第四章：

中纬度的经向环流系统(IV)

- Ferrel cell, baroclinic eddies and the westerly jet

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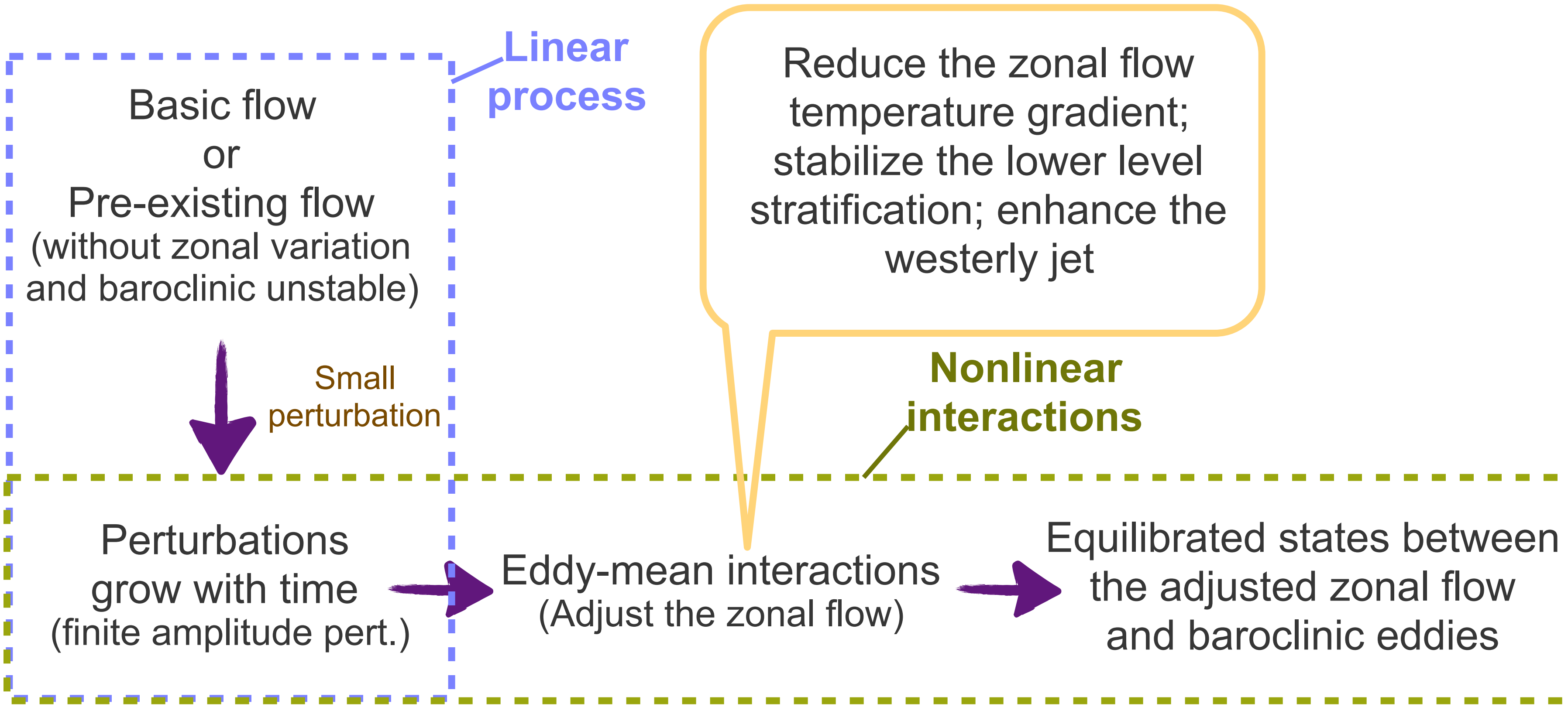
Baroclinic eddies

Review

- From linear to nonlinear



Baroclinic eddies

- E-P flux

Review

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

- In a QG, **steady**, **adiabatic** and **frictionless** flow:

$$[v] = \frac{1}{f}\frac{\partial}{\partial y}([u^* v^*]) \qquad [\omega] = -\frac{\partial}{\partial y}\left(\frac{[\theta^* v^*]}{\partial\theta_s/\partial p}\right) \qquad \nabla\cdot\mathcal{F} = 0$$

- In a QG, **steady** flow:

$$f[v] - \frac{\partial([u^* v^*])}{\partial y} + [F_x] = 0$$

$$[\omega]\frac{\partial\theta_s}{\partial p} + \frac{\partial([\theta^* v^*])}{\partial y} - \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p} = 0$$

The **meridional overturning flow**, in addition to the **eddy forcing**, has to balance the **external forcing**.

E-P flux, TEM and Residual Circulation - Summary

- E-P flux: $$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$
- In a **steady**, **adiabatic** and **frictionless** flow:

$$[v] = \frac{1}{f}\frac{\partial}{\partial y}([u^* v^*]) \qquad [\omega] = -\frac{\partial}{\partial y}\left(\frac{[\theta^* v^*]}{\partial\theta_s/\partial p}\right) \qquad \nabla\cdot\mathcal{F} = 0$$

- Residual mean circulations:

$$[\tilde{\omega}] = [\omega] + \frac{\partial}{\partial y}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right), \qquad [\tilde{v}] = [v] - \frac{\partial}{\partial p}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right)$$

- TEM equations: $$\frac{\partial[u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x], \qquad \frac{\partial[\theta]}{\partial t} = -[\tilde{\omega}]\frac{\partial\theta_s}{\partial p} + \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p}$$

Baroclinic eddies

- TEM

Review

$$\tilde{\psi} = \psi_m + \frac{[v^*\theta^*]}{\partial\theta_s/\partial p}$$

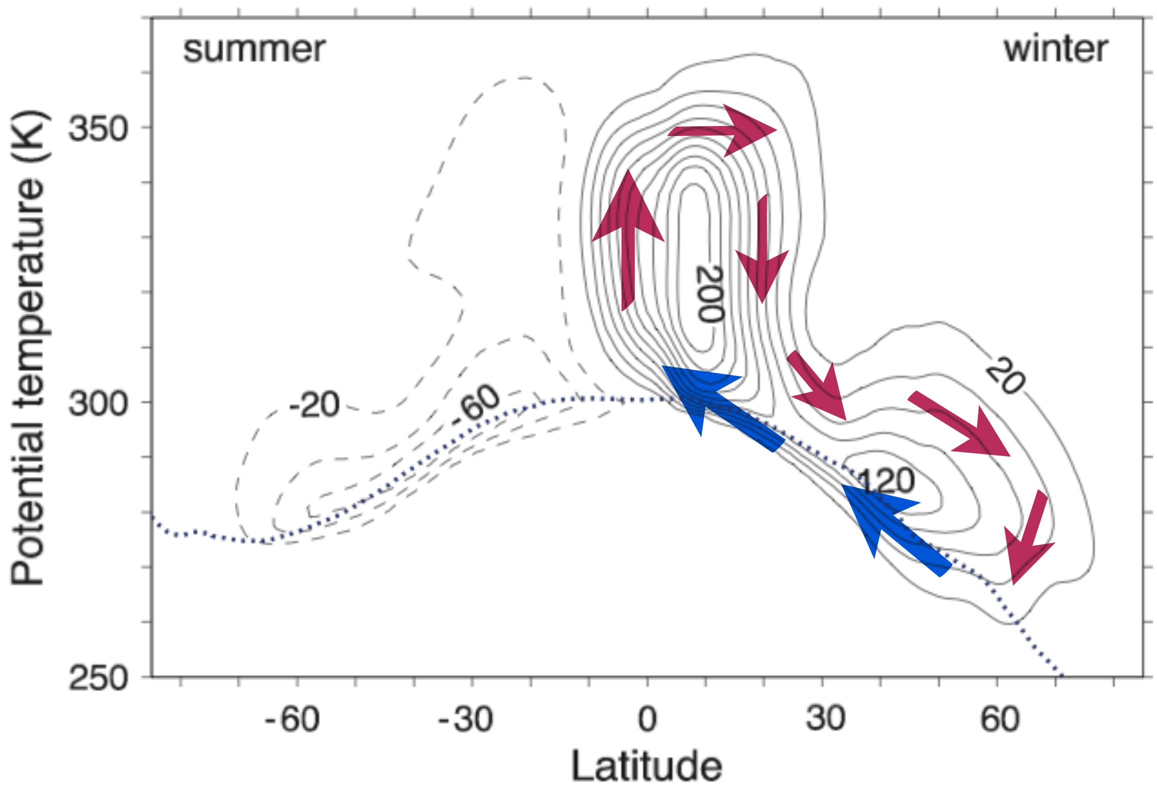
- In **isentropic** coordinate

$$\begin{aligned}\frac{D}{Dt} &= \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla_\theta + \frac{D\theta}{Dt}\frac{\partial}{\partial\theta} \\ &= \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla_\theta + \dot{\theta}\frac{\partial}{\partial\theta}\end{aligned}$$

zero for adiabatic flow

The Ferrel cell in the isentropic coordinate is essentially reflect the *Residual Mean Circulation*.

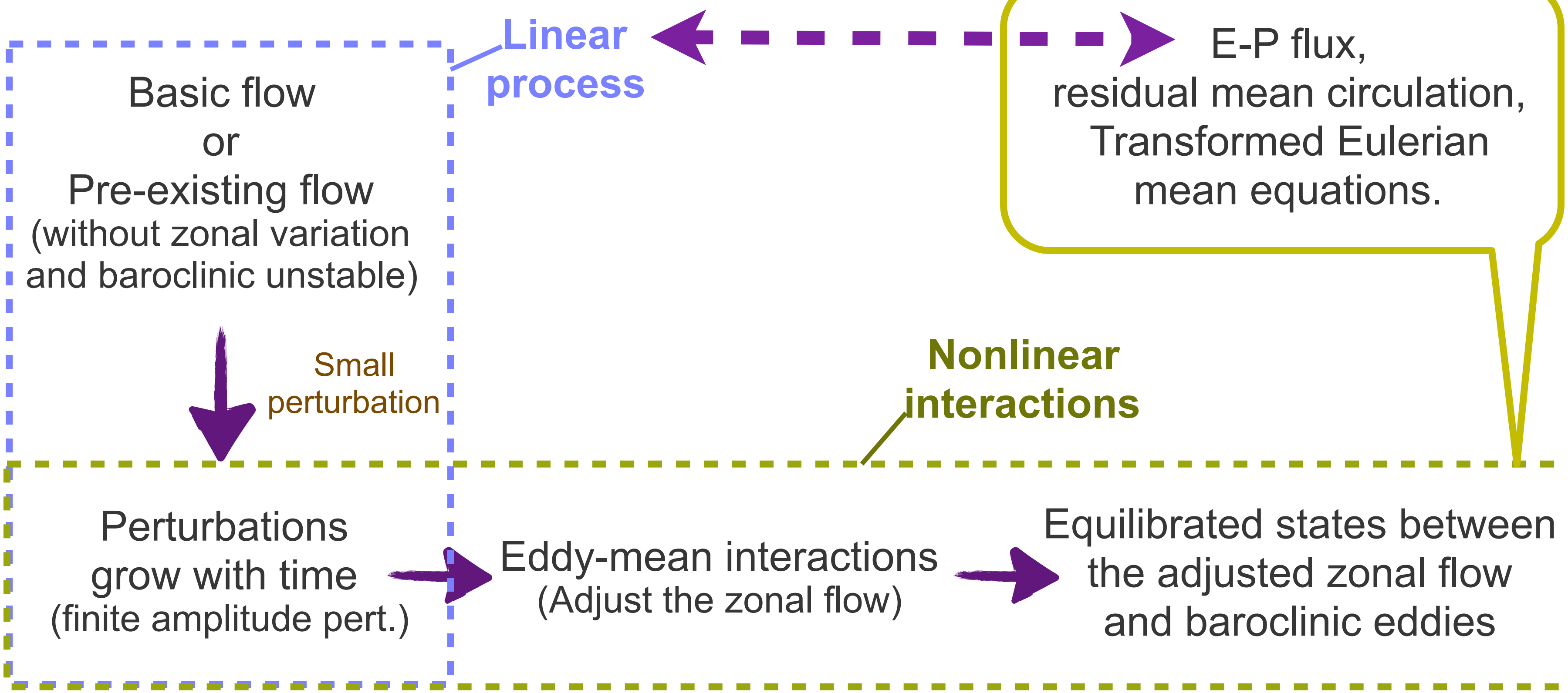
Case 2: Observed circulation



(Fig.11.4, Vallis, 2006)

Baroclinic eddies

- From linear to nonlinear



Baroclinic eddies

- E-P flux: a second view

Review

- E-P flux and the Quasi-geostrophic potential vorticity

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

From the definition of QG potential vorticity:

$$\bar{q} = \frac{\partial^2\bar{\psi}}{\partial y^2} + \beta y + f_o^2\frac{\partial}{\partial p}\left(\frac{1}{s}\frac{\partial\bar{\psi}}{\partial p}\right)$$

$$q' = \frac{\partial^2\psi'}{\partial x^2} + \frac{\partial^2\psi'}{\partial y^2} + f_o^2\frac{\partial}{\partial p}\left(\frac{1}{s}\frac{\partial\psi'}{\partial p}\right)$$

$$\zeta' \qquad\qquad f_o\frac{\partial}{\partial p}\left(\frac{\theta'}{\partial\theta_s/\partial p}\right)$$

$$(u,v) = \left(-\frac{\partial\psi}{\partial y}, \frac{\partial\psi}{\partial x}\right) \qquad \theta = -\kappa f_o\frac{\partial\psi}{\partial p}$$

$$s = -\frac{1}{\kappa}\frac{\partial\theta_s}{\partial p} \qquad \kappa = \frac{p}{R}\left(\frac{p_o}{p}\right)^{R/c_p}$$

PV flux:

$$v'q' = v'\zeta' + \frac{f_o}{\partial\theta_s/\partial p}v'\frac{\partial\theta'}{\partial p}$$

$$v\zeta = v\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = -\frac{\partial}{\partial y}uv + \frac{1}{2}\frac{\partial}{\partial x}(v^2 - u^2)$$

$$v\frac{\partial\theta}{\partial p} = \frac{\partial}{\partial p}v\theta - \theta\frac{\partial v}{\partial p}$$

thermal wind relation for meridional wind

$$+\frac{\theta}{\kappa f_o}\frac{\partial\theta}{\partial x}$$

$$v\frac{\partial\theta}{\partial p} = \frac{\partial}{\partial p}v\theta + \frac{1}{2\kappa f_o}\frac{\partial}{\partial x}\theta^2$$

Note: ‘ denotes small perturbation

Baroclinic eddies

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From the definition of QG potential vorticity:

$$\begin{aligned} v'q' &= v'\zeta' + \frac{f_o}{\partial\theta_s/\partial p} v' \frac{\partial\theta'}{\partial p} \\ &= \frac{1}{2}\frac{\partial}{\partial x}\left(v'^2 - u'^2 + \frac{1}{\kappa}\frac{\theta'^2}{\partial\theta_s/\partial p}\right) \\ &\quad + \frac{\partial}{\partial y} u'v' \\ &\quad + f_o \frac{\partial}{\partial p}\frac{v'\theta'}{\partial\theta_s/\partial p} \end{aligned}$$

#1

Zonally averaged PV flux by eddies:

$$\begin{aligned} [v^* q^*] &= \frac{\partial}{\partial y}[u^* v^*] + f_o \frac{\partial}{\partial p}\frac{[v^*\theta^*]}{\partial\theta_s/\partial p} \\ &= \nabla\cdot\mathcal{F} \end{aligned}$$

Baroclinic eddies
- E-P flux: a second view

Review

- E-P flux and the Eliassen-Palm relation

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

Linearized PV equation (q=PV):

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) q' + \frac{\partial \psi'}{\partial x}\frac{\partial \bar{q}}{\partial y} = 0$$

Multiplying by q' and zonally average:

$$\frac{1}{2}\frac{\partial}{\partial t}[q'^2] + [v'q']\frac{\partial \bar{q}}{\partial y} = 0$$

Define *wave activity density:*

$$\mathcal{A} = \frac{[q'^2]}{2\partial\bar{q}/\partial y}$$

#2

$$\frac{\partial \mathcal{A}}{\partial t} + \nabla \cdot \mathcal{F} = 0$$

Eliassen-Palm relation

Baroclinic eddies

- E-P flux: a second view

Review

- E-P flux and the Rossby waves

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

Linearized PV equation (q=PV):

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) q' + \frac{\partial\psi'}{\partial x}\frac{\partial\bar{q}}{\partial y} = 0$$

Assume U is fixed, and $\frac{\partial\bar{q}}{\partial y} = \beta$

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left[\nabla^2\psi' + f_o^2\frac{\partial}{\partial p}\left(\frac{1}{s}\frac{\partial\psi'}{\partial p}\right)\right] + \beta\frac{\partial\psi'}{\partial x} = 0$$

Exist solutions of the form

$$\psi' = \mathrm{Re}\Psi e^{i(kx+ly+mp-\omega t)}$$

Dispersion relation of Rossby waves:

$$\omega = Uk - \frac{\beta k}{K^2}$$

$$K^2 = k^2 + l^2 + m^2 f_o^2/s$$

with group velocity

$$c_{gy} = \frac{2\beta kl}{K^4} \quad c_{gp} = \frac{2\beta km f_o^2/s}{K^4}$$

Baroclinic eddies

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Review

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$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) q' + \frac{\partial \psi'}{\partial x}\frac{\partial \bar{q}}{\partial y} = 0$$

with group velocity

$$c_{gy} = \frac{2\beta k l}{K^4} \quad c_{gp} = \frac{2\beta k m f_o^2/s}{K^4}$$

$$K^2 = k^2 + l^2 + m^2 f_o^2/s$$

$$\hat{u} = -\mathrm{Re}\, i l \Psi,\ \hat{v} = \mathrm{Re}\, i k \Psi$$

$$\hat{\theta} = -\mathrm{Re}\, i m \kappa f_o \Psi,\ \hat{q} = -\mathrm{Re} K^2 \Psi$$

Wave activity density:

$$\mathcal{A} = \frac{[q'^2]}{2\beta} = \frac{K^4}{4\beta}|\Psi^2|$$

$$-[u'v'] = \frac{1}{2} k l |\Psi^2| = c_{gy}\mathcal{A}$$

$$f_o\frac{[v'\theta']}{\partial\theta_s/\partial p} = \frac{f_o^2}{2s} k m |\Psi^2| = c_{gz}\mathcal{A}$$

#3

$$\vec{\mathcal{F}} = \vec{\mathbf{c}_\mathbf{g}}\mathcal{A}$$

E-P flux, TEM and Residual Circulation - Summary

- E-P flux: $$\mathcal{F} \equiv -[u^*v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

- In a **steady**, **adiabatic** and **frictionless** flow:

$$[v] = \frac{1}{f}\frac{\partial}{\partial y}([u^*v^*]) \qquad [\omega] = -\frac{\partial}{\partial y}\left(\frac{[\theta^*v^*]}{\partial\theta_s/\partial p}\right) \qquad \nabla\cdot\mathcal{F} = 0$$

#1
$$[v^*q^*] = \frac{\partial}{\partial y}[u^*v^*] + f_o\frac{\partial}{\partial p}\frac{[v^*\theta^*]}{\partial\theta_s/\partial p} = \nabla\cdot\mathcal{F}$$

#2
$$\frac{\partial\mathcal{A}}{\partial t} + \nabla\cdot\mathcal{F} = 0$$

#3
$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A} \quad\longrightarrow\quad \frac{\partial\mathcal{A}}{\partial t} + \nabla\cdot(\mathcal{A}\vec{\mathbf{c_g}}) = 0$$

- Residual mean circulations:

$$[\tilde{\omega}] = [\omega] + \frac{\partial}{\partial y}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right), \qquad [\tilde{v}] = [v] - \frac{\partial}{\partial p}\left(\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\right)$$

- TEM equations: $$\frac{\partial[u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x], \qquad \frac{\partial[\theta]}{\partial t} = -[\tilde{\omega}]\frac{\partial\theta_s}{\partial p} + \left(\frac{p_o}{p}\right)^{R/c_p}\frac{[Q]}{c_p}$$

Outline

- Observations
- The Ferrel Cell
- Baroclinic eddies
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction, E-P flux
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- The energy cycle

Baroclinic eddy life cycle

- An E-P flux view

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

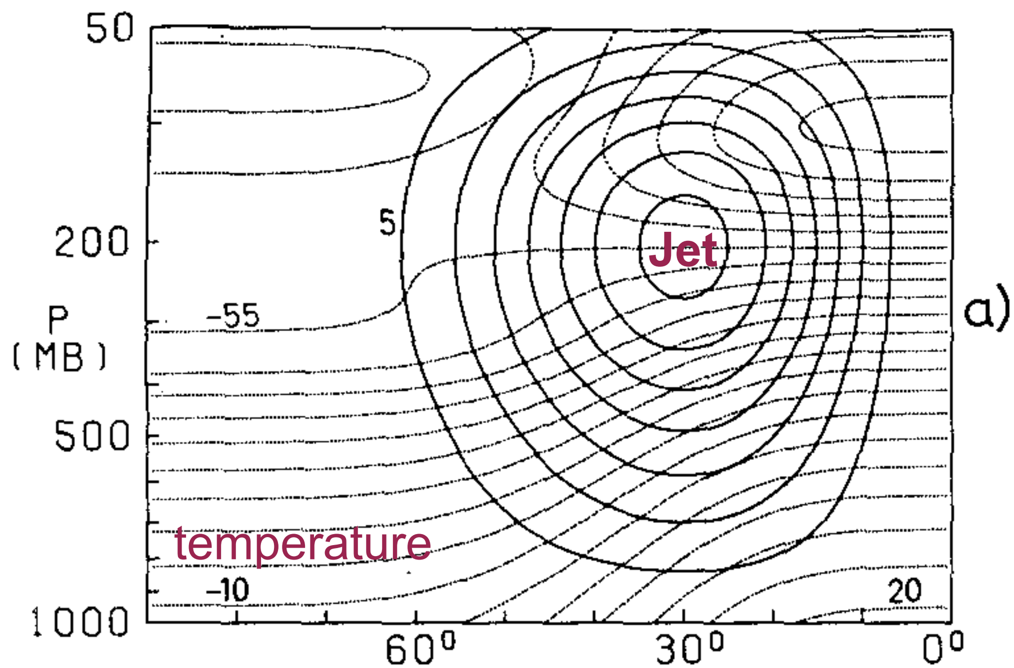
$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

$$\frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x]$$

$$\frac{\partial [\theta]}{\partial t} = -[\tilde{\omega}]\frac{\partial\theta_s}{\partial p} + \left(\frac{p_o}{p}\right)^{R/c_p}$$

Numerical results from
Simmons and Hoskins, 1978, JAS

Initial state:



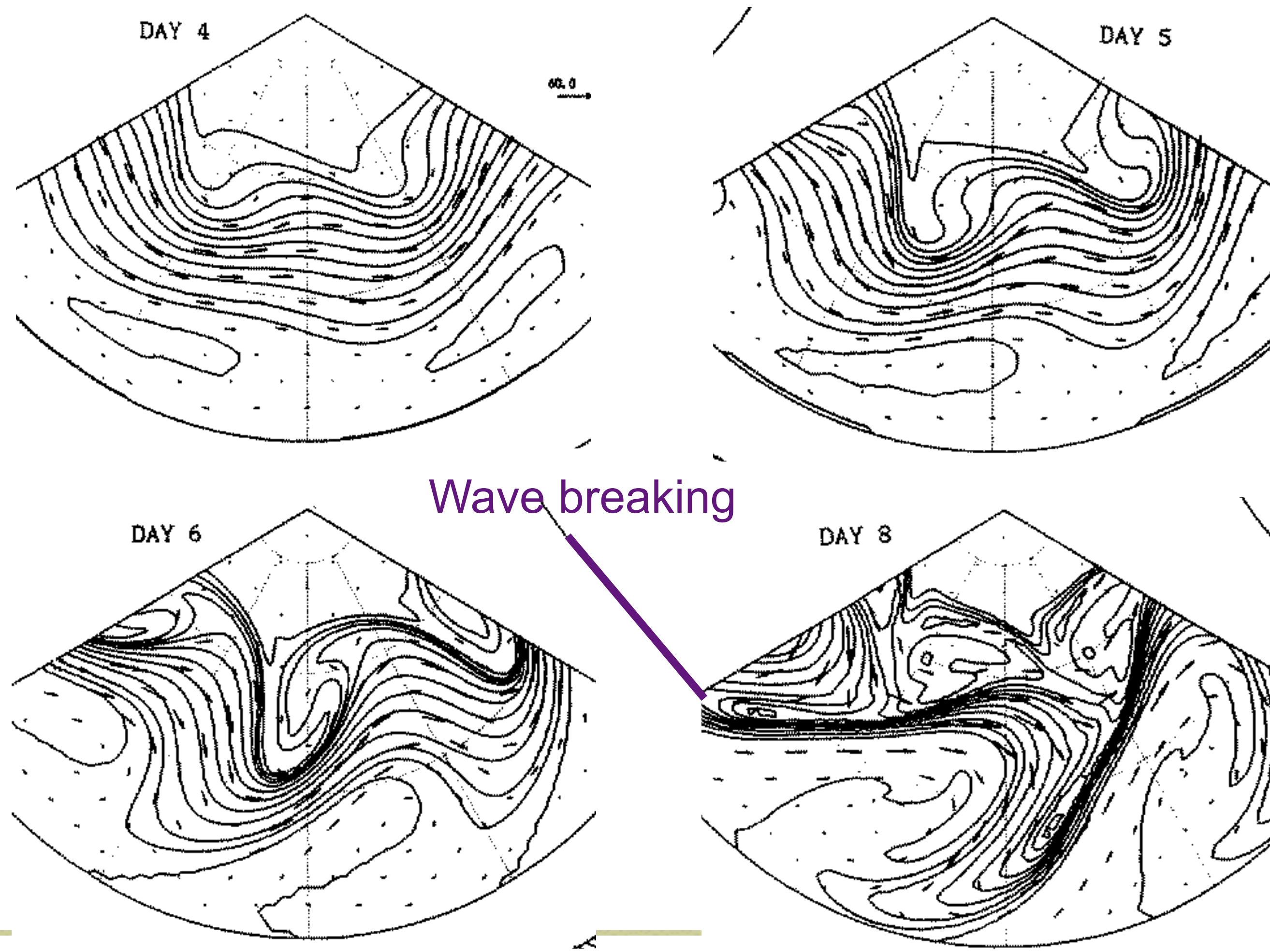
Baroclinic eddies
- baroclinic eddy life cycle

- Eddies' development

Small amplitude perturbations

Finite amplitude perturbations

Wave breaking



(Thorncroft et al, 1993, Q.J.R.)

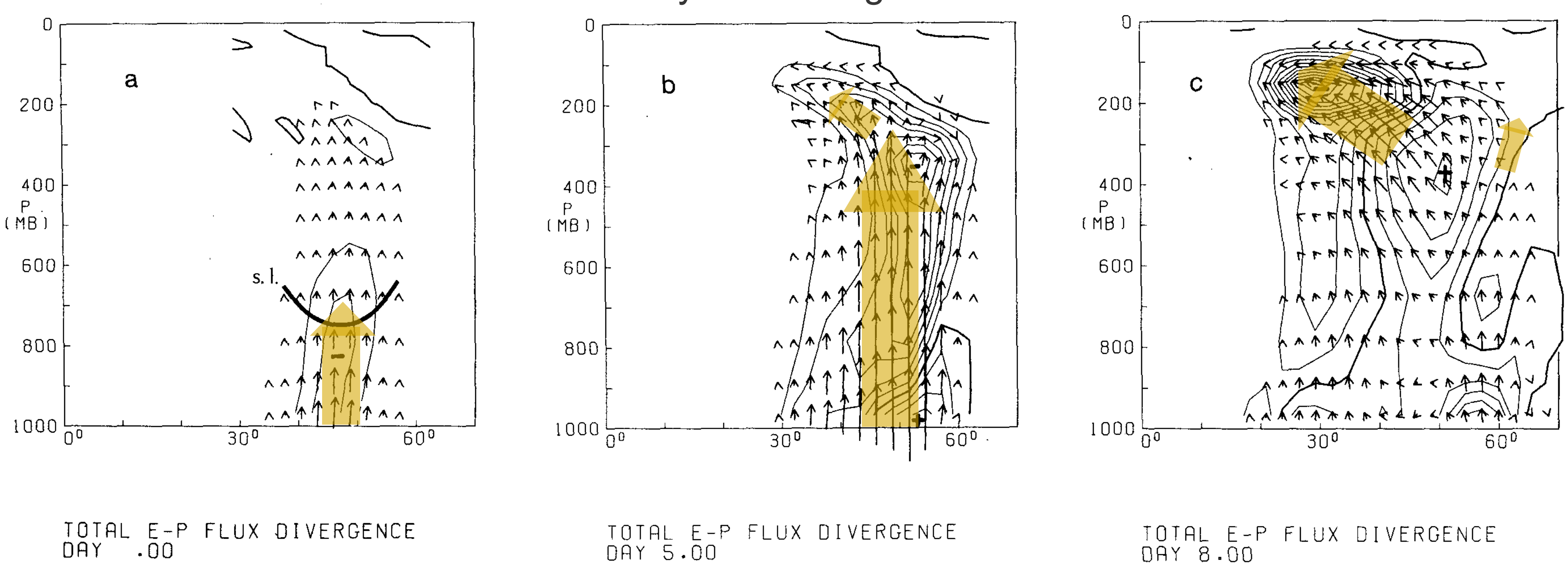
Baroclinic eddy life cycle - An E-P flux view

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

Numerical results from
Simmons and Hoskins, 1978, JAS

Eddies: generate at lower level, propagate **upwards** and **away** from the eddy source region



E-P flux

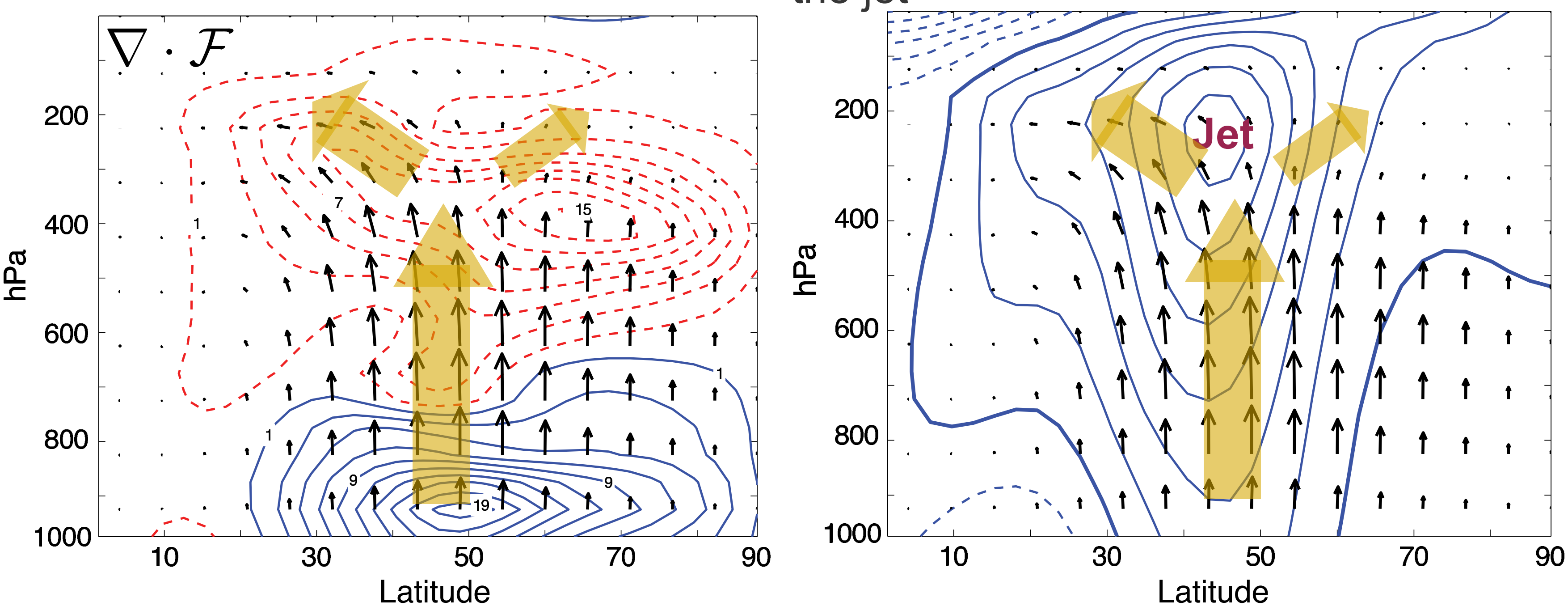
- In the equilibrium state

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

Numerical results from
idealized model with
pure midlatitude jet
(Vallis, 2006)

Wave energies:
propagate **upwards** and
away from the center of
the jet



E-P flux

- The westerly jet

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

$$\frac{\partial[u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x]$$

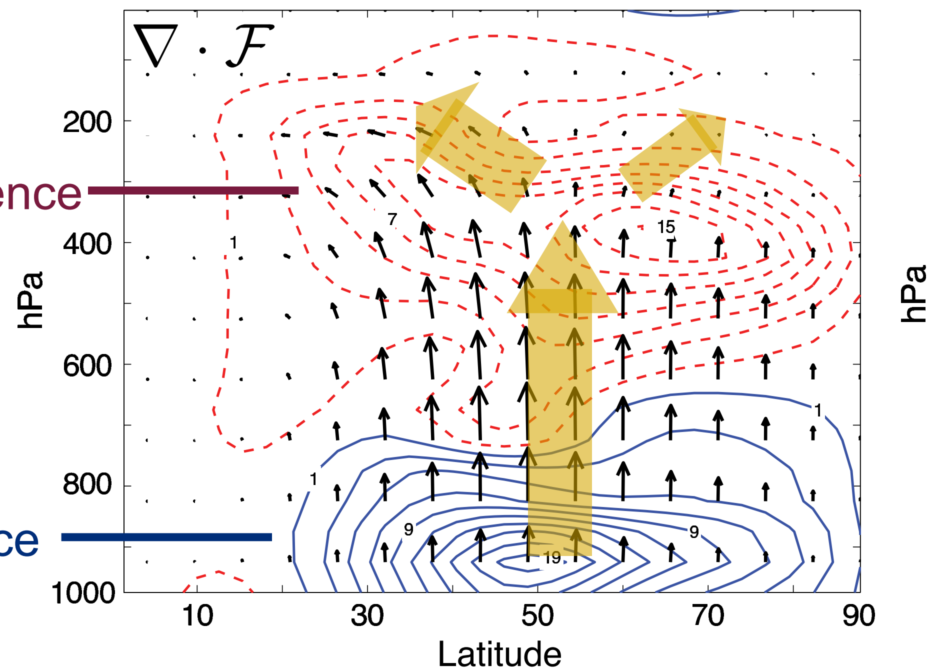
Wave energies: propagate **upwards** and **away** from the center of the jet

In the vertical direction:

Accelerating the lower jet
decelerating the upper jet
reduce the vertical shear of U

Convergence

Divergence



- The westerly jet

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

$$\frac{\partial [u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x]$$

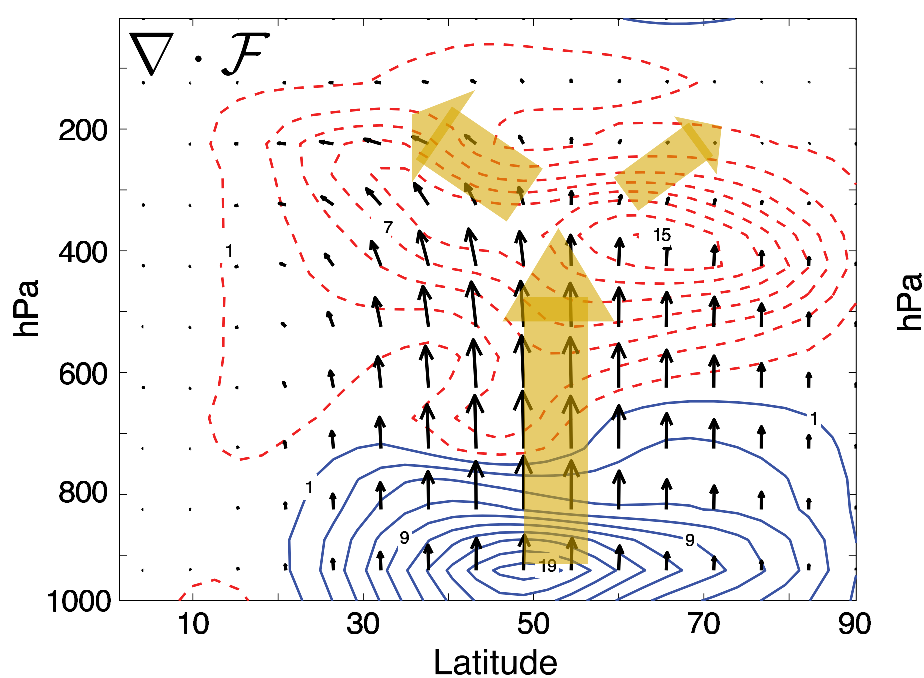
Integrate vertically:

$$\frac{1}{g}\int_0^{p_s} \mathrm{d}p$$

$$\frac{\partial}{\partial t} < [u] >= -\frac{\partial}{\partial y} < [u^* v^*] > -r[u_{\mathrm{surf}}]$$

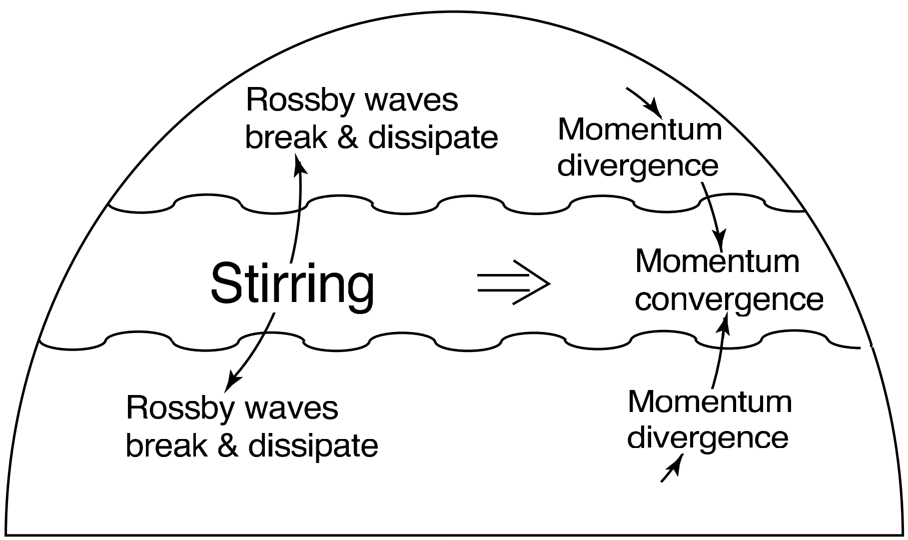
< > means vertical average

Wave energies: propagate **upwards** and **away** from the center of the jet



Eddy-driven jet:
- the momentum budget

$$\mathcal{F} \equiv -[u^*v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$



$$\frac{\partial}{\partial t} < [u] > = -\frac{\partial}{\partial y} < [u^*v^*] > -r[u_{\mathrm{surf}}]$$

< > means vertical average

Wave energies: propagate **upwards** and **away** from the center of the jet

In equilibrium: $\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$

$$r[u_{\mathrm{surf}}] \sim -\frac{\partial}{\partial y} < [u^*v^*] >$$

There MUST be **surface westerlies** at midlatitudes.

E-P flux

- in the real atmosphere

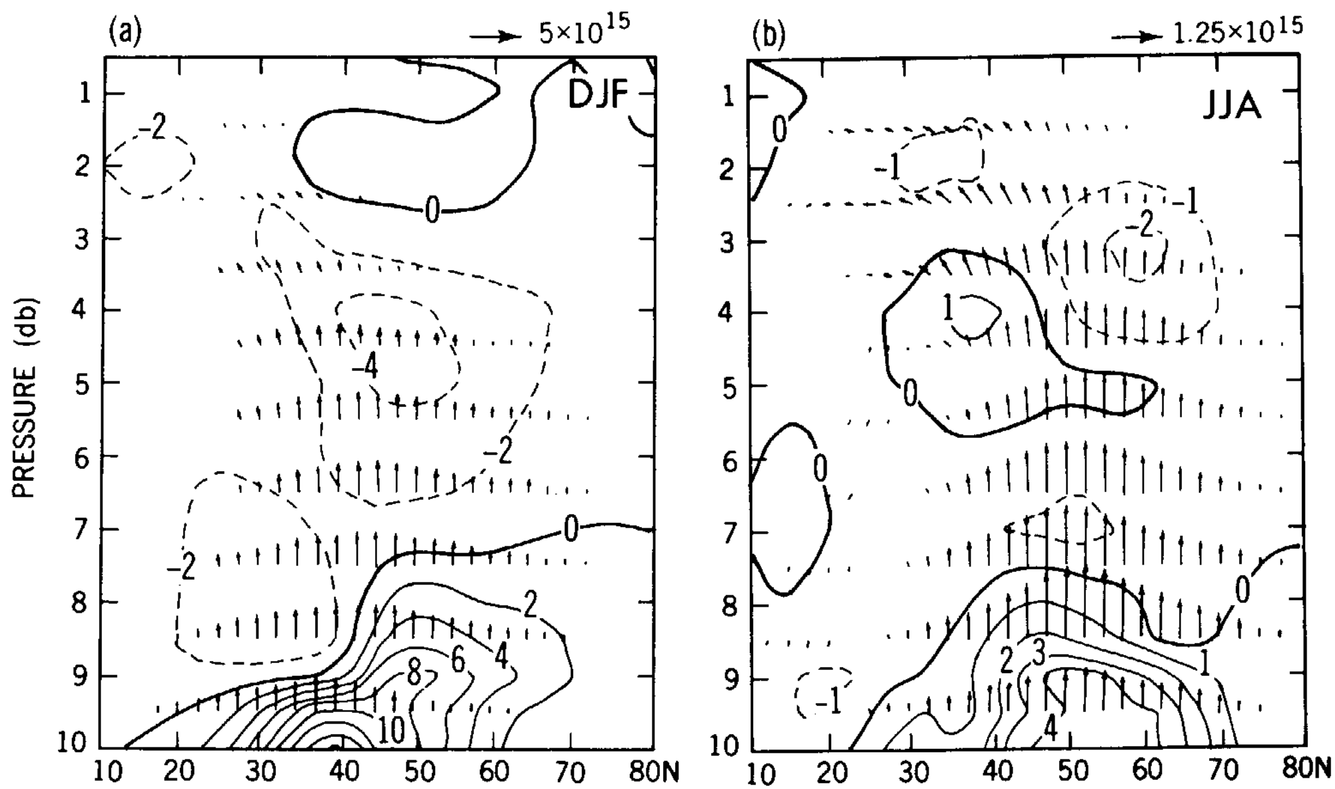
$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

Vertical component is dominant.

EP divergence in the lower layers; convergence in the upper layers.

E—P FLUX TRANSIENT EDDIES



E-P flux and the eddy-driven jet -summary

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

- Numerical results and observations: eddies generate in the lower level, propagate **upwards** and **away** from the eddy source region.

$$\frac{\partial[u]}{\partial t} = f[\tilde{v}] + \nabla\cdot\mathcal{F} + [F_x]$$

- Accelerating the lower jet, decelerating the upper jet, reduce the vertical shear of U

- Momentum budget indicates that there MUST be surface westerlies in the eddy source latitude.

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Energy cycles
in the baroclinic eddy-mean flow interactions

- Basic forms of energy:
  - Kinetic energy (动能): $$K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2)$$
  - Internal energy (内能): $$I = c_v T$$
  - Gravitational-potential energy (位能): $$\Phi = gz$$
  - Latent energy (相变潜热能): $$\mathrm{LH} = Lq$$
- Total energy:

$$E = I + \Phi + \mathrm{LH} + K$$

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  - Kinetic energy (动能): $K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2)$
  - Internal energy (内能): $I = c_v T$
  - Gravitational-potential energy (位能): $\Phi = gz$
  - Total potential energy:

$$\int_0^{\infty} \rho(I + \Phi)\mathrm{d}z = \frac{1}{g}\int_0^{p_s} (c_v T + RT)\mathrm{d}p = \frac{1}{g}\int_0^{p_s} c_p T \mathrm{d}p$$

Energy cycles
in the baroclinic eddy-mean flow interactions

- Basic forms of energy:
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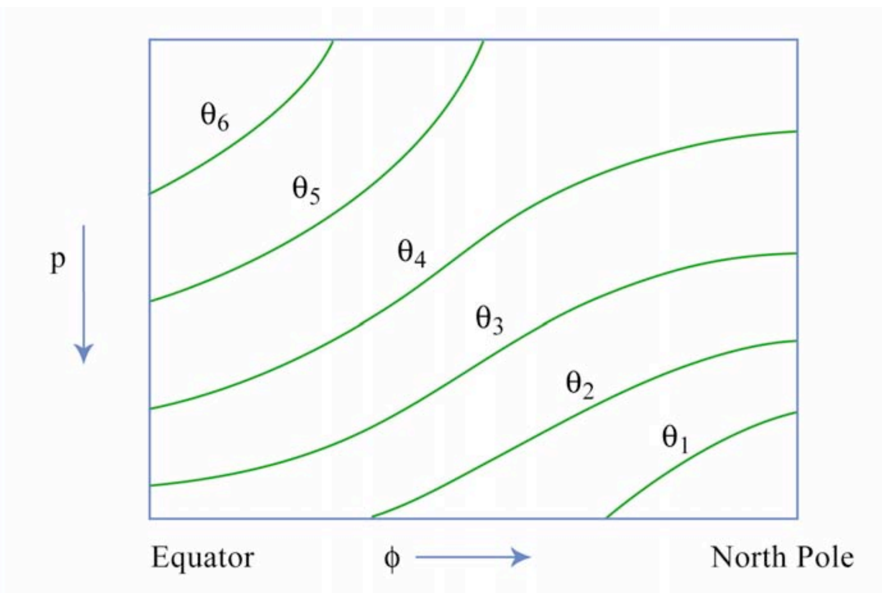
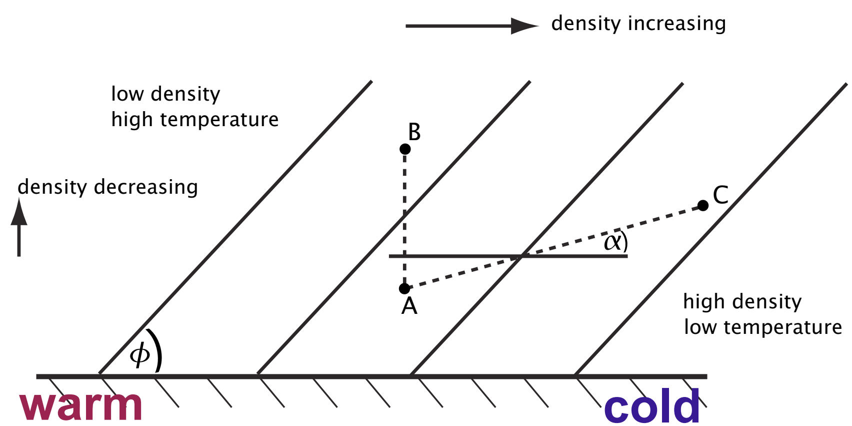
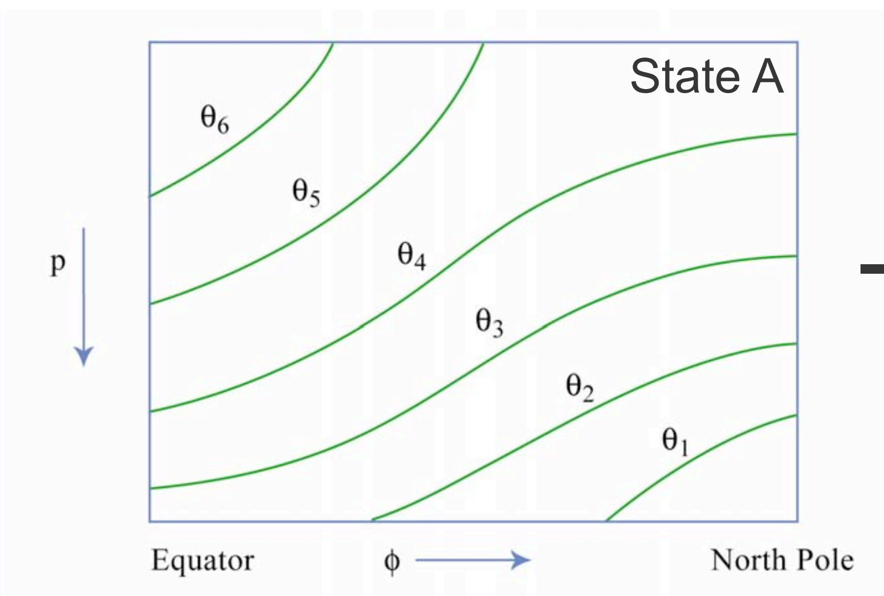


From Stone's class notes



- Basic forms of energy:
  - Kinetic energy (动能): $K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2)$

$$\int_0^{\infty} \rho(I + \Phi)\mathrm{d}z = \frac{1}{g}\int_0^{p_s}(c_v T + RT)\mathrm{d}p = \frac{1}{g}\int_0^{p_s} c_p T \mathrm{d}p$$



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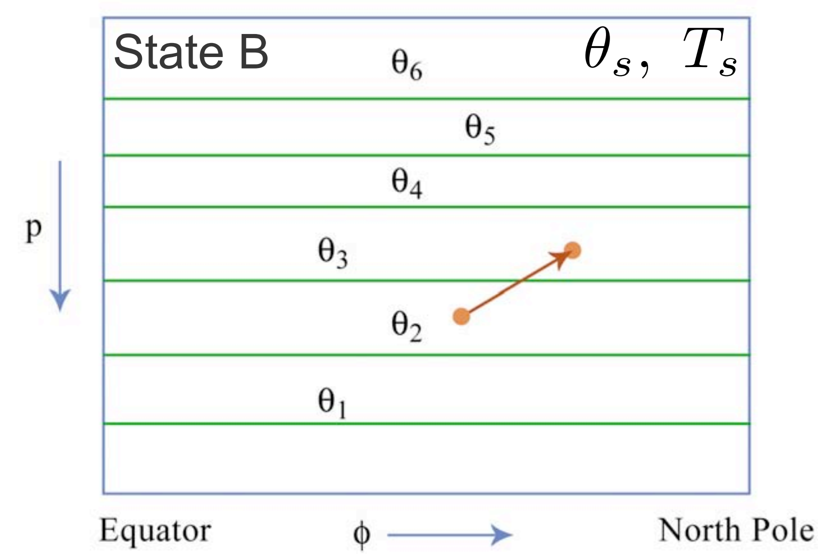


= Available potential energy

Energy cycles
in the baroclinic eddy-mean flow interactions

- Basic forms of energy:

  - Kinetic energy (动能): $K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2)$

$$\int_0^\infty \rho(I + \Phi)\mathrm{d}z = \frac{1}{g}\int_0^{p_s} (c_v T + RT)\mathrm{d}p = \frac{1}{g}\int_0^{p_s} c_p T \mathrm{d}p$$

State A − State B **= Available potential energy**

  - Available potential energy (有效位能):

$$P = \frac{1}{2}\int_0^{p_s} \frac{T_s}{\gamma_d - \gamma_s}\left(\frac{T - T_s}{T_s}\right)^2 \mathrm{d}p = \frac{c_p}{2g}\int_0^{p_s} \Gamma\left(T - T_s\right)^2 \mathrm{d}p$$

$$\begin{aligned} \Gamma &= -\frac{R}{c_p p}\left(\frac{p_s}{p}\right)^{\frac{R}{c_p}}\left(\frac{\partial \theta_s}{\partial p}\right)^{-1} \\ &= \left(\gamma_d / T_s\right)\left(\gamma_d - \gamma_s\right)^{-1} \end{aligned}$$

From the "approximate" expression of Lorenz (1955)

Energy cycles
in the baroclinic eddy-mean flow interactions

- Basic forms of energy:
  - Kinetic energy (动能): $$K = \frac{1}{2}(u^2 + v^2 + w^2) \approx \frac{1}{2}(u^2 + v^2)$$
  - Available potential energy (有效位能):

$$P = \frac{1}{2}\int_0^{p_s} \frac{T_s}{\gamma_d - \gamma_s}\left(\frac{T - T_s}{T_s}\right)^2 \mathrm{d}p = \frac{c_p}{2g}\int_0^{p_s} \Gamma\left(T - T_s\right)^2 \mathrm{d}p$$

- Tendency equations:

$$\frac{\partial}{\partial t}\int K\mathrm{d}m = -R\int \frac{\omega T}{p}\mathrm{d}m + \int (uF_x + vF_y)\mathrm{d}m$$

$$\frac{\partial}{\partial t}\int P\mathrm{d}m = R\int \frac{\omega T}{p}\mathrm{d}m + \int \Gamma(T - T_s)(Q - Q_s)\mathrm{d}m$$

Q - diabatic heating

Energy cycles

in the baroclinic eddy-mean flow interactions

- Zonal mean and eddy components:
  - Kinetic energy (动能): $K_{\mathrm{M}}=\frac{1}{2}\left([u]^2+[v]^2\right)$ $\quad K_{\mathrm{E}}=\frac{1}{2}\left([u^{*2}]+[v^{*2}]\right)$
  - Available potential energy (有效位能):

$$P_{\mathrm{M}}=\frac{c_p}{2}\Gamma\left([T]-T_s\right)^2 \qquad P_{\mathrm{E}}=\frac{c_p}{2}\Gamma[T^{*2}]$$

- Tendency equations:

$$\frac{\partial}{\partial t}\int K\mathrm{d}m=-R\int\frac{\omega T}{p}\mathrm{d}m+\int(uF_x+vF_y)\mathrm{d}m$$

$$\frac{\partial}{\partial t}\int P\mathrm{d}m=R\int\frac{\omega T}{p}\mathrm{d}m+\int\Gamma(T-T_s)(Q-Q_s)\mathrm{d}m$$

Q - diabatic heating

Energy cycles
in the baroclinic eddy-mean flow interactions

- Equations under the **Quasi-geostrophic** assumption:

$$\frac{\partial}{\partial t}\int K\mathrm{d}m = -R\int\frac{\omega T}{p}\mathrm{d}m + \int(uF_x + vF_y)\mathrm{d}m$$

$$\frac{\partial}{\partial t}\int K_{\mathrm{M}}\mathrm{d}m = -R\int\frac{[\omega][T]}{p}\mathrm{d}m + \int[u^*v^*]\frac{\partial[u]}{\partial y}\mathrm{d}m + \int([u][F_x] + [v][F_y])\mathrm{d}m$$

$$\frac{\partial}{\partial t}\int K_{\mathrm{E}}\mathrm{d}m = -R\int\frac{[\omega^*T^*]}{p}\mathrm{d}m - \int[u^*v^*]\frac{\partial[u]}{\partial y}\mathrm{d}m + \int([u^*F_x^* + v^*F_y^*])\mathrm{d}m$$

$$\frac{\partial}{\partial t}\int P\mathrm{d}m = R\int\frac{\omega T}{p}\mathrm{d}m + \int\Gamma(T - T_s)(Q - Q_s)\mathrm{d}m$$

$$\frac{\partial}{\partial t}\int P_{\mathrm{M}}\mathrm{d}m = R\int\frac{[\omega][T]}{p}\mathrm{d}m + c_p\int\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}\mathrm{d}m + \int\Gamma([T] - T_s)([Q] - Q_s)\mathrm{d}m$$

$$\frac{\partial}{\partial t}\int P_{\mathrm{E}}\mathrm{d}m = R\int\frac{[\omega^*T^*]}{p}\mathrm{d}m - c_p\int\Gamma[v^*T^*]\frac{\partial[T]}{\partial y}\mathrm{d}m + \int\Gamma[T^*Q^*]\mathrm{d}m$$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Equations under the **Quasi-geostrophic** assumption:

$$\frac{\partial}{\partial t}\int K\mathrm{d}m = -R\int \frac{\omega T}{p}\mathrm{d}m + \int (uF_x + vF_y)\mathrm{d}m$$

$$\frac{\partial}{\partial t}\int K_{\mathrm{M}}\mathrm{d}m = -<P_{\mathrm{M}}, K_{\mathrm{M}}> + <K_{\mathrm{E}}, K_{\mathrm{M}}> + D(K_{\mathrm{M}})$$

$$\frac{\partial}{\partial t}\int K_{\mathrm{E}}\mathrm{d}m = -<P_{\mathrm{E}}, K_{\mathrm{E}}> - <K_{\mathrm{E}}, K_{\mathrm{M}}> + D(K_{\mathrm{E}})$$

$$\frac{\partial}{\partial t}\int P\mathrm{d}m = R\int \frac{\omega T}{p}\mathrm{d}m + \int \Gamma(T - T_s)(Q - Q_s)\mathrm{d}m$$

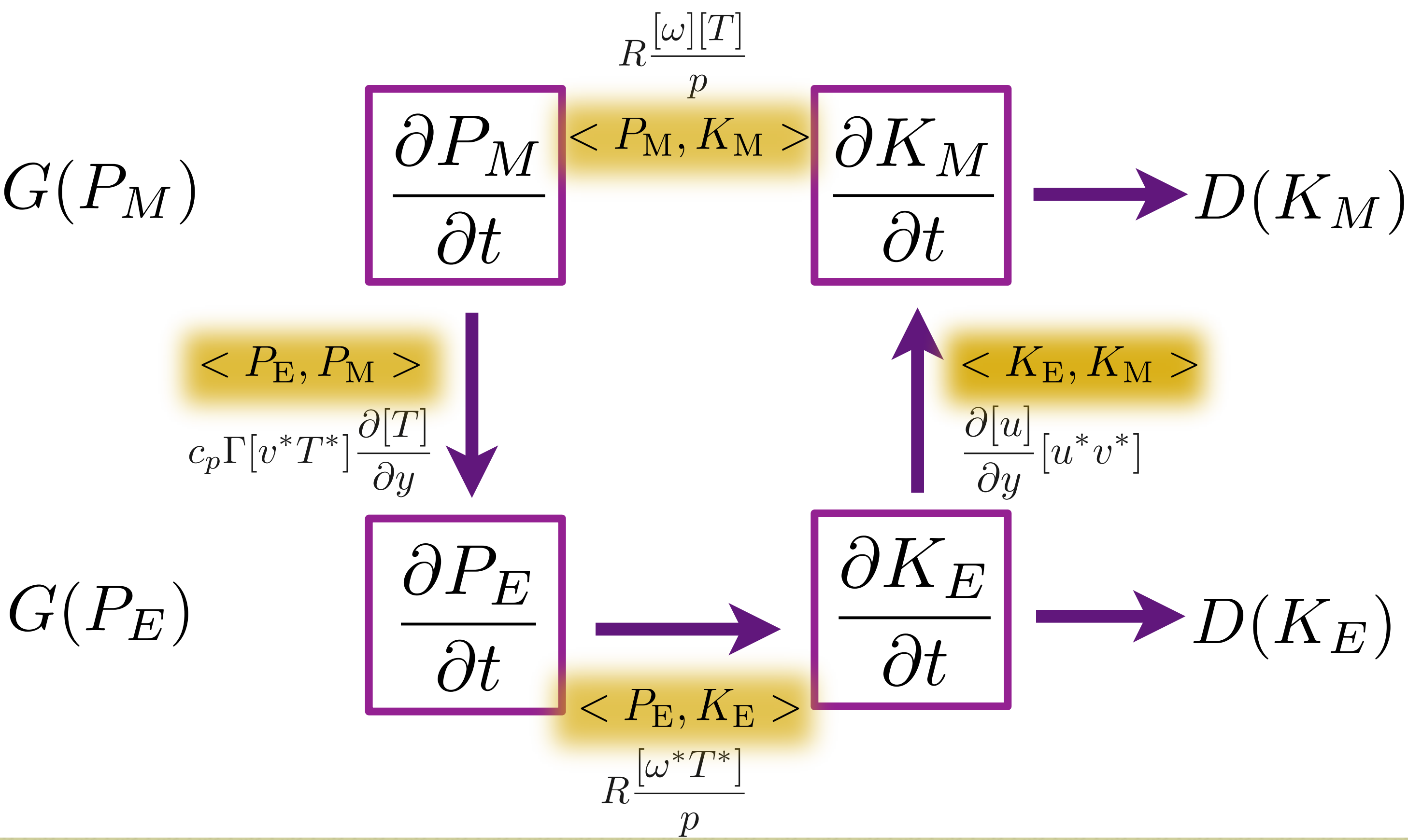
$$\frac{\partial}{\partial t}\int P_{\mathrm{M}}\mathrm{d}m = <P_{\mathrm{M}}, K_{\mathrm{M}}> + c_p <P_{\mathrm{E}}, P_{\mathrm{M}}> \mathrm{d}m + G(P_{\mathrm{M}})$$

$$\frac{\partial}{\partial t}\int P_{\mathrm{E}}\mathrm{d}m = <P_{\mathrm{E}}, K_{\mathrm{E}}> - c_p <P_{\mathrm{E}}, P_{\mathrm{M}}> \mathrm{d}m + G(P_{\mathrm{E}})$$

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in eddy life cycle: PE → KE



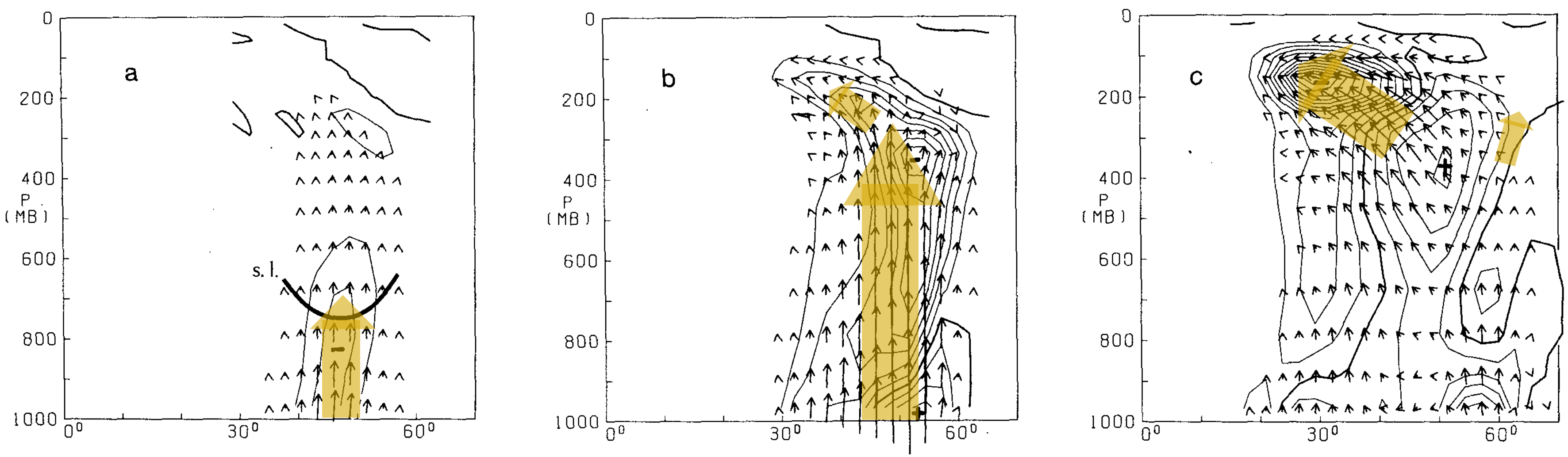
Baroclinic eddy life cycle
- An E-P flux view

$$\mathcal{F} \equiv -[u^* v^*]\,\mathbf{j} + f\,\frac{[v^*\theta^*]}{\partial\theta_s/\partial p}\,\mathbf{k}$$

$$\vec{\mathcal{F}} = \vec{\mathbf{c_g}}\mathcal{A}$$

Numerical results from
Simmons and Hoskins, 1978, JAS

Eddies: generate at lower level, propagate **upwards** and **away** from the eddy source region



TOTAL E-P FLUX DIVERGENCE
DAY .00

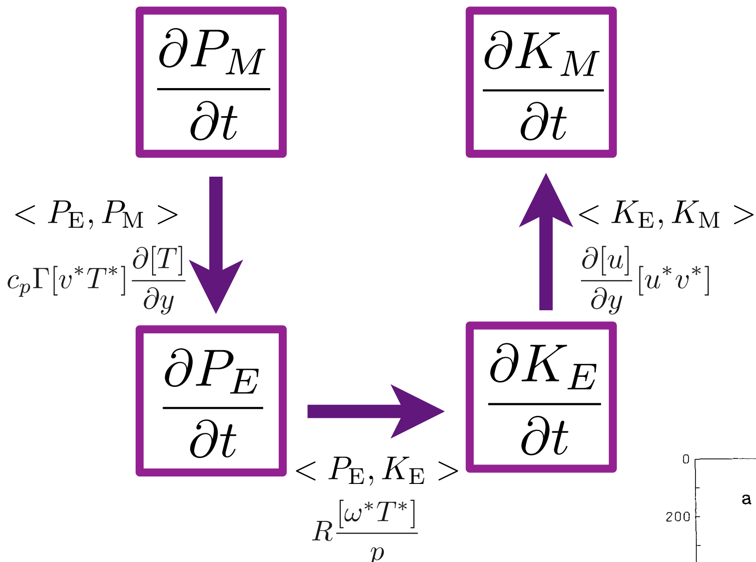
TOTAL E-P FLUX DIVERGENCE
DAY 5.00

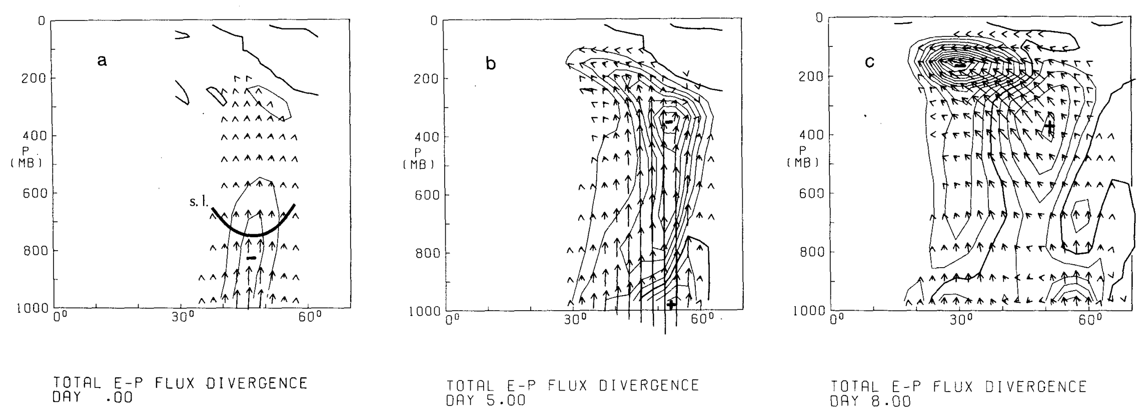
TOTAL E-P FLUX DIVERGENCE
DAY 8.00

Baroclinic eddies

- baroclinic eddy life cycle

- Westerly jet and energy cycle:

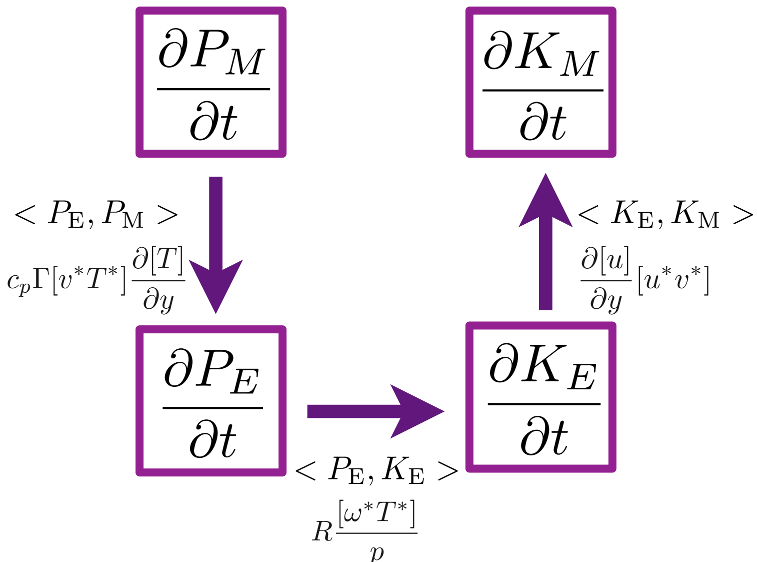


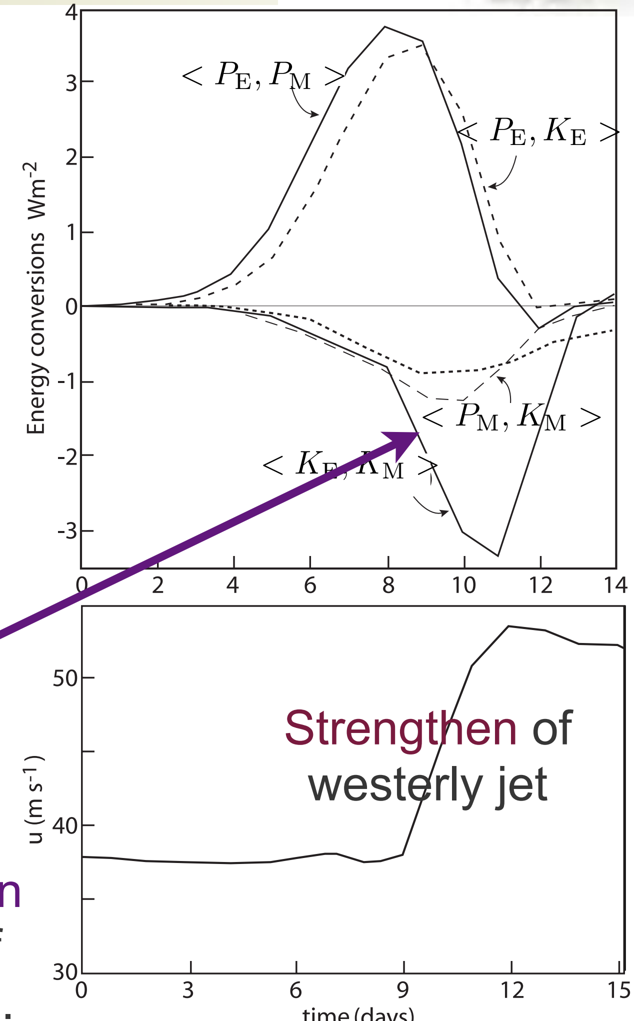


Baroclinic eddies

- baroclinic eddy life cycle

- Westerly jet and energy cycle:





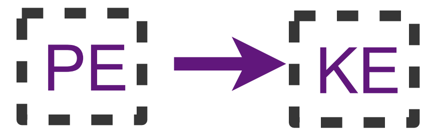
Numerical results from
Simmons and Hoskins, 1978, JAS

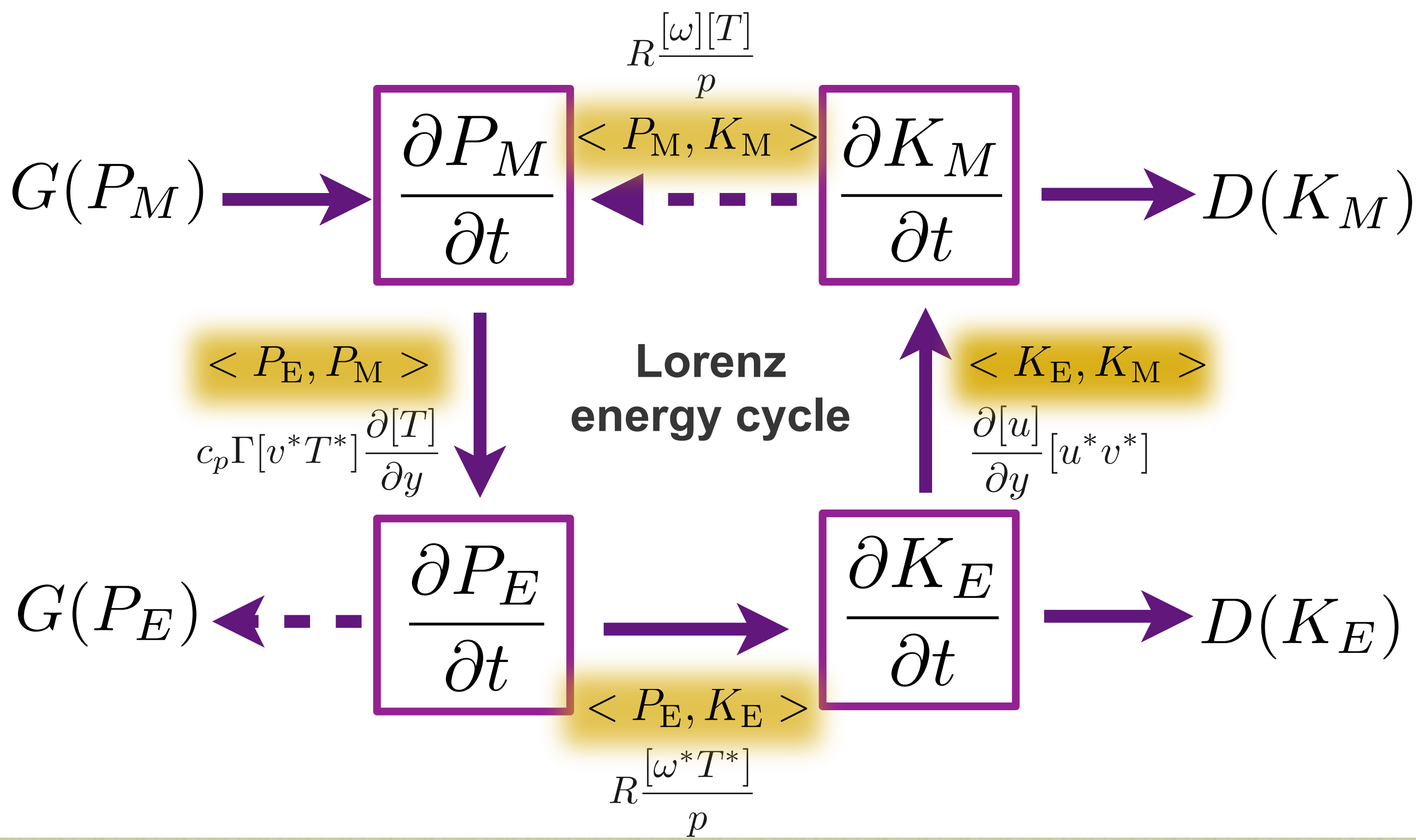
Eddy momentum flux grows, which extracts kinetic energy from the zonal mean flow, then the growth of the eddy energy ceases.

Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in equilibrium:





Energy cycles in Hadley Cell

PE → KE

$$G(P_M) \rightarrow \frac{\partial P_M}{\partial t} \xrightarrow{<P_{\rm M}, K_{\rm M}>} \frac{\partial K_M}{\partial t} \rightarrow D(K_M)$$

$$R\frac{[\omega][T]}{p}$$

If assume no eddies.

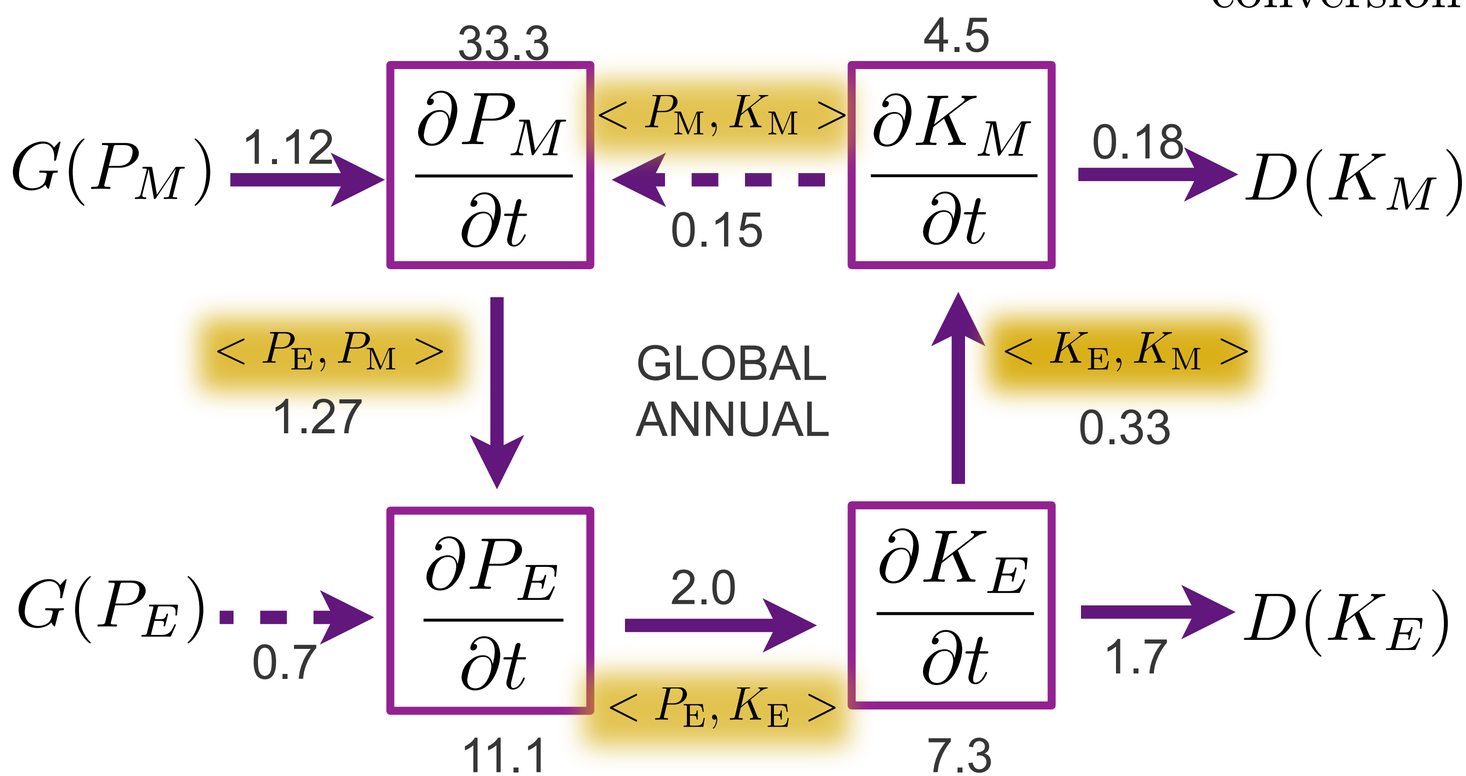
Energy cycles

in the baroclinic eddy-mean flow interactions

- Energy cycles in real atmosphere:

energy: $10^5 Jm^{-2}$

conversion: Wm^{-2}



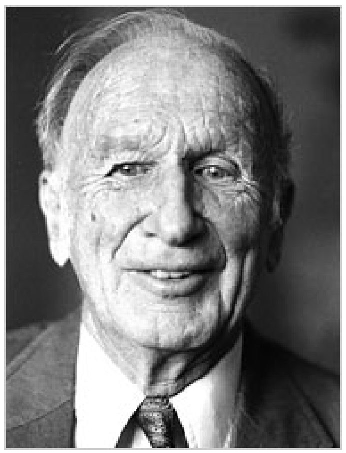
The New York Times

U.S.

Edward N. Lorenz, a Meteorologist and a Father of Chaos Theory, Dies at 90

By KENNETH CHANG
Published: April 17, 2008

Edward N. Lorenz, a meteorologist who tried to predict the weather with computers but instead gave rise to the modern field of chaos theory, died Wednesday at his home in Cambridge, Mass. He was 90.



M.I.T. News Office

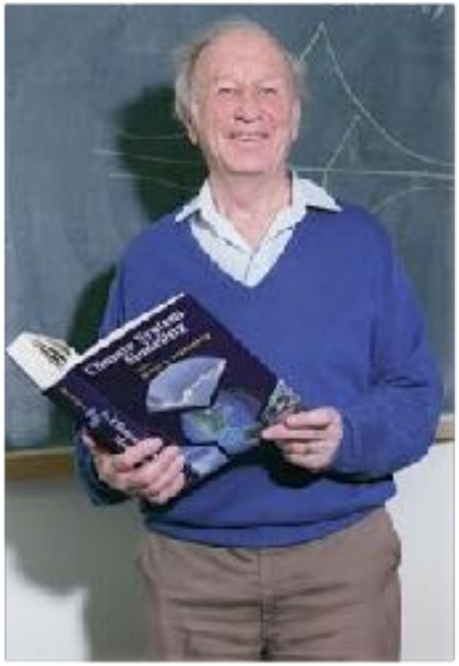
Edward N. Lorenz

The cause was cancer, said his daughter Cheryl Lorenz.

In discovering "deterministic chaos," Dr. Lorenz established a principle that "profoundly influenced a wide range of basic sciences and brought about one of the most dramatic changes in mankind's view of nature since Sir Isaac Newton," said a committee that awarded him the 1991 Kyoto Prize for basic sciences.

Dr. Lorenz is best known for the notion of the "butterfly effect," the idea that a small disturbance like the flapping of a butterfly's wings can induce enormous consequences.

As recounted in the book "Chaos" by James Gleick, Dr. Lorenz's accidental discovery of chaos came in the winter of 1961. Dr. Lorenz was running simulations of weather using a simple computer model. One day, he wanted to repeat one of the simulations for a longer time, but instead of repeating the whole simulation, he started the second run in the middle, typing in numbers from the first run for the initial conditions.



Summary & Discussion

- Observations
- The Ferrel Cell
- Baroclinic eddies
  - Review: baroclinic instability and baroclinic eddy life cycle
  - Eddy-mean flow interaction, E-P flux
  - Transformed Eulerian Mean equations
- Eddy-driven jet
- The energy cycle

1. The role of moisture;

2. Quantify (parameterize) the relation between eddies and mean flow;

3. Zonal variations.