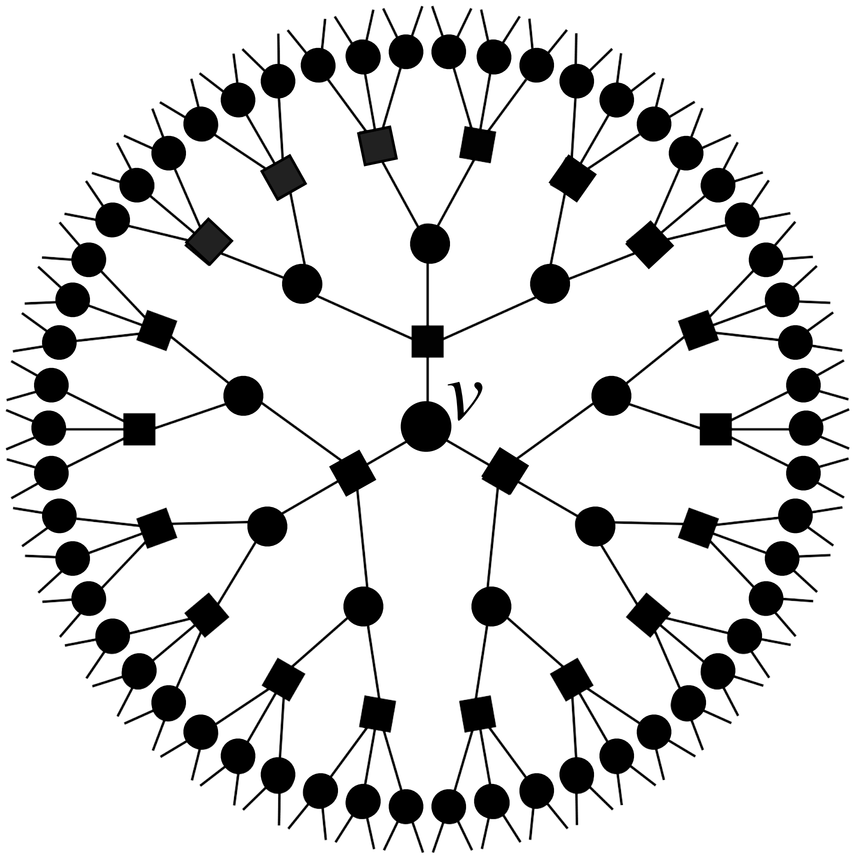
$$\mu[\,\text{all the neighbors of } v \text{ are unoccupied}\,] = \mu[\,v \text{ is occupied}\,] + \mu[\,v \text{ is unoccupied}\,] \prod_{i=1}^{d+1} \left(1 - \sum_{j=1}^{k} \mu[\,v_{ij} \text{ is occupied} \mid v \text{ is unoccupied}\,]\right)$$

(μ is Simple)

Gibbs Measures

T: the infinite $(k+1)$-uniform $(d+1)$-regular hypertree

μ is a simple Gibbs measure over independent sets of T



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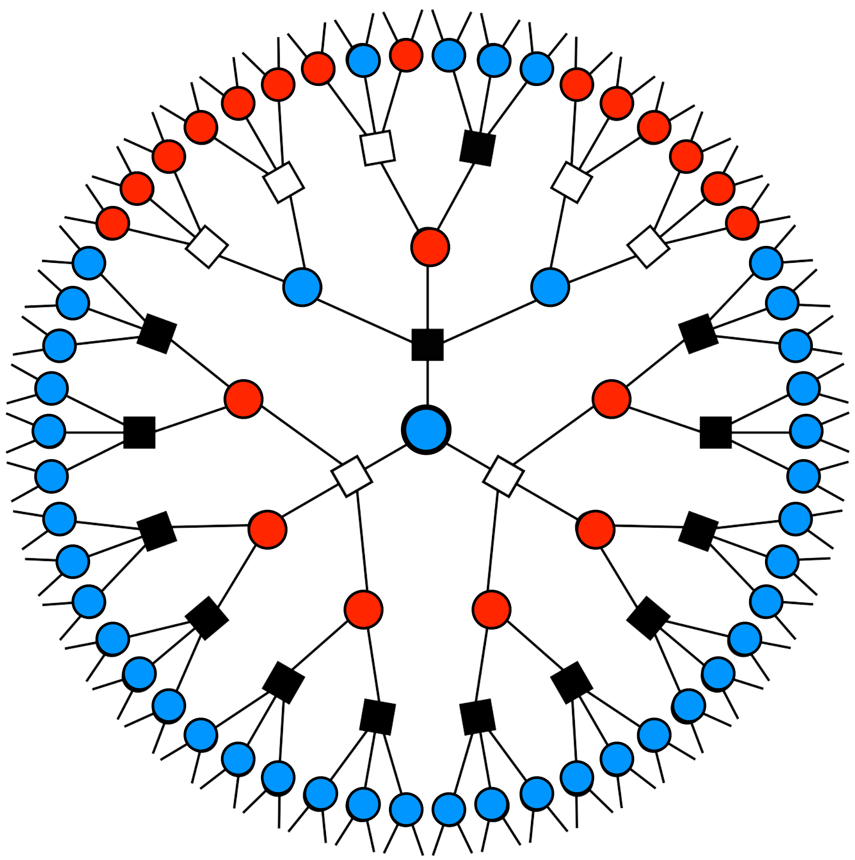
$$\mu[\,\text{all the neighbors of } v \text{ are unoccupied}\,] = \mu[\,v \text{ is occupied}\,] + \mu[\,v \text{ is unoccupied}\,] \prod_{i=1}^{d+1} \left(1 - \sum_{j=1}^{k} \mu[\,v_{ij} \text{ is occupied} \mid v \text{ is unoccupied}\,] \right)$$

(μ is Simple)

$$p_v = \lambda (1 - p_v)^{-d} \prod_{i=1}^{d+1} \left(1 - p_v - \sum_{j=1}^{k} p_{v_{ij}} \right)$$

where $p_v = \mu[\,v \text{ is occupied}\,]$

Uniqueness



$$p_v = \lambda(1-p_v)^{-d}\prod_{i=1}^{d+1}\left(1-p_v-\sum_{j=1}^{k}p_{v_{ij}}\right)$$

where $p_v = \mu[\,v \text{ is occupied}\,]$

assuming a symmetry:

- every blue vertex is incidents to 1 black edge and d white edges;
- every red vertex is incidents to 1 white edge and d black edges;
- every black edge contains k blue vertices and 1 red vertex;
- every white edge contains k red vertices and 1 blue vertex;

the system becomes:
$$\begin{cases} p_b = \lambda(1-p_b)^{-d}(1-k\,p_b-p_r)(1-p_b-k\,p_r)^d \\ p_r = \lambda(1-p_r)^{-d}(1-k\,p_r-p_b)(1-p_r-k\,p_b)^d \end{cases}$$

$\lambda > \lambda_c(k,d) = \dfrac{d^d}{k(d-1)^{d+1}}$ ⇒ has three solutions $(p^*,p^*), (p^+,p^-), (p^-,p^+)$

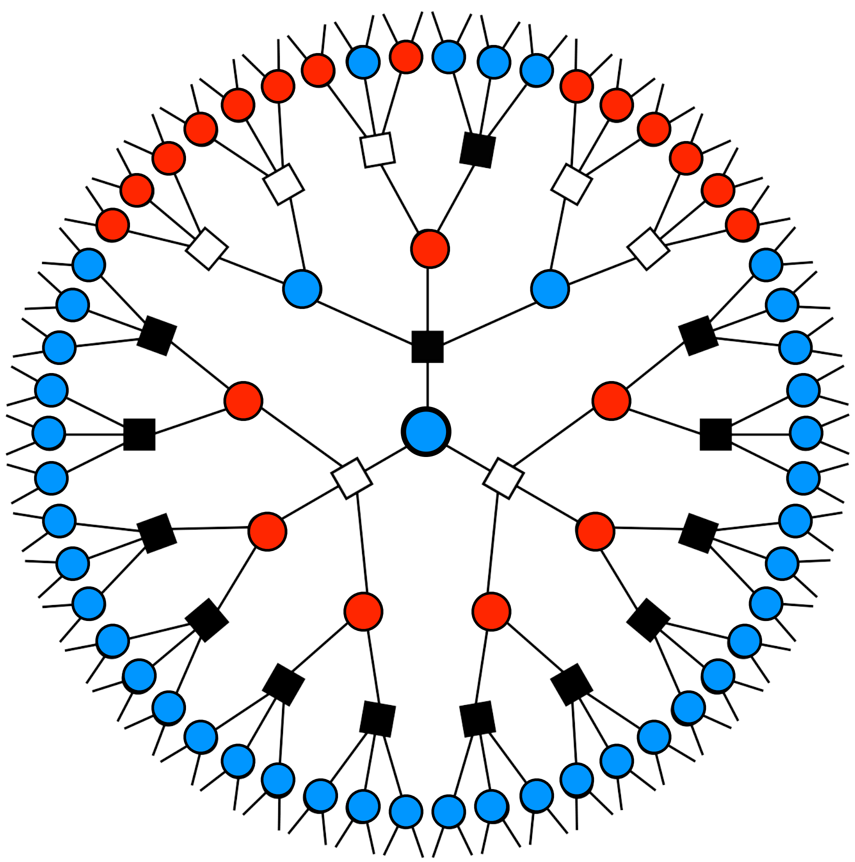
non-uniqueness!

$\lambda \le \lambda_c(k,d) = \dfrac{d^d}{k(d-1)^{d+1}}$ ⇒ has a unique solution (p^*,p^*)

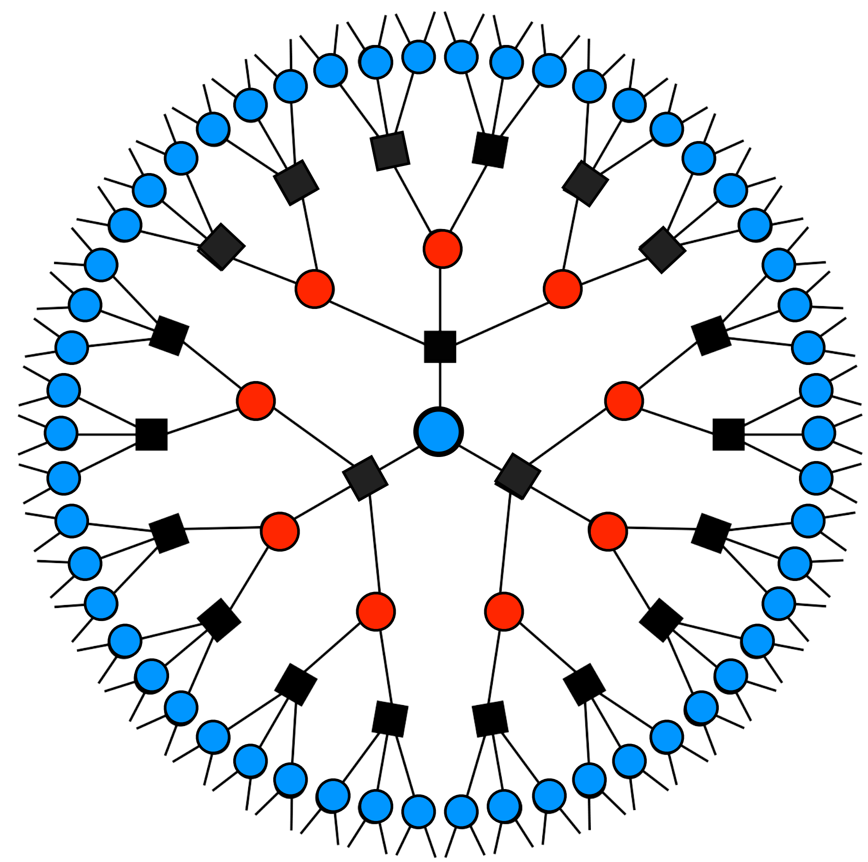
Symmetry

Gibbs measure μ is invariant under automorphisms from a group *G*

action of *G* classifies vertices and hyperedges into types (orbits)



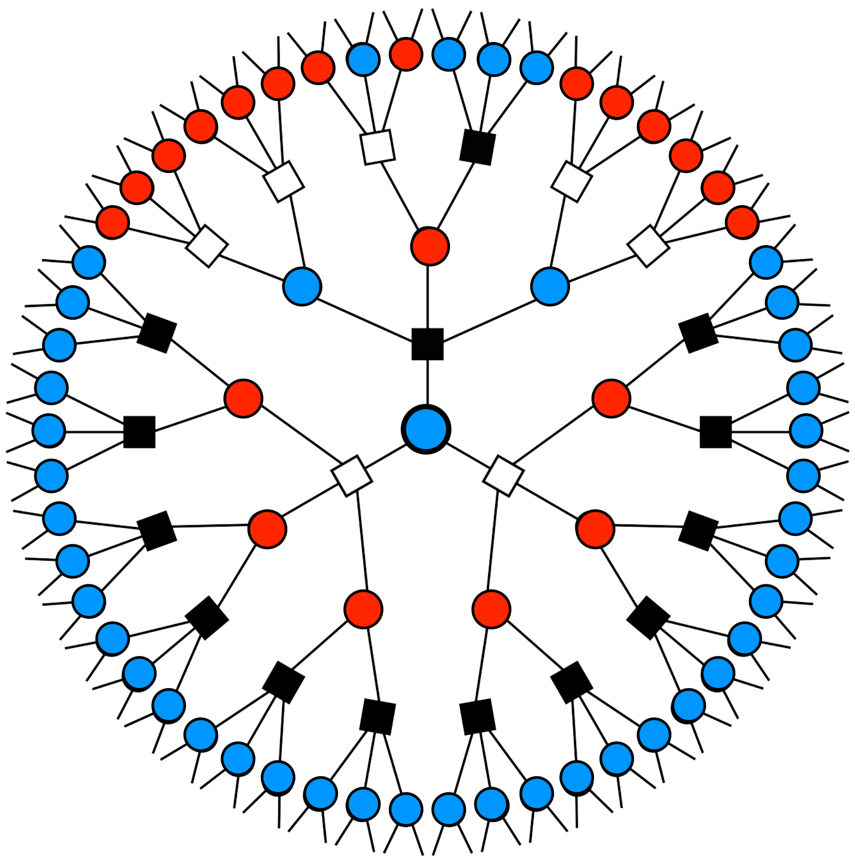
vs.



Symmetry

Gibbs measure μ is invariant under automorphisms from a group *G*

action of *G* classifies vertices and hyperedges into types (orbits)



τ_v : # of types(oribits) for vertices

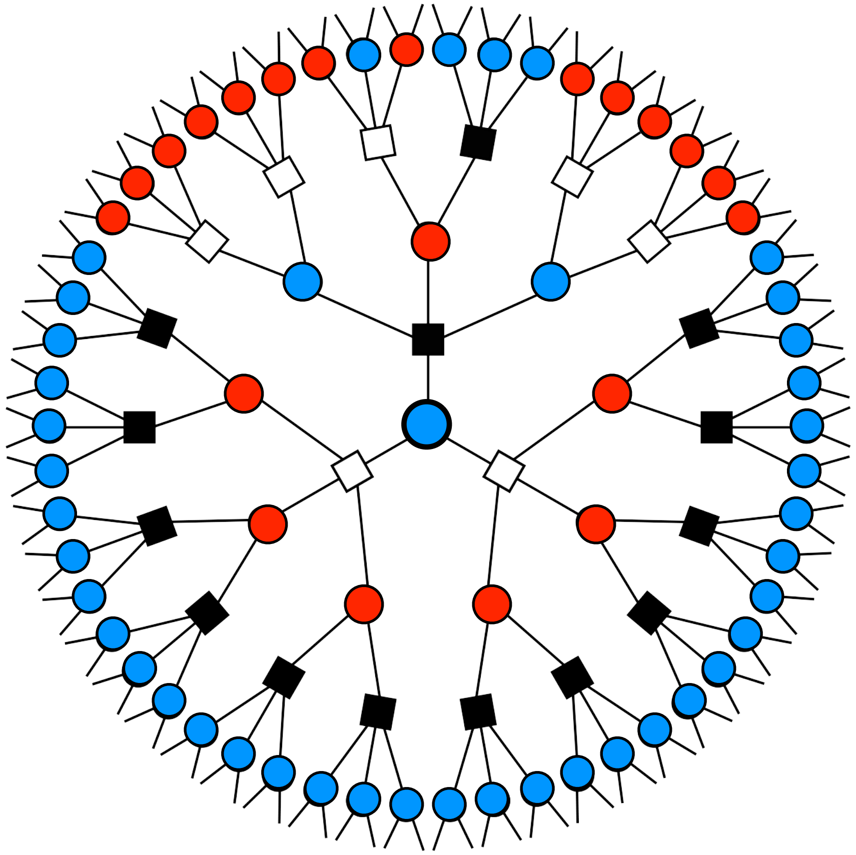
τ_e : # of types(oribits) for hyperedges

hypergraph branching matrices:

$$\mathbf{D} = (d_{ij})^{\tau_v \times \tau_e} \qquad \mathbf{K} = (k_{ji})^{\tau_e \times \tau_v}$$

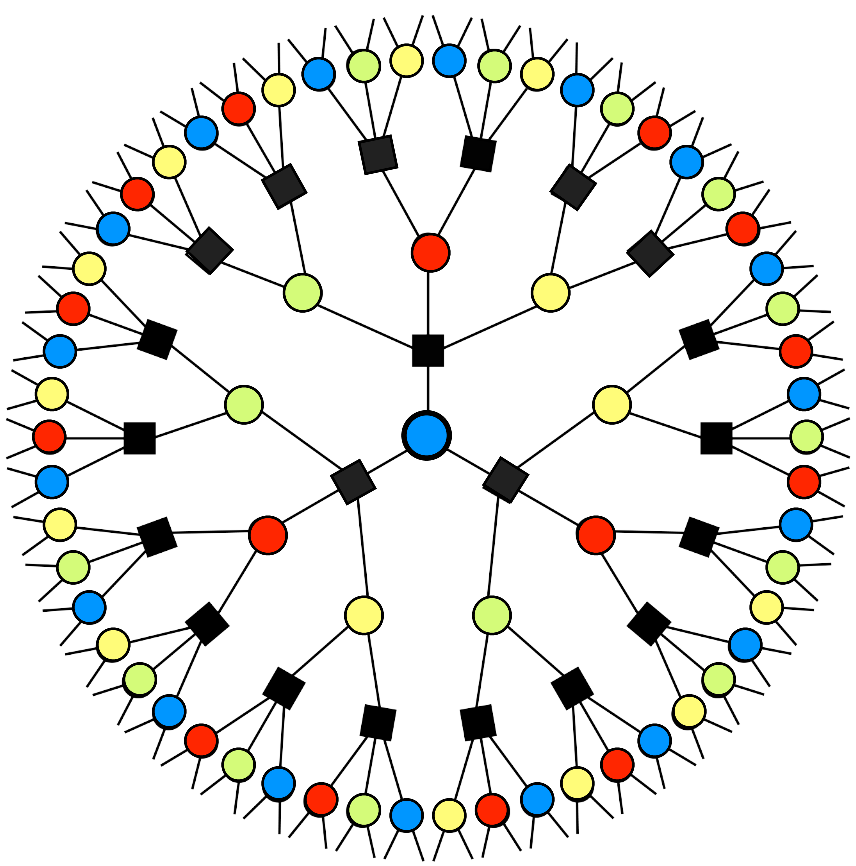
- each type-i vertex is incident to d_{ij} hyperedges of type-j
- each type-j hyperedge contains k_{ji} vertices of type-i

branching matrices completely characterize orbits of hypergraph automorphism groups



- every blue vertex is incidents to 1 black edge and d white edges;
- every red vertex is incidents to 1 white edge and d black edges;
- every black edge contains k blue vertices and 1 red vertex;
- every white edge contains k red vertices and 1 blue vertex;

$$\mathbf{D} = \begin{bmatrix} 1 & d \\ d & 1 \end{bmatrix} \qquad \mathbf{K} = \begin{bmatrix} k & 1 \\ 1 & k \end{bmatrix}$$



- there are k+1 types of vertices;
- there is only 1 type of hyperedges;
- each hyperedge has 1 vertex for each type;

$$\mathbf{D} = \left.\begin{bmatrix} d+1 \\ \vdots \\ d+1 \end{bmatrix}\right\} k+1 \qquad \mathbf{K} = \underbrace{\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}}_{k+1}$$

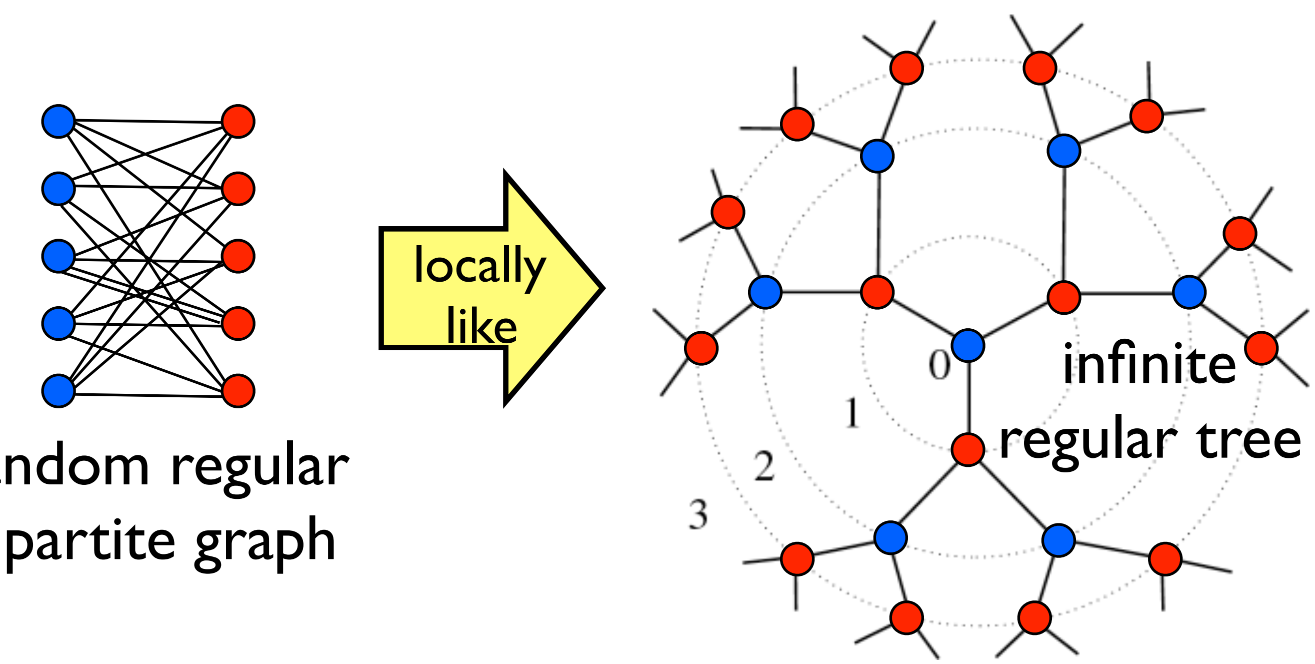
Local Convergence

fix a locally finite infinite hypergraph $\mathbb{T}$ and a labeling(orbits) $\mathscr{C}$ for vertices and hyperedges:

Definition (Local Convergence)

a sequence of (random) finite hypergraph $\mathcal{H}_n$ locally converges to $(\mathbb{T}, \mathscr{C})$ if there exists a labeling of vertices and hyperedges of $\mathcal{H}_n$ such that for any t>0, for random vertex v in $\mathcal{H}_n$ and random vertex-type x in $(\mathbb{T}, \mathscr{C})$ the t-neighborhoods $N_t(v, \mathcal{H}_n)$ converges to $N_t(v, \mathbb{T})$ in distribution.

defined in [Montanari, Mossel, Sly 2012] [Sly, Sun 2012]

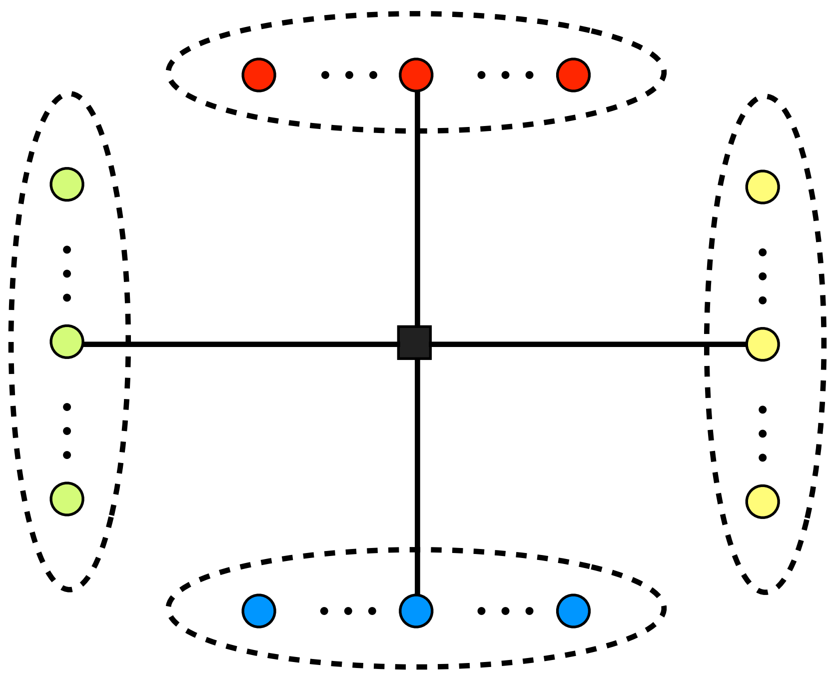


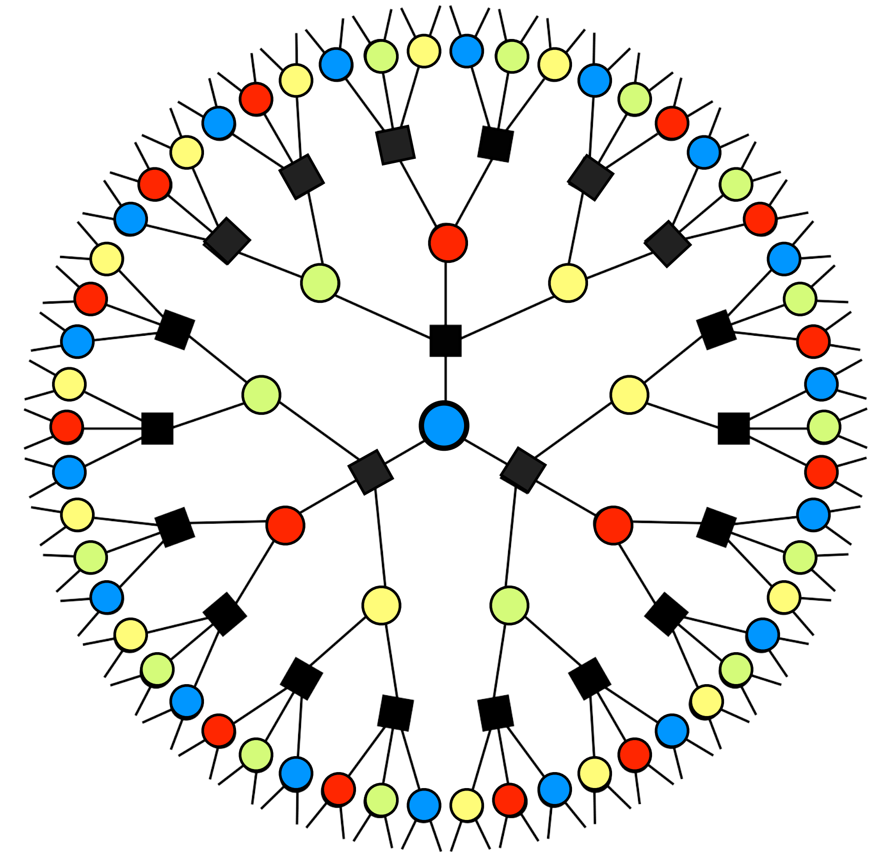
random regular bipartite graph

with parity labeling

plays a crucial role in establishing sharp transition of computational complexity:

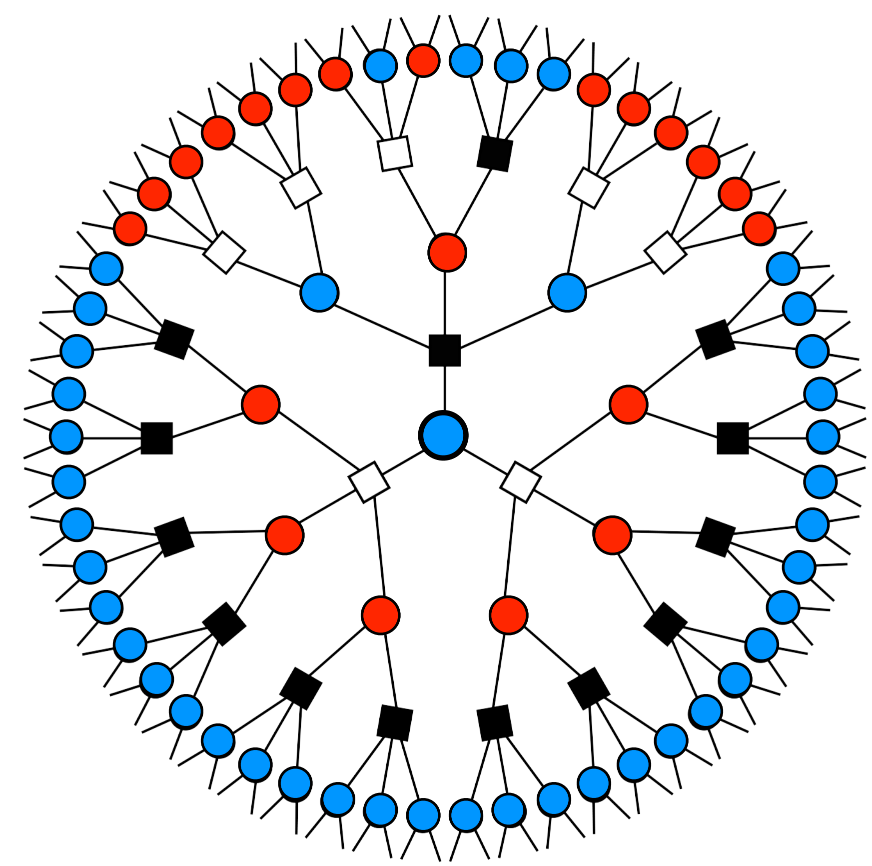
[Dyer, Frieze, Jerrum '02]
[Mossel, Weitz, Wormald '09]
[Sly '10] [Sly, Sun '12]
[Galanis, Ge, Štefankovič, Vigoda, Yang '11]
[Galanis, Štefankovič, Vigoda '12 '14]
... ...





random $(k+1)$-uniform $(d+1)$-regular
$(k+1)$-partite hypergraph

?



Local Convergence

Theorem:

There exists a sequence of finite hypergraphs $\mathcal{H}_n$ locally convergent to $(k{+}1)$-uniform $(d{+}1)$-regular infinite hypertree with branching matrices $\mathbf{D}, \mathbf{K}$

if and only if Markov chain $\begin{bmatrix} \mathbf{0} & \frac{1}{d+1}\boldsymbol{D} \\ \frac{1}{k+1}\boldsymbol{K} & \mathbf{0} \end{bmatrix}$ is time-reversible.

∃ distributions $\boldsymbol{p}$ over vertex orbits and $\boldsymbol{q}$ over hyperedge orbits

satisfying the detailed balanced equation:

$$p_i d_{ij} = q_j k_{ji}$$

$\boldsymbol{p}$ must be a left Perron Eigenvector of $\mathbf{DK}$

$\boldsymbol{q}$ must be a left Perron Eigenvector of $\mathbf{KD}$

Local Convergence

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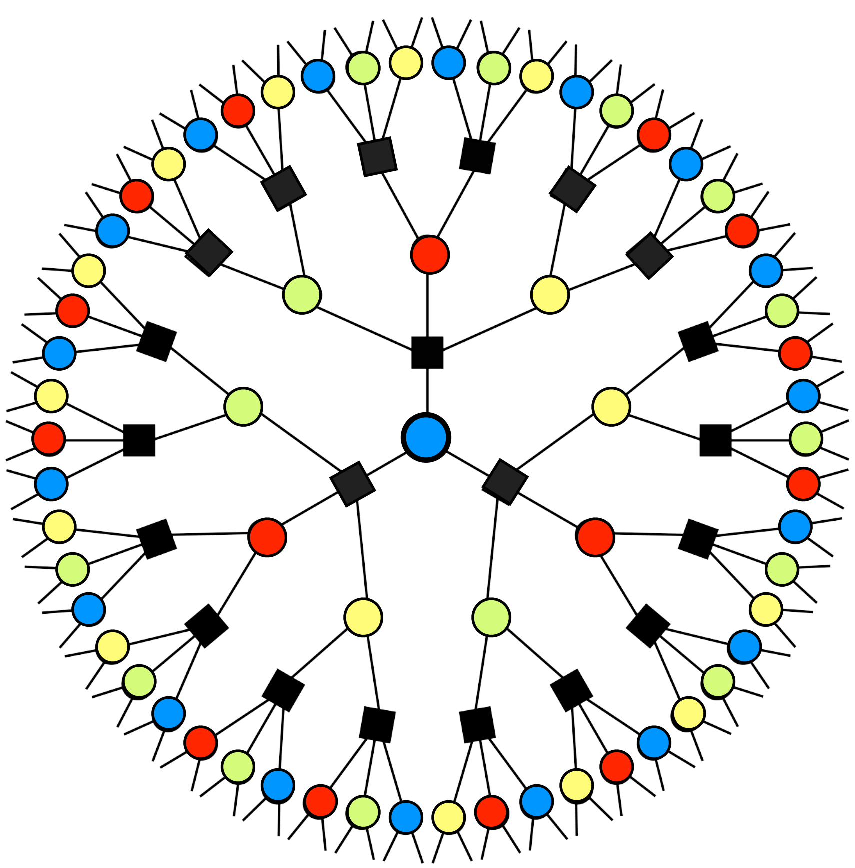
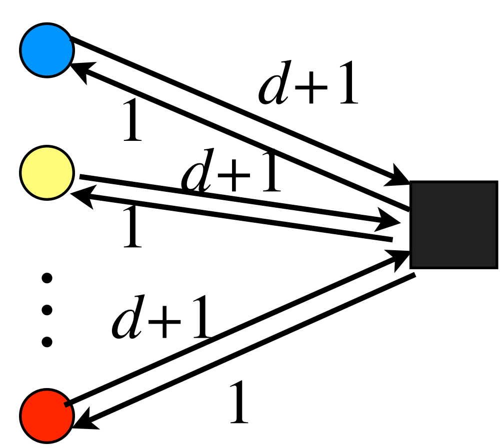


$$\mathbf{D} = \left.\begin{bmatrix} d+1 \\ \vdots \\ d+1 \end{bmatrix}\right\} k+1 \qquad \mathbf{K} = \underbrace{\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}}_{k+1}$$



time-reversible

Local Convergence

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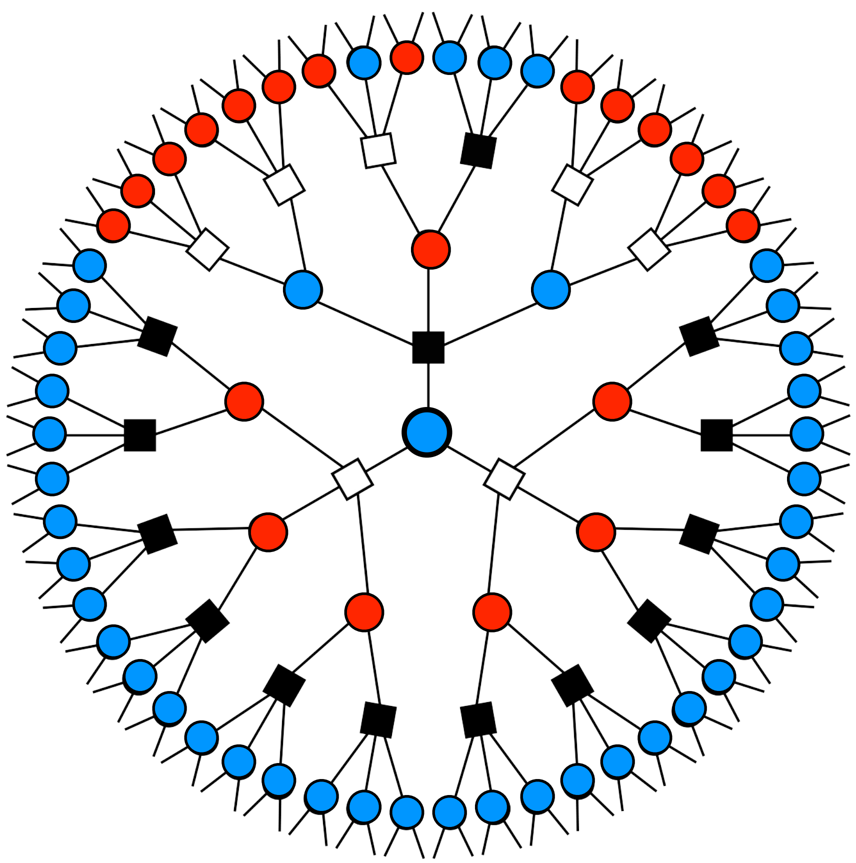

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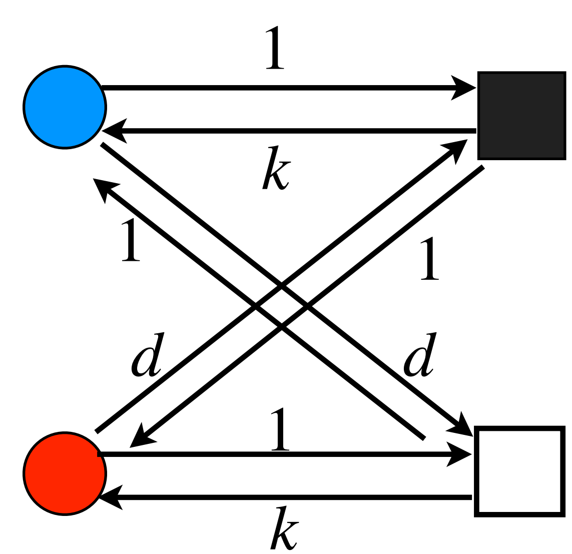


not time-reversible

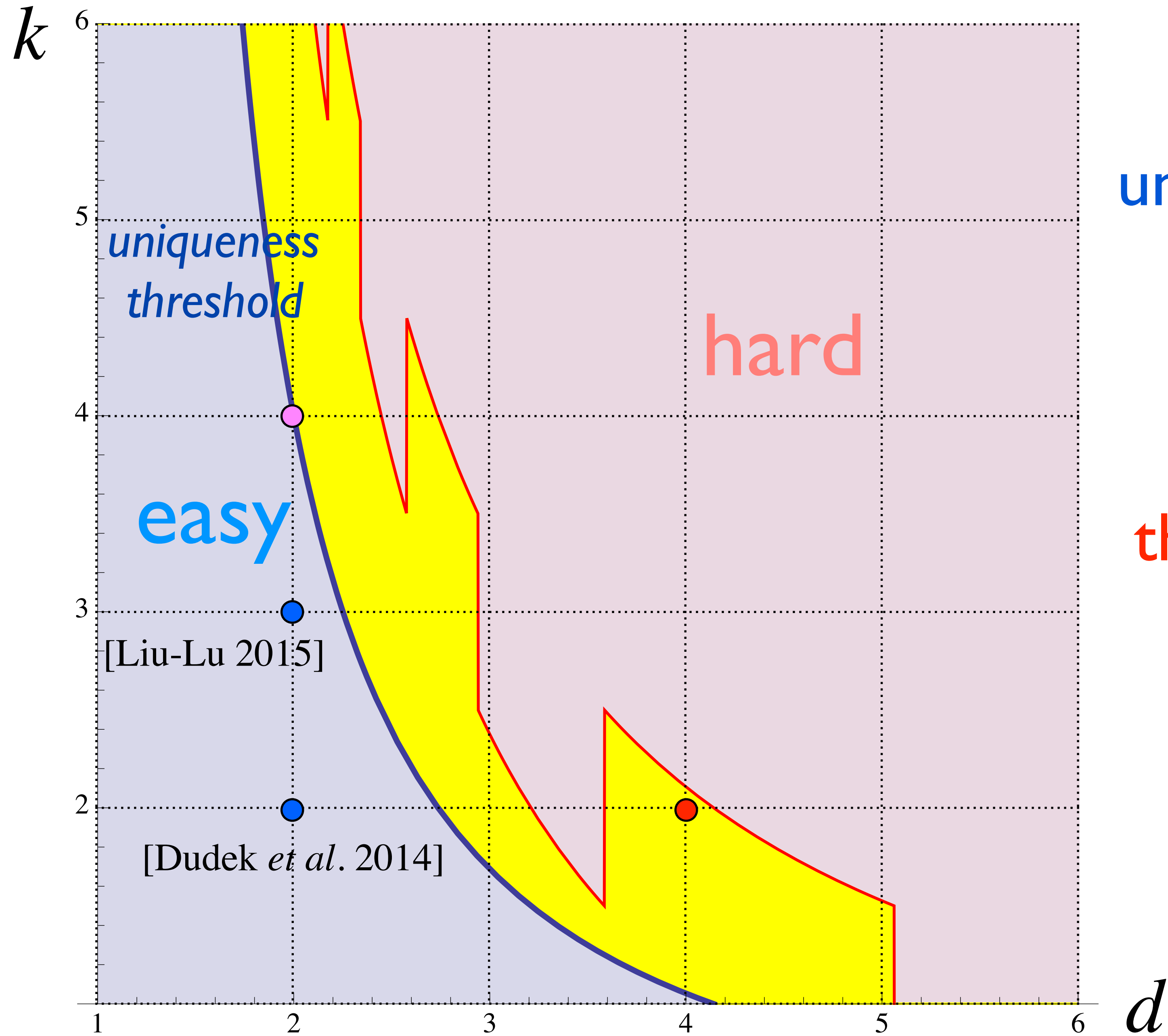
Summary

independent sets of hypergraphs of max-degree $(d+1)$ and max-edge-size $(k+1)$

- uniqueness threshold for $(k+1)$-uniform $(d+1)$-regular infinite hypertree:
$$\lambda_c(k,d) = \frac{d^d}{k(d-1)^{d+1}}$$
- SAW-tree holds for the model
  - hypertree are the worst-case for SSM
  - $\lambda<\lambda_c$: FPTAS for the partition function
- $\lambda>2\lambda_c$: inapproximable (by simulating hardcore)
- local convergence exists if and only if time-reversibility is satisfied
  - the extremal Gibbs measures achieving the uniqueness threshold are not realizable by finite hypergraphs

$\lambda = 1$: matchings of hypergraphs of max-degree $(k+1)$ and max-edge-size $(d+1)$

independent sets of hypergraphs of max-degree $(d+1)$ and max-edge-size $(k+1)$



uniqueness threshold:

$$\lambda_c = \frac{d^d}{k(d-1)^{(d+1)}}$$

threshold for hardness:

$$\frac{2k+1+(-1)^k}{k+1}\lambda_c \approx 2\lambda_c$$

- algorithmic technique which does not rely on decay of correlation?
- inapproximability which does not need local convergence?
- other extremal Gibbs measures with the same threshold?

Thank you!

Any questions?