第四章:

中纬度的经向环流系统(II)

- Ferrel cell, baroclinic eddies and the westerly jet

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2019. 10. 30

Observations

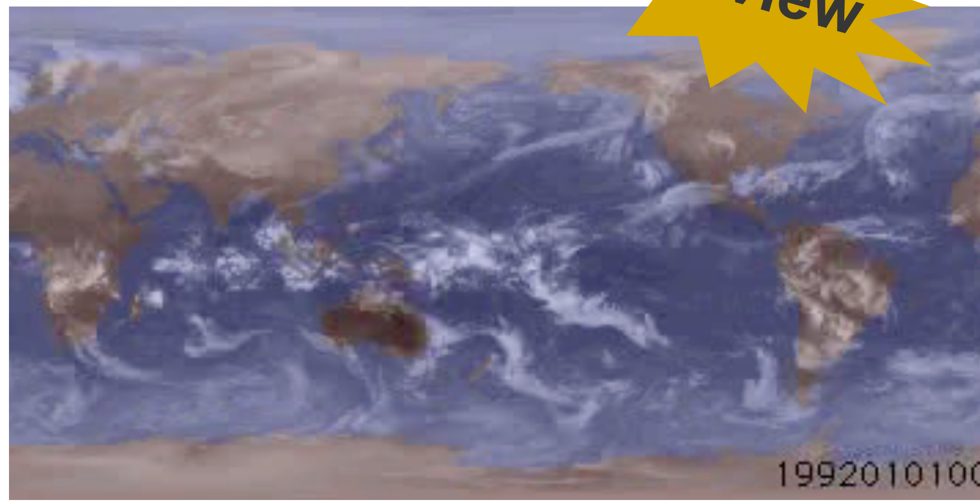
- Eddy fields

Review

Strong **baroclinic eddy** activity

$L_R \sim O(1000km)$

Synoptic time scale (2-8 days)



$$
\begin{aligned}
[\overline{AB}] &= [\overline{(\bar{A}+A')(\bar{B}+B')}] = [\bar{A}\bar{B}] + [\overline{A'B'}] \\
&= [([\bar{A}] + \bar{A}^*)([\bar{B}] + \bar{B}^*)] + [\overline{A'B'}] \\
&= [\bar{A}][\bar{B}] + [\bar{A}^*\bar{B}^*] + [\overline{A'B'}]
\end{aligned}
$$

$$A = [\bar{A}] + \bar{A}^* + A'$$

Observations

- Summary:
  - Zonal-mean flow:
    - Ferrel Cell: an indirect cell centered at 40-60 degree, with strong seasonal variation in N.H.
    - Westerly jet: surface westerlies centered at 40-60 degree
  - Eddies: transient eddies are dominant with stationary eddies only obvious in N.H.
    - Kinetic energy
    - Momentum flux
    - Heat flux

The Ferrel Cell

eddy-zonal flow interaction (I)

- Start from the equations:
  - Momentum equation:

$$\left(\frac{du}{dt}\right)_p - fv = -\left(\frac{\partial \Phi}{\partial x}\right)_p + F_x$$

  - Continuity equation:

$$\nabla_p \cdot \boldsymbol{v} + \frac{\partial \omega}{\partial p} = 0$$

  - Thermodynamic equation:

$$\left(\frac{d \ln \theta}{dt}\right)_p = \frac{Q}{c_p T}$$

$$\left(\frac{d}{dt}\right)_p = \left(\frac{\partial}{\partial t}\right)_p + u\left(\frac{\partial}{\partial x}\right)_p + v\left(\frac{\partial}{\partial y}\right)_p + \omega\frac{\partial}{\partial p}$$

Decompose into zonal mean and eddy components:

$$A = [A] + A^*$$

The Ferrel Cell

eddy-zonal flow interaction (I)

- Start from the equations:
  - Momentum equation:

$$\frac{\partial[u]}{\partial t}+\frac{\partial([u][v])}{\partial y}+\frac{\partial([u][\omega])}{\partial p}=-\frac{\partial([u^*v^*])}{\partial y}-\frac{\partial([u^*\omega^*])}{\partial p}+f[v]+[F_x]$$

  - Continuity equation:

$$\frac{\partial[v]}{\partial y}+\frac{\partial[\omega]}{\partial p}=0$$

  - Thermodynamic equation:

$$\frac{\partial[\theta]}{\partial t}+\frac{\partial([v][\theta])}{\partial y}+\frac{\partial([\omega][\theta])}{\partial p}=-\frac{\partial([\theta^*v^*])}{\partial y}-\frac{\partial([\theta^*\omega^*])}{\partial p}+\left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p}$$

$$\left(\frac{d}{dt}\right)_p=\left(\frac{\partial}{\partial t}\right)_p+u\left(\frac{\partial}{\partial x}\right)_p+v\left(\frac{\partial}{\partial y}\right)_p+\omega\frac{\partial}{\partial p}$$

Under the quasi-geostrophic approximation ($R_o \ll 1$), above equations can be simplified.

The Ferrel Cell
eddy-zonal flow interaction (I)

- Start from the equations:

$$\frac{\partial [u]}{\partial t} + \frac{\partial([u][v])}{\partial y} + \frac{\partial([u][\omega])}{\partial p} = -\frac{\partial([u^* v^*])}{\partial y} - \frac{\partial([u^* \omega^*])}{\partial p} + f[v] + [F_x]$$

$$\frac{\partial [\theta]}{\partial t} + \frac{\partial([v][\theta])}{\partial y} + \frac{\partial([\omega][\theta])}{\partial p} = -\frac{\partial([\theta^* v^*])}{\partial y} - \frac{\partial([\theta^* \omega^*])}{\partial p} + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q]}{c_p}$$

$$\frac{\partial [v]}{\partial y} + \frac{\partial [\omega]}{\partial p} = 0$$

- Simplification:
  - For midlatitude large scale flow, the eddy components of the meridional heat and momentum transports are dominant. (recall the observations)

$$\frac{\partial}{\partial y}[u^* v^*] \gg \frac{\partial}{\partial y}([u][v]) \qquad \frac{\partial}{\partial y}[\theta^* v^*] \gg \frac{\partial}{\partial y}([\theta][v])$$

  - From the QG approximation, $\frac{\partial \omega^*}{\partial p} \sim R_o \frac{\partial v^*}{\partial y} \rightarrow \frac{\partial}{\partial y}[u^* v^*] \gg \frac{\partial}{\partial p}[u^* \omega^*]$
  - Horizontal variation of the stratification is small: $\frac{\partial}{\partial p}([\theta][\omega]) \approx [\omega] \frac{\partial \theta_s}{\partial p}$

The Ferrel Cell

eddy-zonal flow interaction (I)

- The simplified equations:
  - Momentum equation:

$$\frac{\partial [u]}{\partial t} = -\frac{\partial([u^* v^*])}{\partial y} + f[v] + [F_x]$$

  - Continuity equation:

$$\frac{\partial [v]}{\partial y} + \frac{\partial [\omega]}{\partial p} = 0$$

  - Thermodynamic equation:

$$\frac{\partial [\theta]}{\partial t} + [\omega]\frac{\partial \theta_s}{\partial p} = -\frac{\partial([\theta^* v^*])}{\partial y} + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q]}{c_p}$$

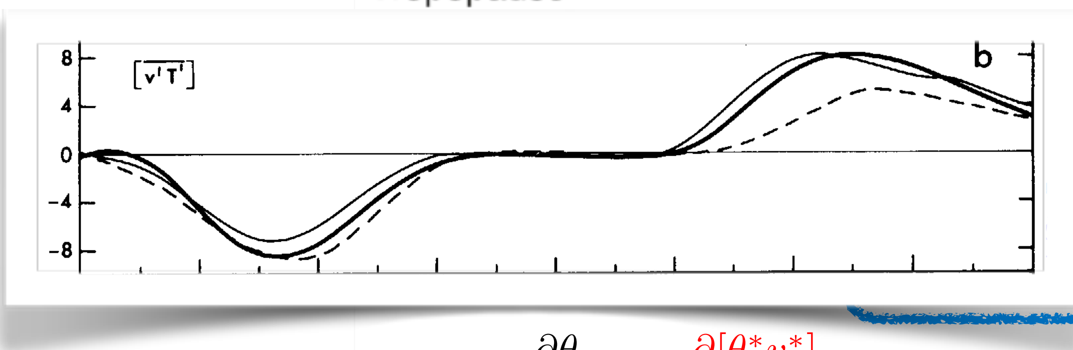
$$\left(\frac{d}{dt}\right)_p = \left(\frac{\partial}{\partial t}\right)_p + u\left(\frac{\partial}{\partial x}\right)_p + v\left(\frac{\partial}{\partial y}\right)_p + \omega\frac{\partial}{\partial p}$$

Under the quasi-geostrophic approximation ($R_o \ll 1$)

The Ferrel Cell

- The balance equations:

Tropopause



$$\ldots [u^*v^*]) \over \partial y + f[v] + [F_x]$$

$$\omega \frac{\partial \theta_s}{\partial p} \sim -\frac{\partial [\theta^* v^*]}{\partial y} < 0$$

$$\omega \frac{\partial \theta_s}{\partial p} \sim -\frac{\partial [\theta^* v^*]}{\partial y} > 0$$

$$\frac{\partial [\theta]}{\partial t} + [\omega] \frac{\partial \theta_s}{\partial p} = -\frac{\partial ([\theta^* v^*])}{\partial y} + \left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q]}{c_p}$$

Boundary layer

Ground

Subtropics Latitude Subpolar

The Ferrel Cell

- The balance equations:

Tropopause

$$fv \sim \frac{\partial [u^* v^*]}{\partial y} < 0$$

$$\cancel{\frac{\partial [u]}{\partial t}} = -\frac{\partial ([u^* v^*])}{\partial y} + f[v] + [F_x]$$

$$\omega \frac{\partial \theta_s}{\partial p} \sim -\frac{\partial [\theta^* v^*]}{\partial y} < 0 \qquad \omega \frac{\partial \theta_s}{\partial p} \sim -\frac{\partial [\theta^* v^*]}{\partial y} > 0$$

$$\cancel{\frac{\partial [\theta]}{\partial t}} + [\omega]\frac{\partial \theta_s}{\partial p} = -\frac{\partial ([\theta^* v^*])}{\partial y} + \cancel{\left(\frac{p_o}{p}\right)^{R/c_p} \frac{[Q]}{c_p}}$$

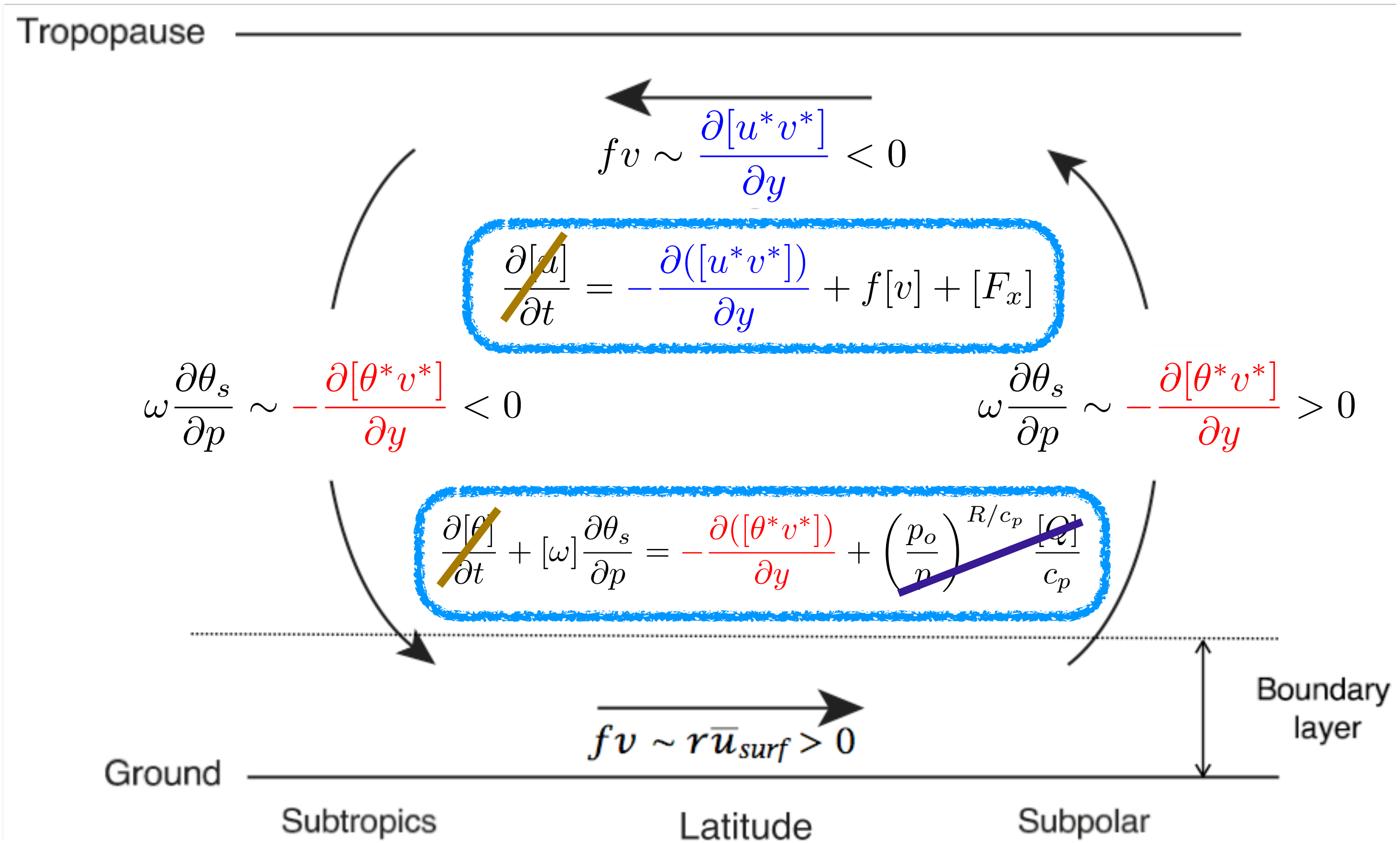
Boundary layer

Ground

Subtropics | Latitude | Subpolar

The Ferrel Cell

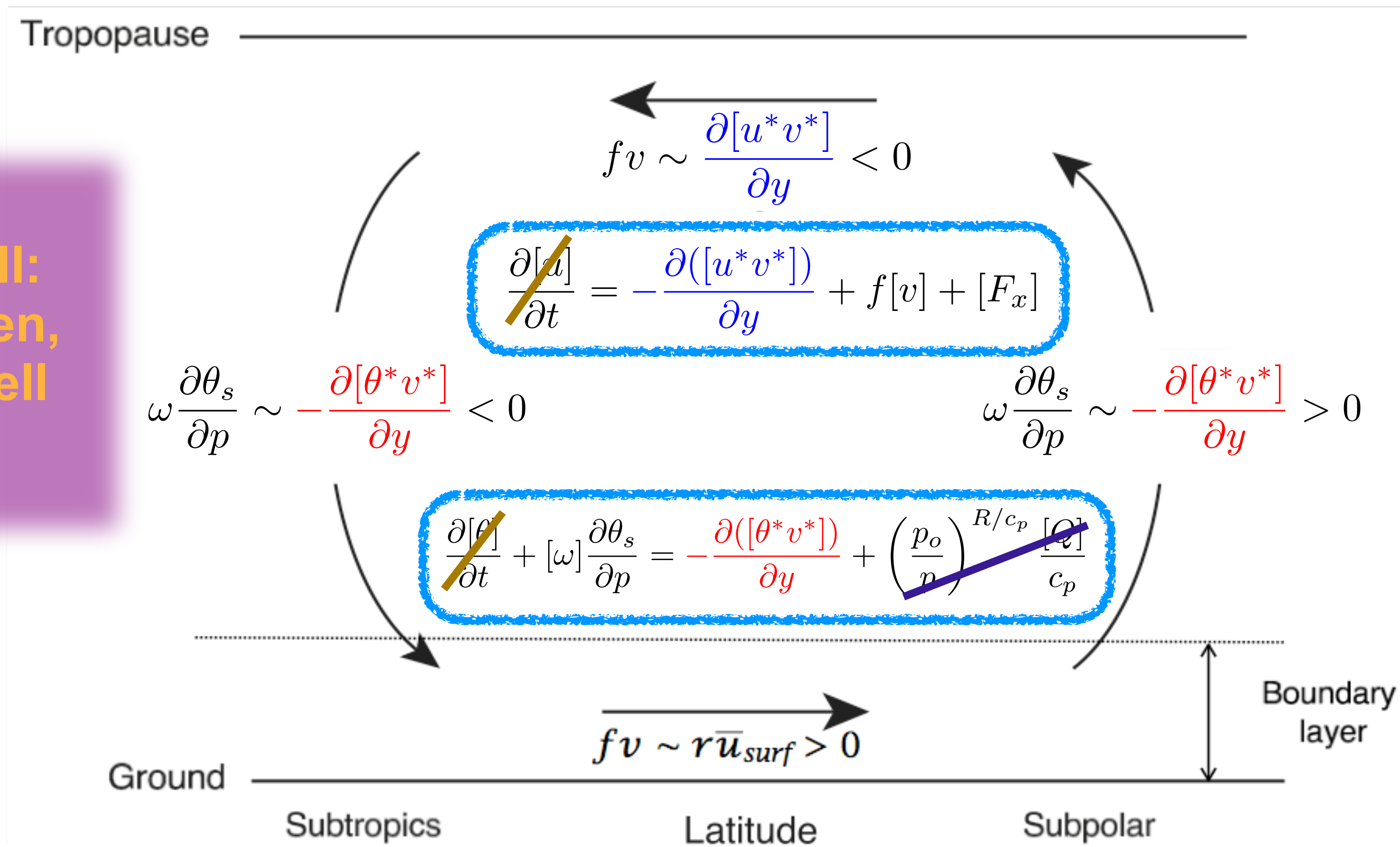
- The balance equations:



The Ferrel Cell

- The balance equations:

Ferrel Cell: eddy-driven, indirect cell



The Ferrel Cell

- In **isentropic** coordinate

$$(x, y, z) \Leftrightarrow (x, y, \theta)$$

$$\frac{D\theta}{Dt} = \dot{\theta}$$

$$\begin{aligned}\frac{D}{Dt} &= \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla_\theta + \frac{D\theta}{Dt}\frac{\partial}{\partial \theta} \\ &= \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla_\theta + \dot{\theta}\frac{\partial}{\partial \theta}\end{aligned}$$

zero for adiabatic flow

Isentrope: An isopleth of entropy. In meteorology it is usually identified with an isopleth of potential temperature.

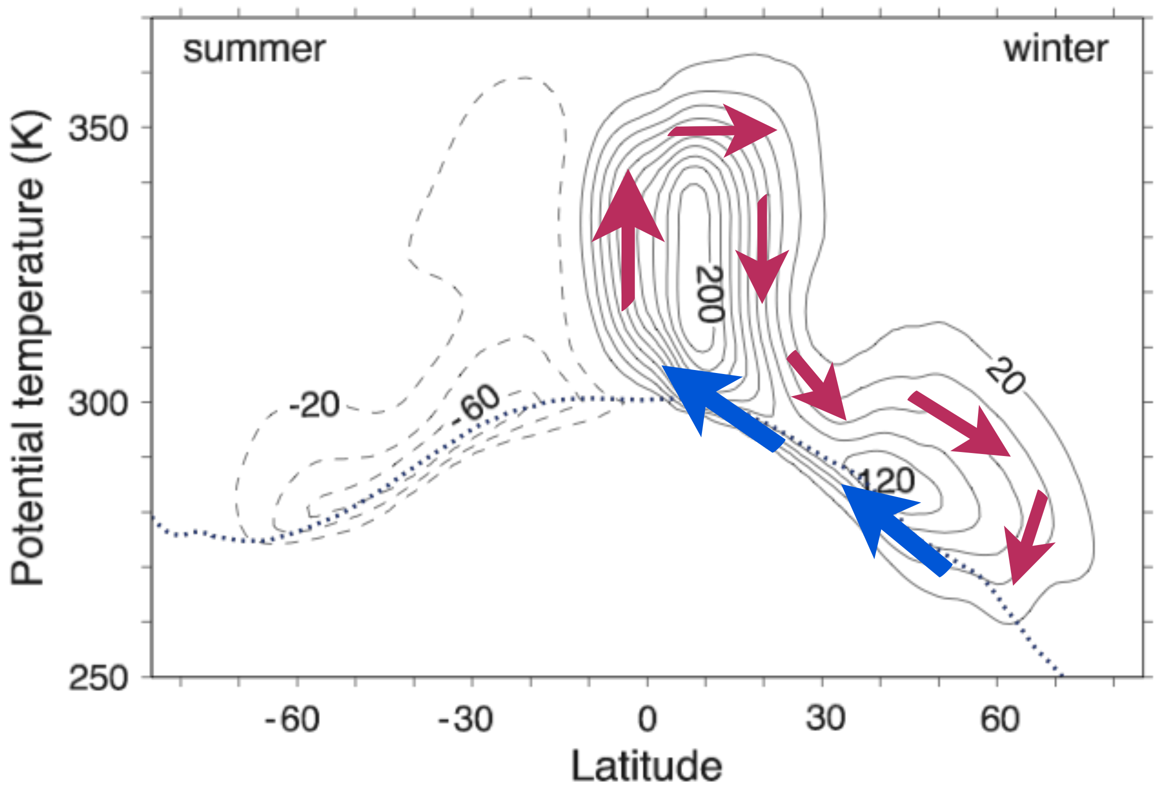
The Ferrel Cell

- In **isentropic** coordinate

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla_\theta + \frac{D\theta}{Dt}\frac{\partial}{\partial\theta}$$
$$= \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla_\theta + \dot{\theta}\frac{\partial}{\partial\theta}$$

The **direction** of Ferrel cell is **reversed** in the isentropic coordinate.

Interactions between Hadley and Ferrel cells are expected.



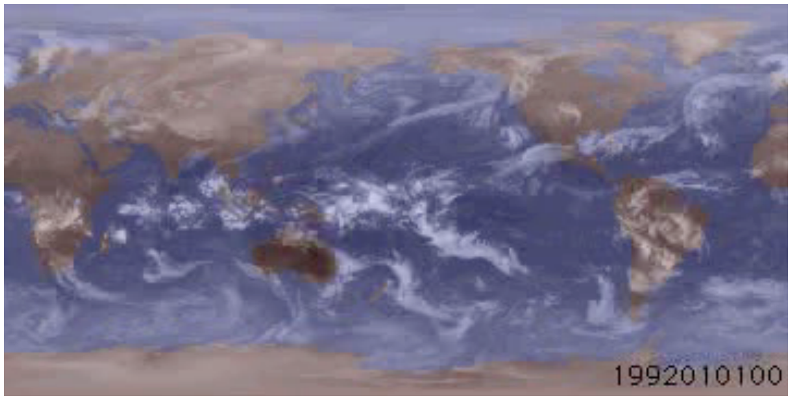
(Fig.11.4, Vallis, 2006)

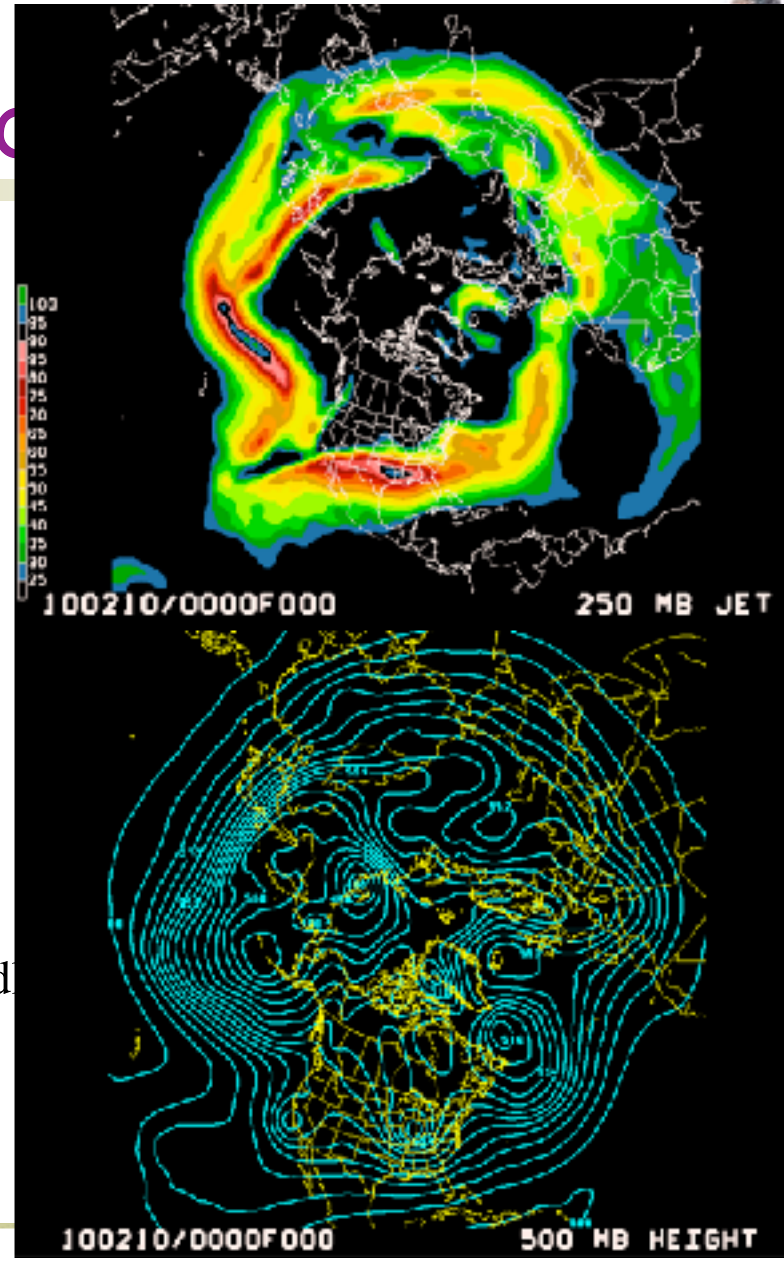
Outline

- Observations
- The Ferrel Cell
- **Baroclinic eddies**
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction
  - Transformed Eulerian Mean equation
- Eddy-driven jet
- The energy cycle

Observed

Baroclinic ec



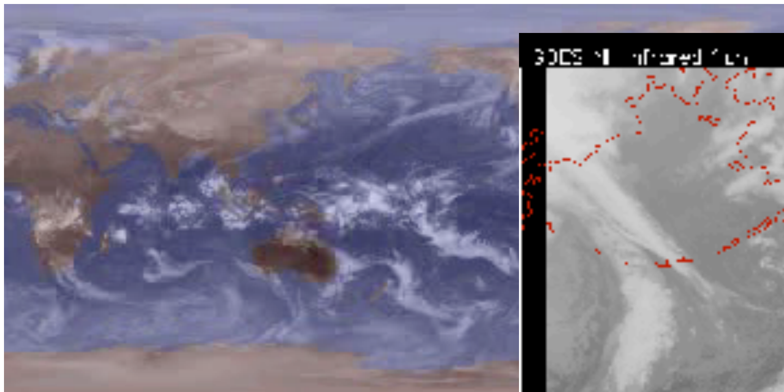


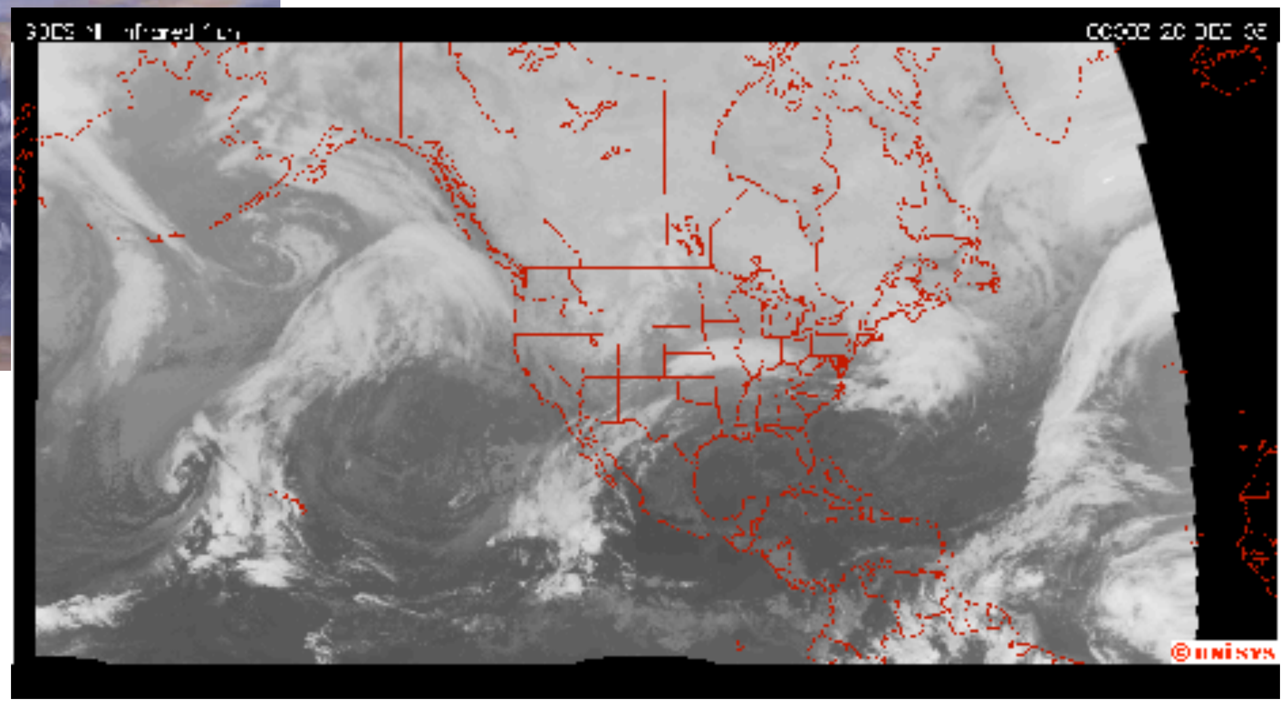
Strong **baroclinic eddy** activity in the mid

Synoptic time scale (2-8 days)

$L_R \sim O(1000km)$

Observed
Baroclinic eddies





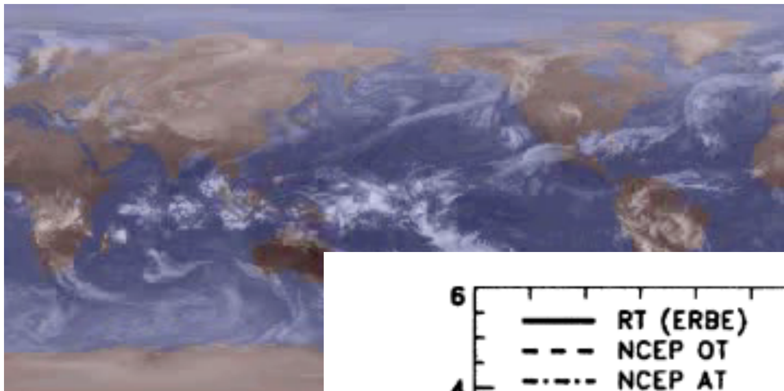
Strong **baroclinic eddy** activity in the midlatitudes

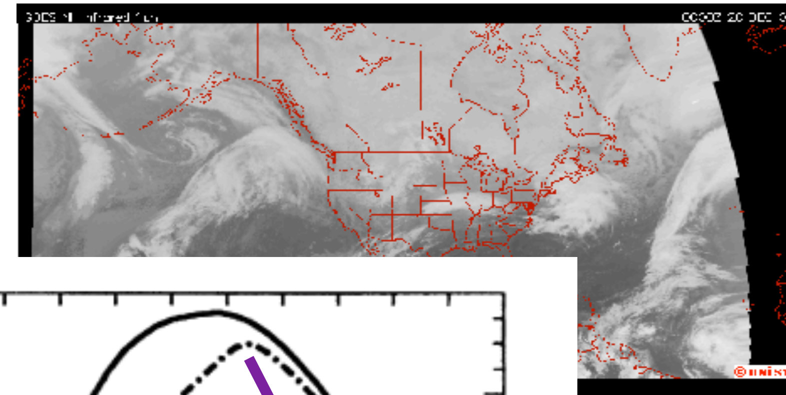
Synoptic time scale (2-8 days)

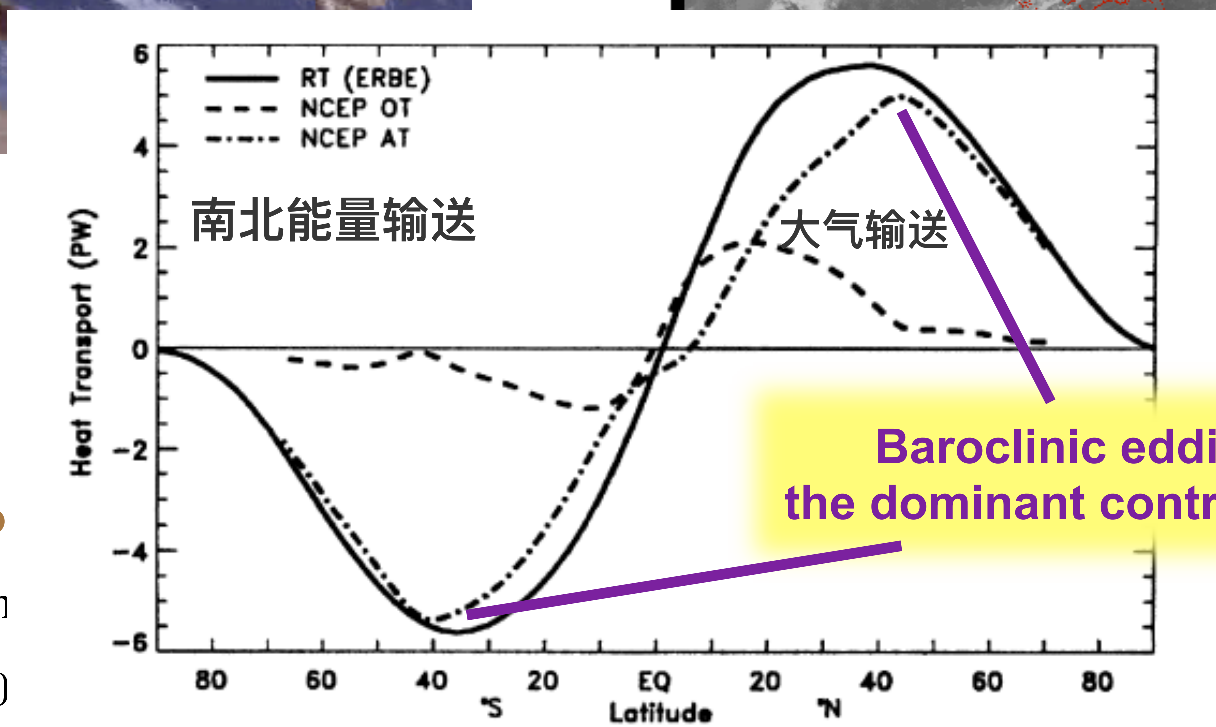
$L_R \sim O(1000km)$

Observed

Baroclinic eddies







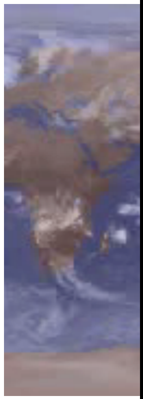
Strong **baro**

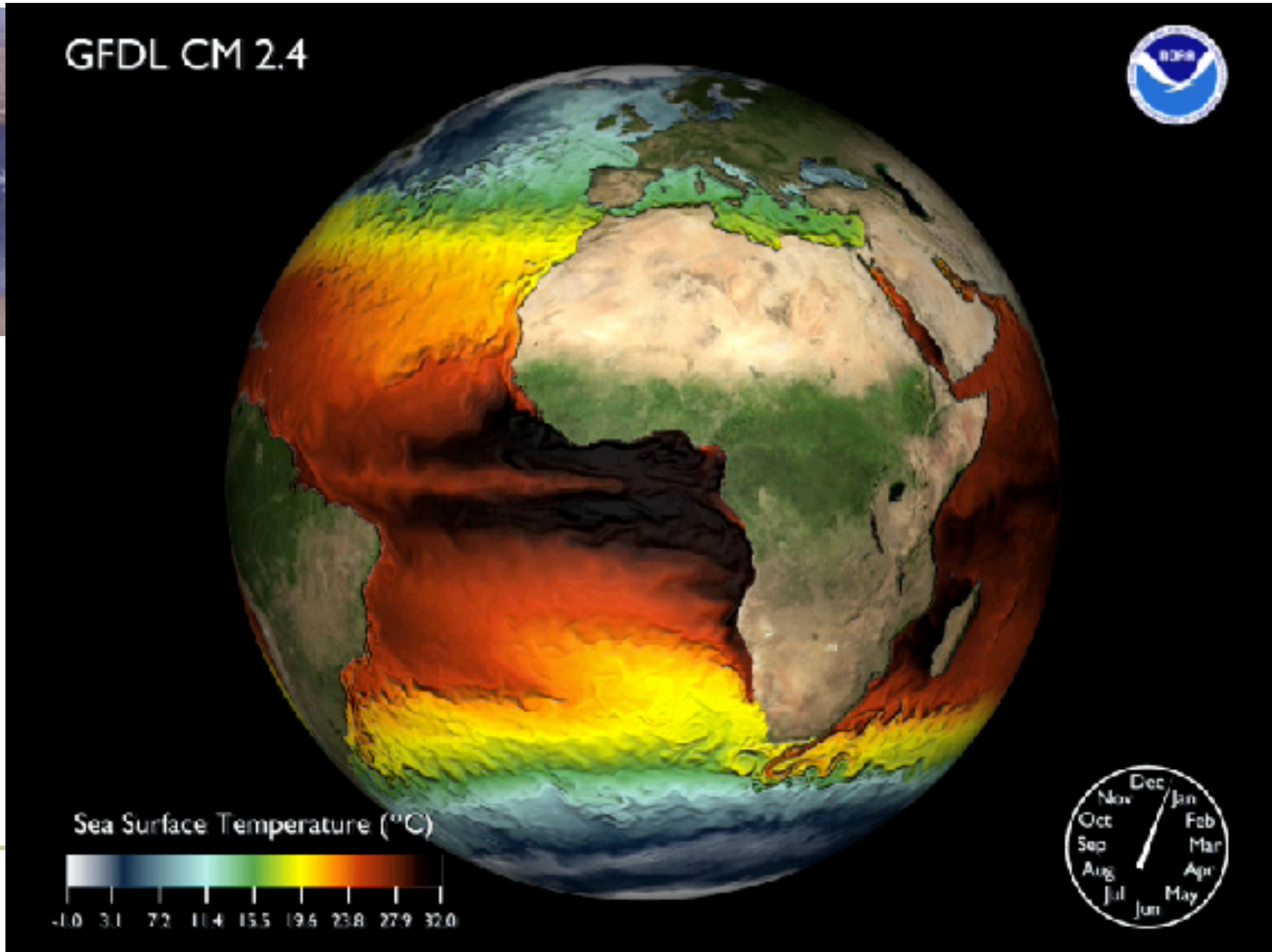
Synoptic tin

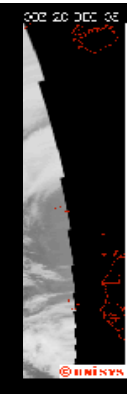
$L_R \sim O(10$

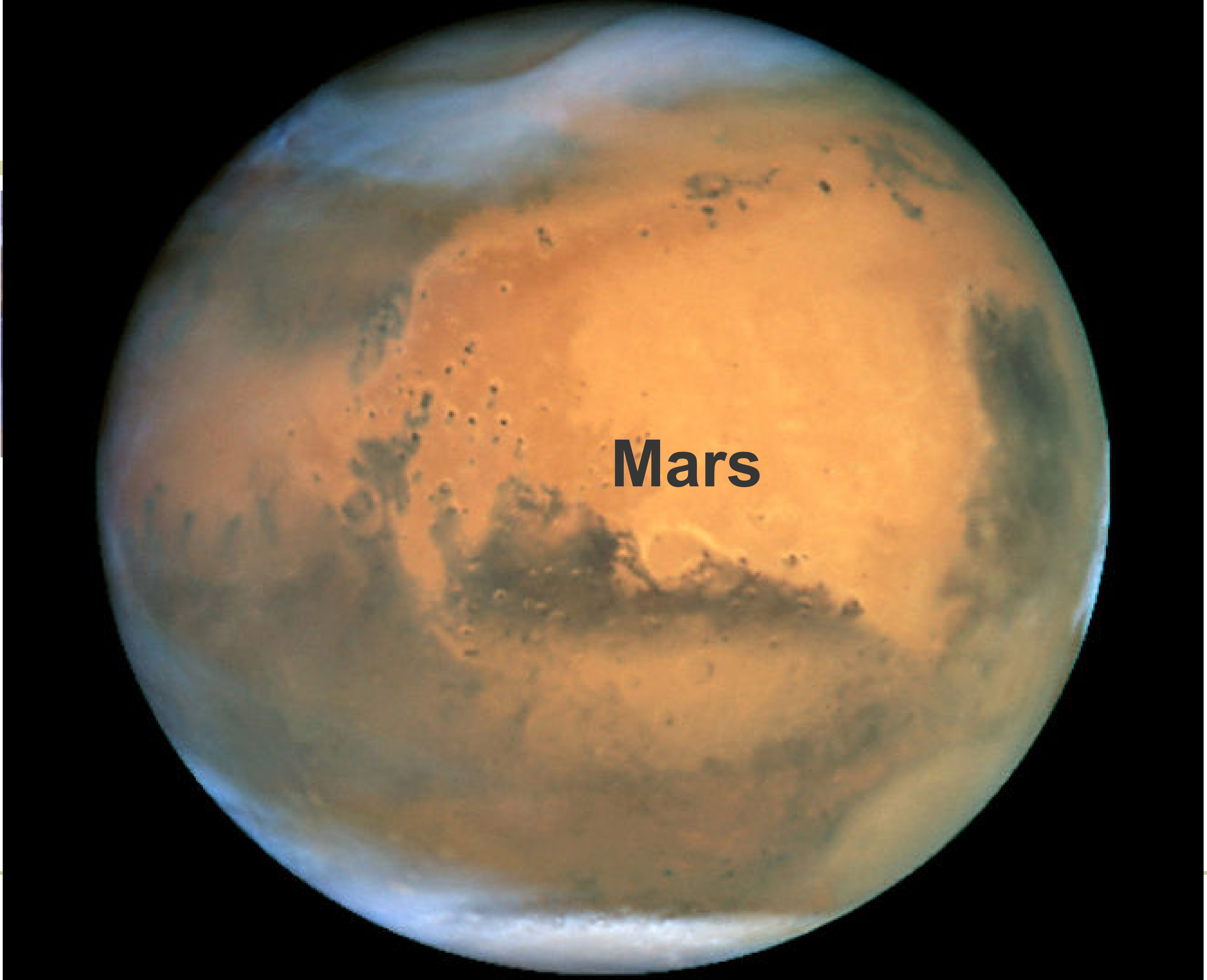
Observed

Baroclinic eddies



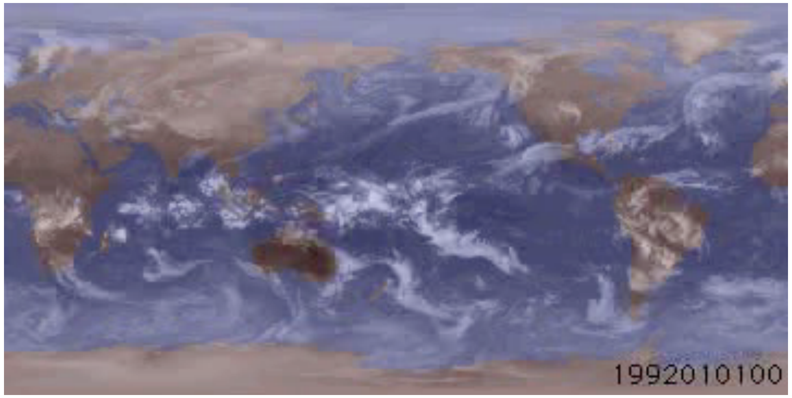


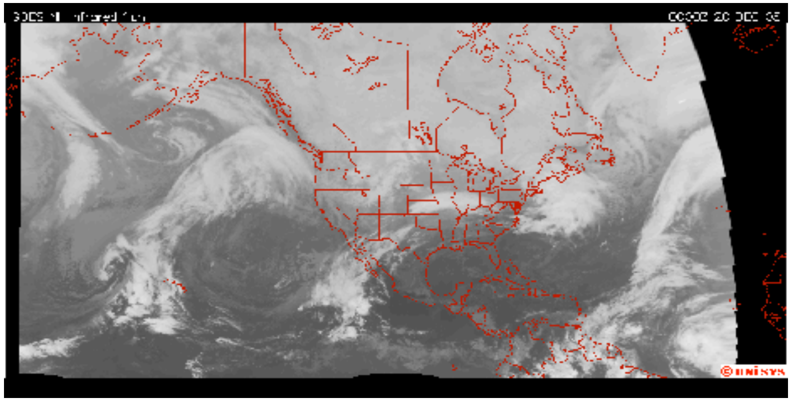



Mars

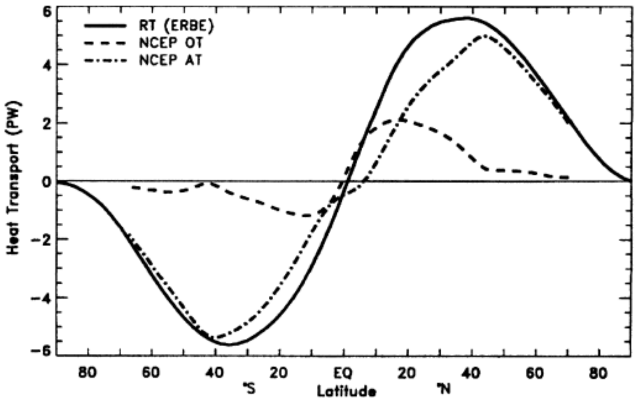
Observed

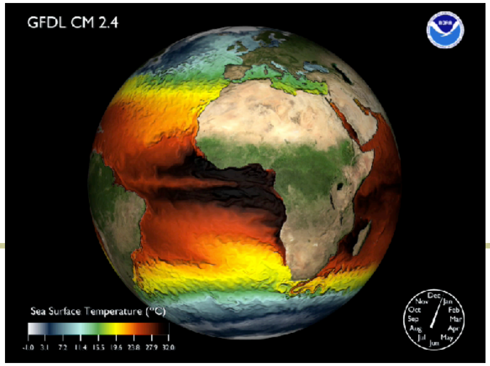
Baroclinic eddies





斜压扰动是中纬度各种日常天气现象背后主要的物理过程，更是维持大气环流和地球大气能量南北输送以及现在的地球气候状态的最主要动力机制，此外斜压扰动也是海洋和其他一些行星大气中的主要动力过程。





Outline

- Observations
- The Ferrel Cell
- Baroclinic eddies
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction, E-P flux
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- Energy cycle

Baroclinic eddies

- baroclinic instability

- Instability:
  - **Phenomenon:** Given a *basic flow* with *perturbations* at the initial moment, if the perturbation *grows with time*, the basic flow is always taken *unstable*.
  - **Mathematics:** $P \propto Ae^{\alpha t}, \exists\, \alpha > 0$ (相对于波动解:$P \propto Ae^{i\omega t}$)
  - **Energy:** 能量源 → 扰动动能
- **Linear Instability:** the instability that arises in *a linear system*.

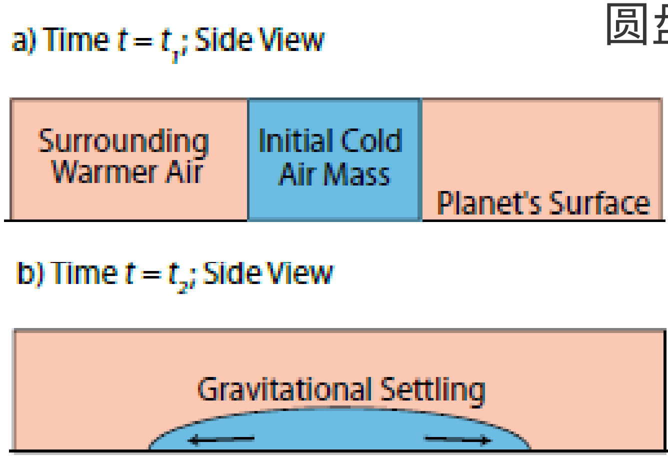
Baroclinic eddies

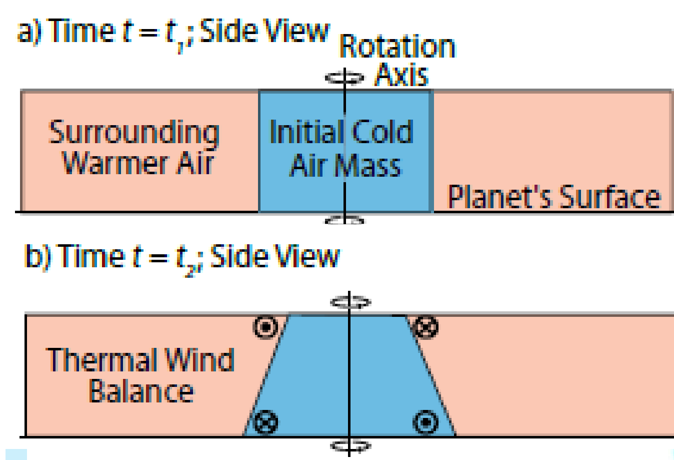
- baroclinic instability

- Baroclinic Instability - “is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*”.

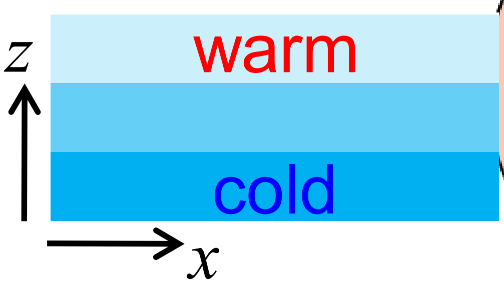
- **Baroclinic Instability** - "is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*".

圆盘实验

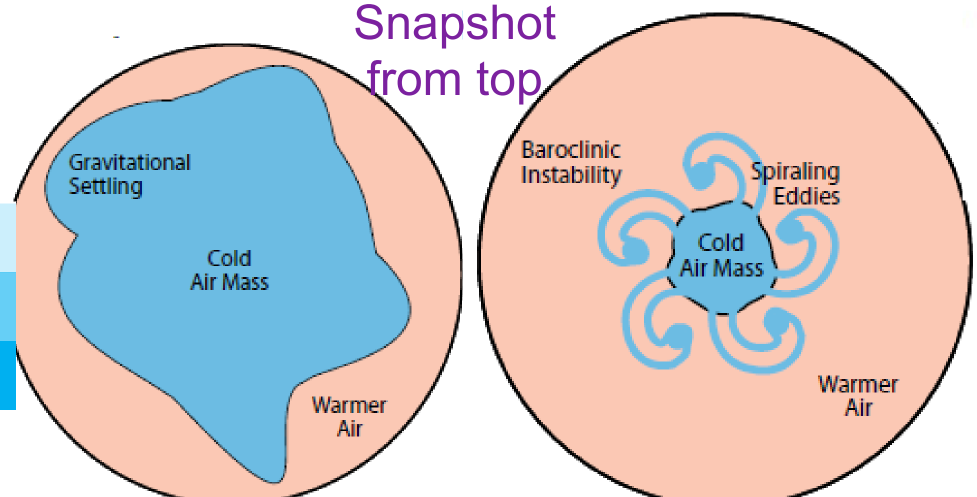


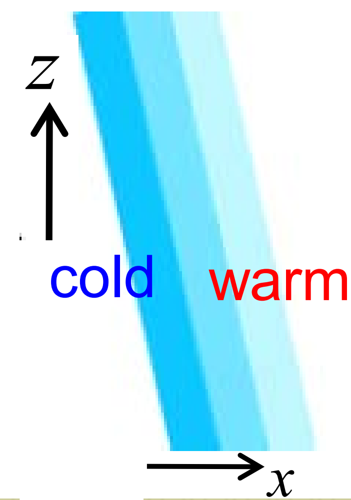


In equilibrium



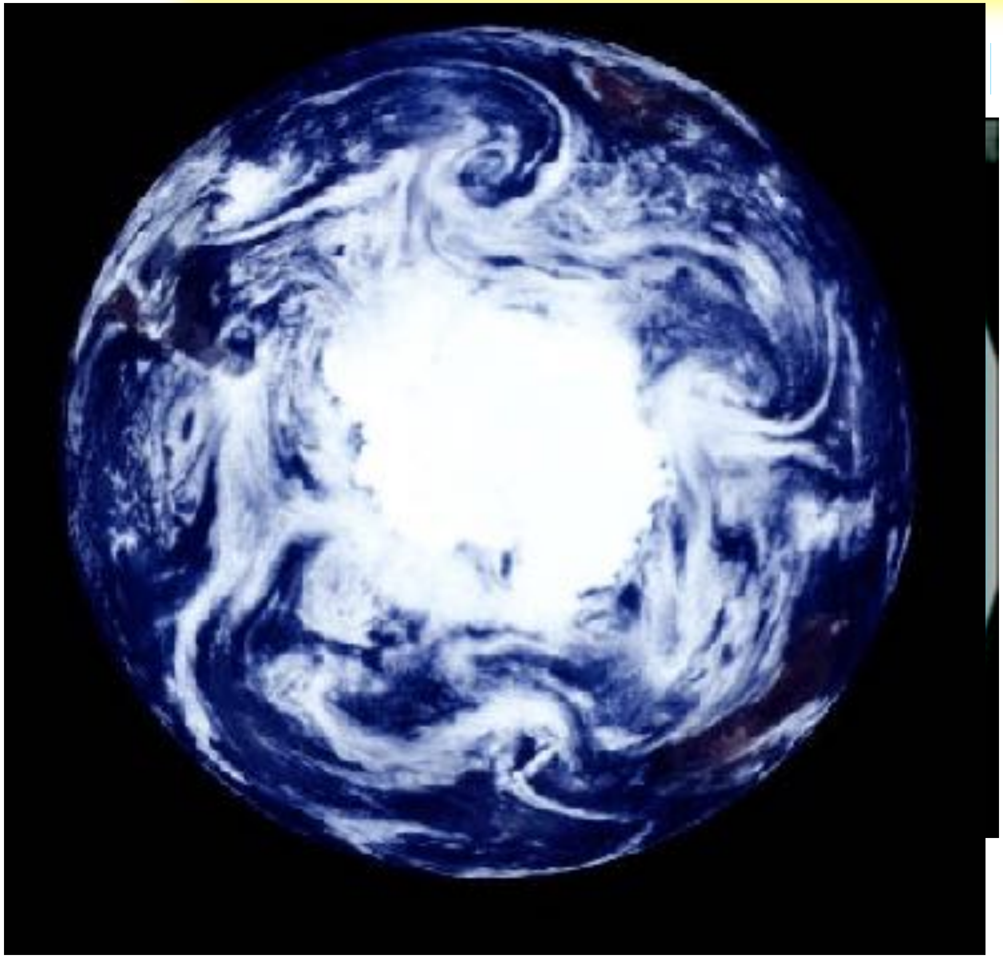
Snapshot from top



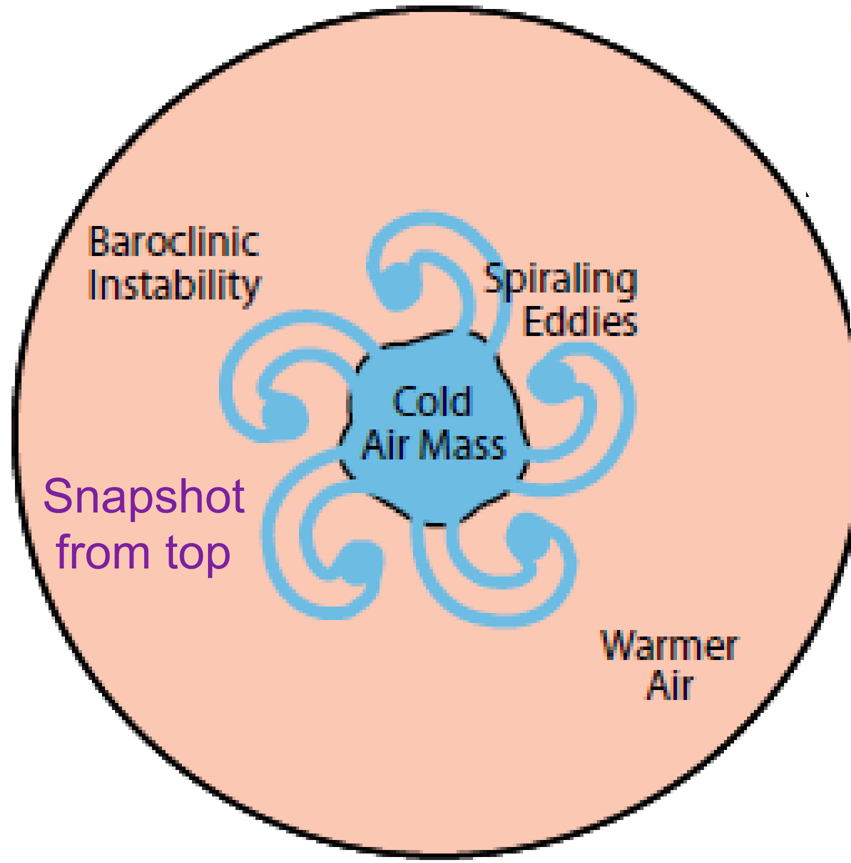


Above from Prof. Fang Juan's class slides

- **Baroclinic Instability** - "is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*".



Satellite view at south pole, from NASA.



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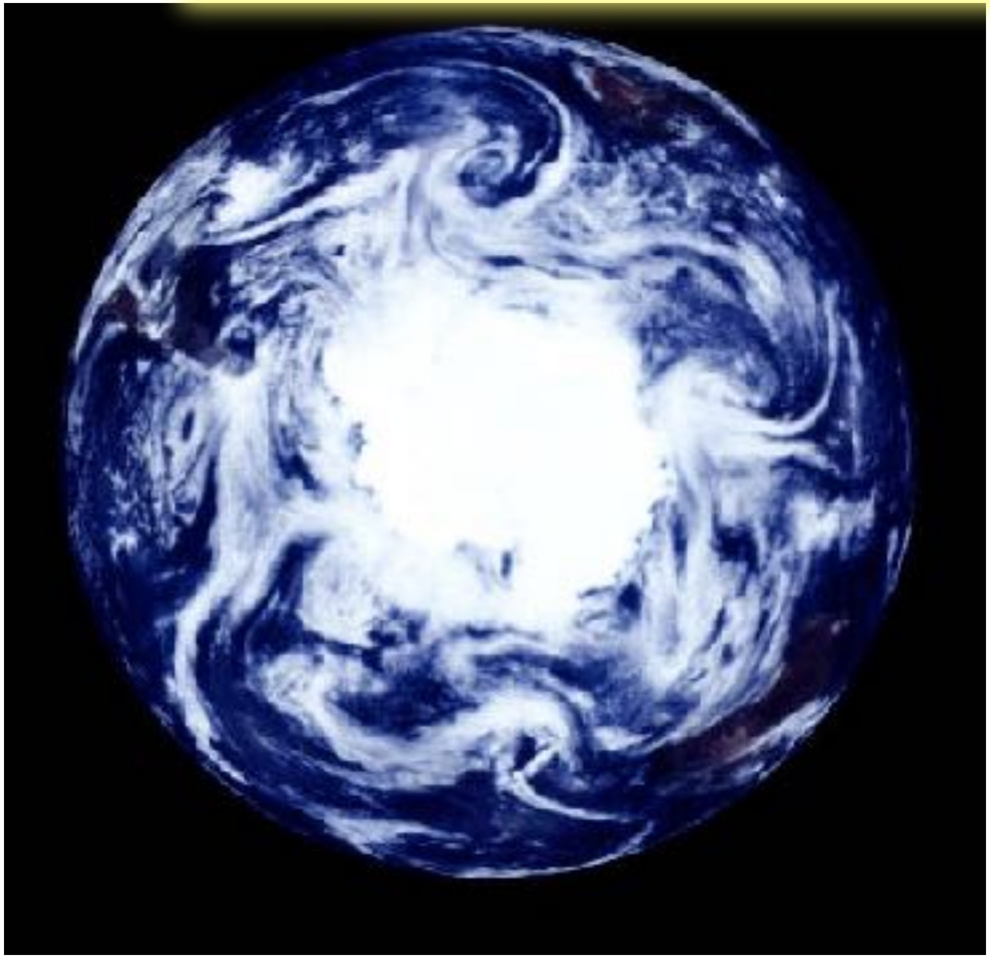
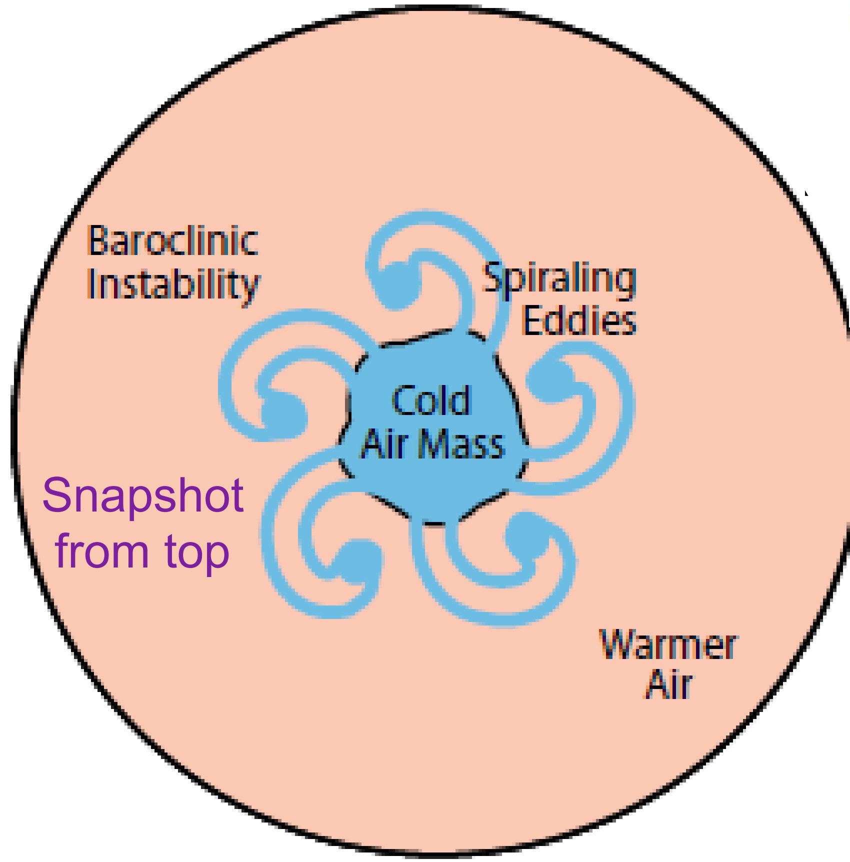


Satellite view at south pole, from NASA.



Baroclinic eddies

- baroclinic instability

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Baroclinic eddies

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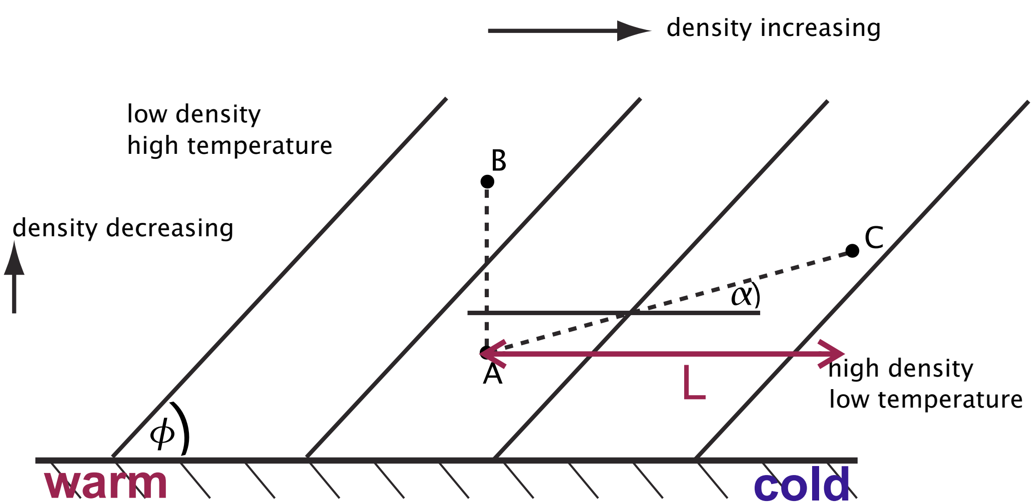


From A to B: negative buoyant

If A and C are interchanged:

$$PE = \int \rho g dz$$

$$\begin{aligned}\Delta PE &= g(\rho_A z_A + \rho_C z_C - \rho_C z_A - \rho_A z_C) \\ &= g(z_A - z_C)(\rho_A - \rho_C) \\ &= g\Delta\rho\Delta z\end{aligned}$$

$$\Delta PE = g\left(L\frac{\partial\rho}{\partial y} + L\tan\alpha\frac{\partial\rho}{\partial z}\right)L\tan\alpha$$

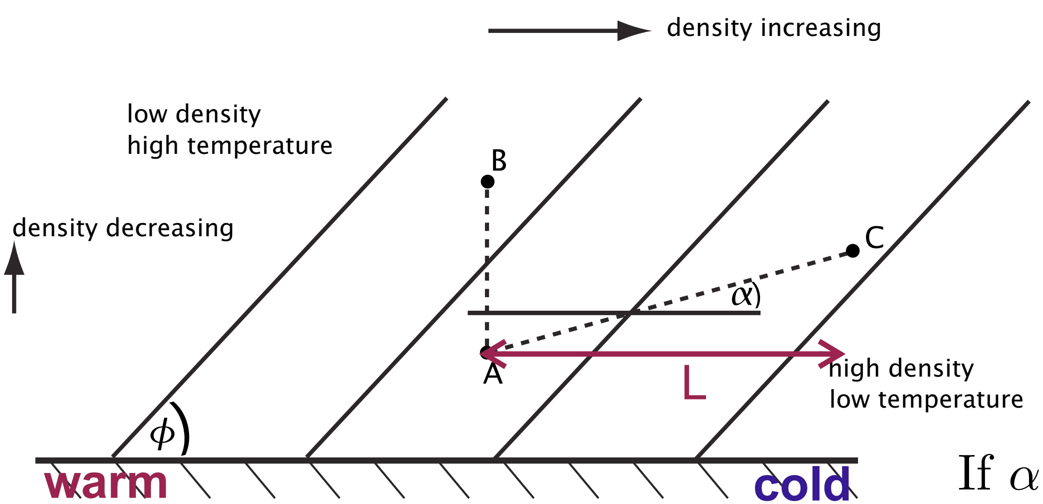
Assume small α and ϕ

$$\Delta PE = gL^2\frac{\partial\rho}{\partial y}\alpha\left(1 - \frac{\alpha}{\phi}\right)$$

Baroclinic eddies

- baroclinic instability

- **Baroclinic Instability** - “is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*”.



From A to B: negative buoyant

If A and C are interchanged:

$$PE = \int \rho g dz$$

$$\Delta PE = gL^2 \frac{\partial \rho}{\partial y} \alpha \left(1 - \frac{\alpha}{\phi}\right)$$

α is called mixing slope.

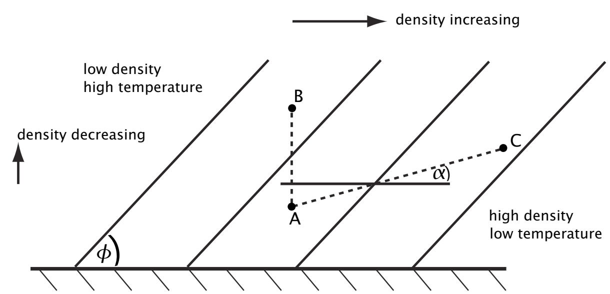
If $\alpha < \phi$, a loss of potential energy.

If $\alpha = \frac{1}{2}\phi$, ΔPE is strongest.

Baroclinic eddies
- baroclinic instability

- Baroclinic Instability - "is an instability that arises in *rotating*, *stratified* fluids that are subject to a *horizontal temperature gradient*".

- Energetics: PE → KE

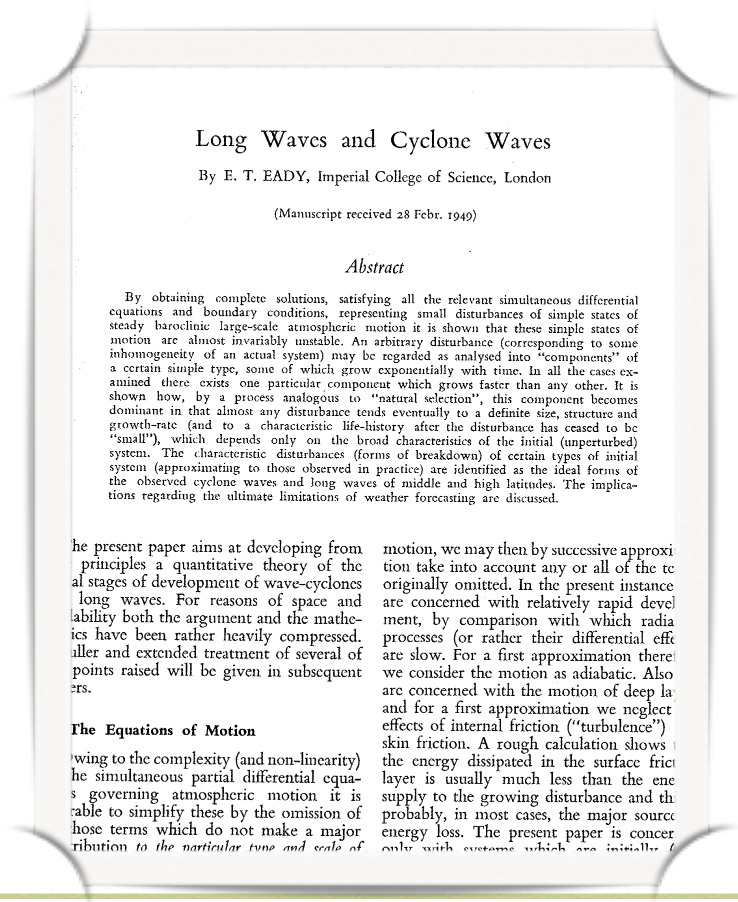


- Mathematics:
  - Linear Baroclinic Instability
    - Linear baroclinic system →
      - Eady's model (1949)
      - Charney's model (1947)

Baroclinic eddies

- linear baroclinic instability

- Eady's model (1949)
- Charney's model (1947)



Long Waves and Cyclone Waves

By E. T. EADY, Imperial College of Science, London

(Manuscript received 28 Febr. 1949)

Abstract

By obtaining complete solutions, satisfying all the relevant simultaneous differential equations and boundary conditions, representing small disturbances of simple states of steady baroclinic large-scale atmospheric motion it is shown that these simple states of motion are almost invariably unstable. An arbitrary disturbance (corresponding to some inhomogeneity of an actual system) may be regarded as analysed into "components" of a certain simple type, some of which grow exponentially with time. In all the cases examined there exists one particular component which grows faster than any other. It is shown how, by a process analogous to "natural selection", this component becomes dominant in that almost any disturbance tends eventually to a definite size, structure and growth-rate (and to a characteristic life-history after the disturbance has ceased to be "small"), which depends only on the broad characteristics of the initial (unperturbed) system. The characteristic disturbances (forms of breakdown) of certain types of initial system (approximating to those observed in practice) are identified as the ideal forms of the observed cyclone waves and long waves of middle and high latitudes. The implications regarding the ultimate limitations of weather forecasting are discussed.

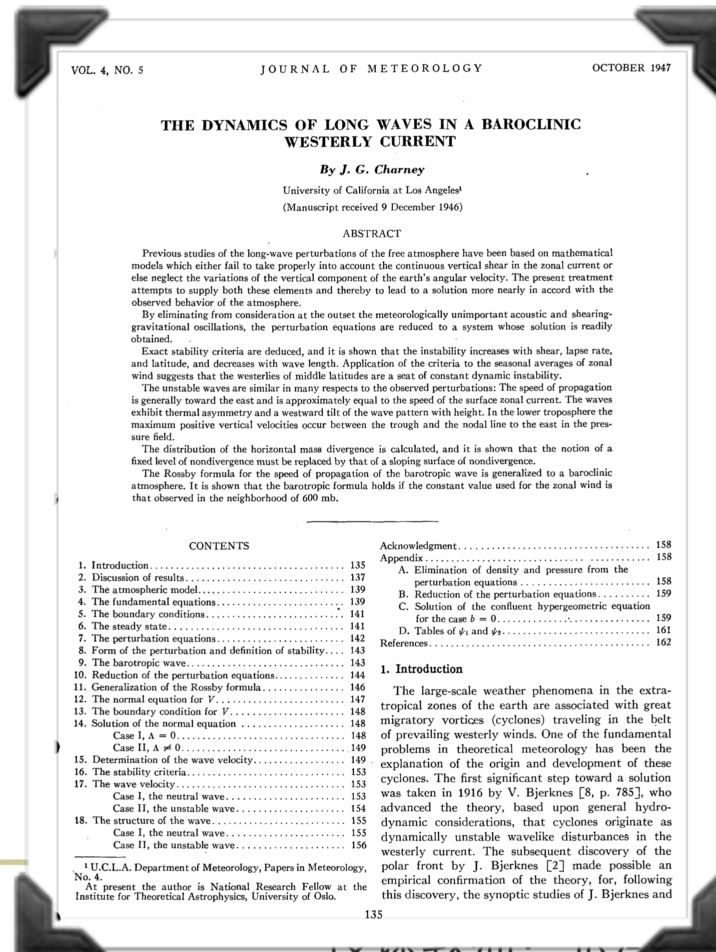
he present paper aims at developing from principles a quantitative theory of the al stages of development of wave-cyclones long waves. For reasons of space and ability both the argument and the mathe- ics have been rather heavily compressed. uller and extended treatment of several of points raised will be given in subsequent ers.

The Equations of Motion

wing to the complexity (and non-linearity) he simultaneous partial differential equa- s governing atmospheric motion it is rable to simplify these by the omission of hose terms which do not make a major ribution *to the particular type and scale of*

motion, we may then by successive approxi tion take into account any or all of the te originally omitted. In the present instance are concerned with relatively rapid devel ment, by comparison with which radia processes (or rather their differential effe are slow. For a first approximation there we consider the motion as adiabatic. Also are concerned with the motion of deep la and for a first approximation we neglect effects of internal friction ("turbulence") skin friction. A rough calculation shows the energy dissipated in the surface frict layer is usually much less than the ene supply to the growing disturbance and th probably, in most cases, the major source energy loss. The present paper is concer only with systems which are initially

Eady, E. 1949. Tellus, 1, 33-52



VOL. 4, NO. 5 JOURNAL OF METEOROLOGY OCTOBER 1947

THE DYNAMICS OF LONG WAVES IN A BAROCLINIC WESTERLY CURRENT

By J. G. Charney

University of California at Los Angeles[1]

(Manuscript received 9 December 1946)

ABSTRACT

Previous studies of the long-wave perturbations of the free atmosphere have been based on mathematical models which either fail to take properly into account the continuous vertical shear in the zonal current or else neglect the variations of the vertical component of the earth's angular velocity. The present treatment attempts to supply both these elements and thereby to lead to a solution more nearly in accord with the observed behavior of the atmosphere.

By eliminating from consideration at the outset the meteorologically unimportant acoustic and shearing-gravitational oscillations, the perturbation equations are reduced to a system whose solution is readily obtained.

Exact stability criteria are deduced, and it is shown that the instability increases with shear, lapse rate, and latitude, and decreases with wave length. Application of the criteria to the seasonal averages of zonal wind suggests that the westerlies of middle latitudes are a seat of constant dynamic instability.

The unstable waves are similar in many respects to the observed perturbations: The speed of propagation is generally toward the east and is approximately equal to the speed of the surface zonal current. The waves exhibit thermal asymmetry and a westward tilt of the wave pattern with height. In the lower troposphere the maximum positive vertical velocities occur between the trough and the nodal line to the east in the pressure field.

The distribution of the horizontal mass divergence is calculated, and it is shown that the notion of a fixed level of nondivergence must be replaced by that of a sloping surface of nondivergence.

The Rossby formula for the speed of propagation of the barotropic wave is generalized to a baroclinic atmosphere. It is shown that the barotropic formula holds if the constant value used for the zonal wind is that observed in the neighborhood of 600 mb.

CONTENTS

1. Introduction

The large-scale weather phenomena in the extra-tropical zones of the earth are associated with great migratory vortices (cyclones) traveling in the belt of prevailing westerly winds. One of the fundamental problems in theoretical meteorology has been the explanation of the origin and development of these cyclones. The first significant step toward a solution was taken in 1916 by V. Bjerknes [8, p. 785], who advanced the theory, based upon general hydrodynamic considerations, that cyclones originate as dynamically unstable wavelike disturbances in the westerly current. The subsequent discovery of the polar front by J. Bjerknes [2] made possible an empirical confirmation of the theory, for, following this discovery, the synoptic studies of J. Bjerknes and

[1] U.C.L.A. Department of Meteorology, Papers in Meteorology, No. 4.
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- linear baroclinic instability

- Eady's model

Eric Thomas Eady
(1915-1966)

- Charney's model

Jule Gregory Charney
1917 – 1981

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" **JULE CHARNEY** was one of the dominant figures in atmospheric science in the three decades following World War II. Much of the change in meteorology from an art to a science is due to his scientific vision and his thorough commitment to people and programs in this field."

-- *by Norman Phillips*