Some Realistic Modifications of Simple Climate Models

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ABSTRACT

Various simple climate models of the Budyko and Sellers types are discussed. It is shown how the variation of the equilibrium position of the ice line as a function of available insolation is determined primarily by the range of latitudes over which heat transport is acting to smooth temperatures and the effectiveness with which this smoothing occurs. It is shown that when transports loosely associated with Hadley transports, atmospheric eddy transports and oceanic transports are separately modeled their net effect is to decrease the sensitivity of the climate substantially over what is obtained from existing simple models. It is also shown that the models we introduce in this paper are reasonably consistent with various aspects of the observed temperature and with various paleoclimatological features, while earlier simple models are not.

1. Introduction

In the recent theoretical study of climate a set of relatively simple models originated by Budyko (1969) and Sellers (1969) have assumed considerable popularity [Held and Suarez (1974) and North (1975) represent recent, more sophisticated approaches to such models]. These models may be characterized as follows:

1) Only latitude dependences are considered, i.e., the models are spatially one-dimensional (though time dependence is also sometimes considered).
2) Global energy budgets are assumed to be expressible in terms of surface temperatures.
3) Planetary albedo is taken to depend primarily on ice and/or snow cover or the lack thereof.
4) The convergence of dynamic heat fluxes is generally represented by either a simple diffusion law or by a linear heating law wherein local heating is proportional to deviations of the global mean temperature from the local surface temperature.
5) Generally, only annual mean conditions are considered.

Such models permit the calculation of, among other things, the equilibrium position of the ice line as a function of incoming solar radiation. In all the above models it is found that as the incoming radiation is reduced to a certain level (and correspondingly the ice line advances to a certain latitude), further advances of the ice line may occur even with *increasing* incoming radiation. At this point, these simple models imply that climate has become unstable and will tend toward an ice-covered earth. Although this result is well known to many readers, we will briefly outline its derivation in Section 2.

Perhaps the most interesting point (apart from the very existence of such an instability) is that the reduction in incoming radiation necessary to reach instability is only on the order of 2% for the models considered. It is, in fact, such models that have strongly influenced the development of expectations (or fears) for the catastrophic climatic effects of relatively small envrionmental changes. To be sure most papers on these models have carefully noted the quantitative inadequacies of such models and the unreliability of such predictions. However, the recent paper of North (1975) shows that this result is remarkably insensitive to a large number of very reasonable improvements in parameterizations of such things as albedo and dynamic heat transfer—thus suggesting that the predicted climate sensitivity might be realistic.

The purpose of the present paper is to show that existing simple climate models (including those considered by North) are inconsistent with certain observed features of the current and past climate. We show, moreover, that simple changes in these climate models which bring them into agreement with the observed features can also lead to an *enhancement* in the stability of the climate (i.e., they suggest that reductions in incoming radiation of from 7–20% rather than 2% might be necessary for an ice-covered earth). The most important feature we consider is the fact that the observed surface temperature within 30° of latitude of the equator is far more nearly isothermal than it is in the existing simple models. Fundamental reasons for this near constancy of temperature within 30° of the equator are developed in Schneider and Lindzen (1977) and Schneider (1977). In Section 3 we show how this feature can be incorporated into Budyko-type models (where a linear dynamic heating law is used), and how one then obtains a marked stabilization near 30° latitude. In Section 4 we consider Sellers-type models (where a diffusive law is used for dynamic heating).

In Section 5 we turn to a far less secure feature of climate. In a recent reconstruction of the ice-age earth of 18 000 years ago (CLIMAP, 1976), it was found that the ice line advanced from ~72° to ~60°, but that zonally averaged tropical temperatures changed very little from present values. In existing simple climate models such an ice advance is accompanied by relatively pronounced reductions of tropical temperatures. The simulation of the observed behavior with simple climate models appears to require, among other things, some reduction of the loss of heat from the tropics to polar regions. We discuss the meaning of the various modifications, and show that they lead to a marked stabilization of climate at middle latitudes.

Section 6 briefly assesses simple climate models, including those developed in this paper.

2. Review of existing simple climate models

The basic premises of simple climate models were briefly stated in the Introduction. Item 2) (viz., global energy budgets can be expressed in terms of surface temperature) implies, among other things, that the atmosphere, land surface, ocean and cryosphere are in equilibrium with each other. Given the time scales associated with such equilibration [O(10^2 years) for deep ocean, O(10^4 years) for glaciers], it would appear that simple climate models are useful, at best, only for the description of very long term average behavior. However, the main concern of such models with the cryosphere accrues from its effect on the planetary albedo, and this effect may manifest itself in a relatively short time for a variety of reasons: the snowline and pack ice develop rapidly, glaciers retreat relatively rapidly, puddling of an ice surface markedly reduces its reflectivity, etc. Thus, the restriction to very long term averages may not be as severe as it seems at first sight.

All simple models begin with an equation of the form

$$C\frac{\partial T(x,t)}{\partial t}=\text{incoming solar radiation}-\text{infrared cooling}$$
$$+\text{divergence of atmospheric and oceanic heat flux}, \quad (1)$$

where C is some heat capacity for the atmosphere-ocean system, T the surface temperature (°C), t time and $x=\sin\theta$, where θ is the latitude. It is usually somewhat more convenient to deal with x than θ.

Under the assumption that the *total* global energy budget can be expressed in terms of the surface temperature, the first term on the right-hand side (rhs) of Eq. (1) is generally taken to be the total insolation as might be determined by a satellite above the atmosphere. It is typically written as

$$Qs(x)\mathcal{A}(T), \quad (2)$$

where Q is one-fourth the solar constant and $s(x)$ is a function whose integral from the equator to the pole is unity and which represents the annually averaged latitude distribution of incoming radiation. This function is discussed in more detail in Held and Suarez (1974). Finally, $\mathcal{A}(T)$ is 1 minus the planetary albedo; $\mathcal{A}$ is allowed to depend on temperature. In most simple climate models a temperature T_s is identified with the onset of ice (snow) cover such that for $T>T_s$ there is no ice and for $T<T_s$ there is. The most important change in $\mathcal{A}$ is due to T passing through T_s. We will specify $\mathcal{A}$ more explicitly later.

Again under the assumption that global energy budgets can be expressed in terms of surface temperature, one writes

$$\text{infrared cooling}=I(T). \quad (3)$$

The justification for (3) is that temperature profiles have more or less the same shape at all latitudes. Hence cooling which depends on the temperature at all levels ought to be expressible in terms of surface temperature since the temperature at all levels is related to the surface temperature. From Fig. 1 (which shows vertical temperature profiles characteristic of the tropics, midlatitudes and the arctic) we see that profile shapes are not so similar at all latitudes. Although two of the profiles are not annual means, the differences between profiles are characteristic. Moreover, Held and Suarez (1974) have shown that 500 mb temperatures correlate better with infrared emission than do surface temperatures. Nevertheless, it is the surface temperature which relates to the formation of ice, and therefore must be used in simple climate models. The fact that total infrared emission is not perfectly related to surface temperature is merely an indication that a significant portion of the emitted radiation originates in the atmosphere. Similarly, not all of the incoming radiation is absorbed at the surface; in practice some of the incoming radiation is not directly involved in the surface energy budget. We shall return to this point in Section 6.

As a rule models based on Eq. (1) take little account of clouds and cloud feedbacks. However, to the extent that clouds can be specified in terms of latitude and surface temperature, their effects on incoming radiation and infrared emission can be included in Eqs. (2) and (3).

The divergence of atmospheric and oceanic heat flux must also be expressed in terms of an operator on surface temperature, i.e.,

$$\text{div flux}=F[T], \quad (4)$$

where F is some operator. Usually F is a linear operator although Held and Suarez (1974) and North (1975) have also considered nonlinear operators as suggested by Green (1970) and Stone (1973). The most common choices for $F[T]$ are a linear relation first suggested by Budyko (1969), i.e.,

$$F[T]=C[\bar{T}-T], \tag{4a}$$

where $\bar{T}$ is the average of T over all latitudes,[1] and a diffusion law, first used in this context by Sellers (1969),

$$F[T]=\frac{\partial}{\partial x}(1-x^2)D\frac{\partial T}{\partial x}, \tag{4b}$$

where C and D in (4a) and (4b) are constants; they are generally chosen to simulate some feature of the existing climate. The observed pole-to-equator temperature difference and the observed total heat flux are commonly the features so used. We have, however, generally avoided using these features. Our reasons for doing this are as follows:

1) Various of our approximations [most notably Eq. (3)] are poor in polar regions and tend to make calculations of equator-pole temperature differences very poor.

2) As we have already noted, the assumption that total meridional heat flux is related to surface temperature is not particularly good. Moreover, as we shall note later, the total heat flux is not always a good check on the modeling of the dependence of heat flux on temperature. Instead, we will usually use the position of the ice line to calibrate our models. In practice, the three methods of calibration lead to similar values for C and D.

The most common application of (1) involves assuming a steady state and seeking a relation between the equilibrium position of the ice line and the solar constant. Usually, one linearizes (2) to obtain

$$I=A+BT, \tag{3a}$$

and replaces T by I as defined in (3a). Eq. (1) becomes

$$QS(x)\mathcal{A}(I)-I+F[I]=0. \tag{1a}$$

As mentioned in the Introduction, one identifies the ice line with a temperature T_s or equivalently $I_s=A+BT_s$. We shall use x_s to identify the value of x at $I=I_s$. Moreover, variations in $\mathcal{A}$ are taken to be due solely to whether or not there is an ice surface, i.e.,

$$\mathcal{A}=\mathcal{A}(x,x_s). \tag{5}$$

If we now write

$$I=Q\tilde{I}(x), \tag{6}$$

and assume F to be a linear operator, then we may divide (1a) by Q, yielding

$$F[\tilde{I}]-\tilde{I}=S(x)\mathcal{A}(x,x_s). \tag{7}$$

[1] A general constraint on $F[T]$ is that its integral over the globe be zero.

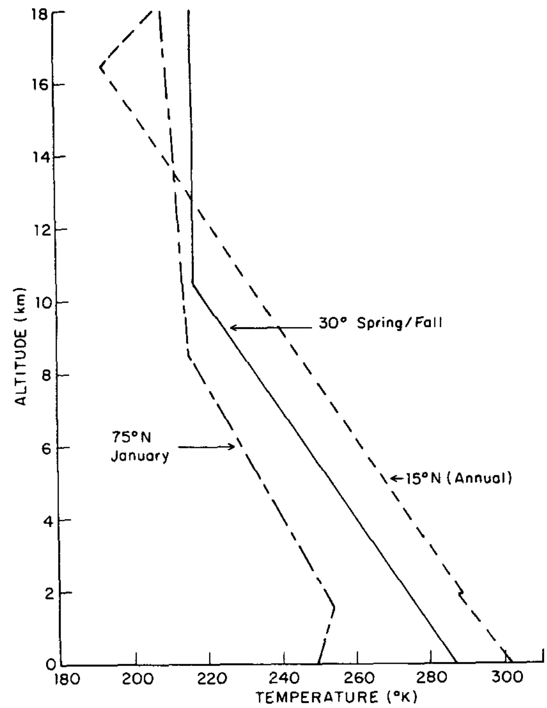

FIG. 1. Vertical temperature profiles for various latitudes. (From *U. S. Standard Atmosphere Supplements, 1966.*)

For any choice of x_s we may solve (7) for $\tilde{I}=\tilde{I}(x,x_s)$. It is now a trivial matter to obtain the solar constant (or equivalently Q) as a function of x_s. We already have

$$I(x_s)=I_s. \tag{8}$$

But since

$$I(x_s)=\tilde{I}(x_s,x_s)Q, \tag{9}$$

by combining (8) and (9) we have

$$\frac{Q}{I_s}=\frac{1}{\tilde{I}(x_s,x_s)}, \tag{10}$$

which is the desired relation.

Normally we expect advancing ice (decreasing x_s) to be associated with decreasing Q. Such a situation is generally stable in the sense that the time-dependent version (1) indicates that perturbations away from the equilibria defined by (10) decay in time. This stability is easy to understand intuitively. If, for example, one decreased x_s while holding Q constant, then Q would be larger than needed for that value of x_s and the resultant warming would cause x_s to increase.

What is remarkable about simple climate models (with heat transport) is that they all imply the existence of values of x_s where a decrease in x_s is associated with an *increase* in Q. Such regions are unstable. Once more, this is intuitively clear. If we decrease x_s while holding Q constant, Q will be insufficient for that value of x_s and the resulting cooling will cause the ice to advance further.

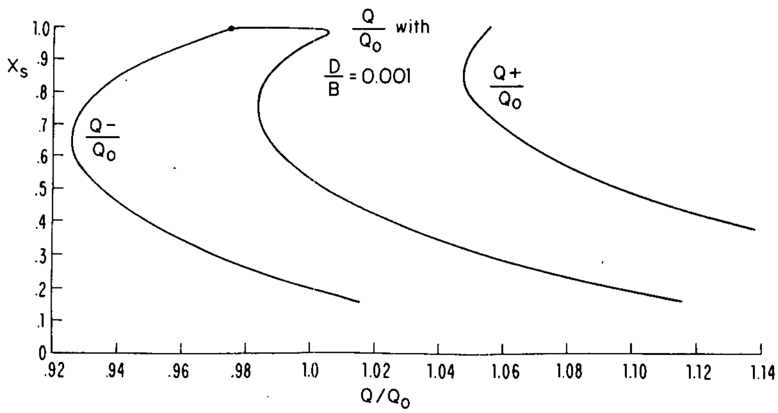

FIG. 2. Equilibrium ice line position x_s as a function of normalized solar constant Q/Q_0 (where Q_0 is the current value of the solar constant) when Budyko-type heat transport is used. The curve Q_-/Q_0 corresponds to $T=T_s$ on the equatorward side of the ice line, while Q_+/Q_0 corresponds to $T=T_s$ on the poleward side. The single remaining curve results from adding a small amount of diffusion to the Budyko-type transport. See text for explanation.

In order to make the above clearer (and also to provide reference results for later sections) we will review results essentially like those obtained by Held and Suarez (1974) and North (1975) for the Budyko and Sellers models, respectively. In all subsequent calculations we will use the following specification for $\mathcal{A}$:

$$\mathcal{A}=\begin{cases}\alpha=0.4 & \text{for } T<T_s\\ \beta=0.7 & \text{for } T>T_s.\end{cases} \tag{11}$$

For T_s we will, for the moment, take $T_s=-10°\text{C}$, and for the constants A and B in (3a) we use $A=211.2$ W m^{-2}, $B=1.55$ W m^{-2} (°C)$^{-1}$. Hence, $I_s=195.7$ W m^{-2}. For $S(x)$ we use the annual average function as approximated by North (1975):

$$S(x)\approx 1-0.241(3x^2-1) \tag{12}$$

For the present solar constant

$$Q=334.4 \text{ W m}^{-2}.$$

For the present climate

$$x_s\approx 0.95 \ (\theta_s\approx 72°).$$

We now turn to the Budyko model wherein dynamic heating is given by Eq. (4a). The use of (4a) in Eq. (1a) leads to an algebraic equation for T which allows discontinuities in T at x_s where $\mathcal{A}$ is discontinuous. This complicates the discussion of stability since now a range of Q's is compatible with a given x_s; all that is required is that T on the ice-free side of x_s be greater than or equal to T_s and that T on the ice side of x_s be less than or equal to T_s. Thus the dependence of Q on x_s is described by two curves: one showing that x_s for which the ice side of x_s is at T_s as a function of Q, and the other showing that x_s for which the ice-free side of x_s is at T_s as a function of Q. Such a pair of curves is shown in Fig. 2. Held and Suarez (1974) have shown that the above degeneracy can be removed by adding a small diffusive term of the form of (4b) to (4a). They have further shown that in the limit of vanishing conductivity the resulting function $Q(x_s)$ is given by

$$Q(x_s)=Q_m(x_s)=\frac{2Q_+(x_s)Q_-(x_s)}{Q_+(x_s)+Q_-(x_s)}. \tag{13}$$

(where Q_+ and Q_- refer to the two curves described above) except in neighborhoods of the poles and the equator. The anomalous behavior at the poles and the equator will be explained in Section 4. We have chosen C in (4a) so that for Q_m equal to its present value, $x_s=0.95$. The result is

$$C/B=2.38 \tag{14}$$

[as contrasted with $C/B=2.1$ as used by Held and Suarez (1974) or $C/B=2.4$ as used by Budyko (1969)].

We do not show Q_m in Fig. 2 because it is indistinguishable from $Q(x_s)$ calculated for the value of C given in Eq. (14) and for a small but finite conductivity[2] [$D/B=0.001$ which is one sixth the value used by Held and Suarez (1974)], which is shown in Fig. 2.

Before discussing the results displayed in Fig. 2, we shall calculate the relation between x_s and Q for the Sellers model where the dynamic heating is given entirely by (4b). We shall choose D so that for the current value of Q, $I=I_s$ at the present value of x_s (0.95). This leads to $D/B=0.3566$ [as compared to North's (1975) choice of $D/B=0.382$]. The resulting curve for Q as a function of x_s is shown in Fig. 3. It

[2] When diffusion is present, Eq. (7) is solved numerically. The algorithm used is described in Lindzen and Kuo (1969).

should be noted that the anomalous behavior of this curve near the pole extends to values of x near 0.95. Hence, our method for choosing D is somewhat questionable. Nevertheless, the value obtained is close to values commonly used in this problem. As we have already stated, we shall defer discussion of the apparent polar cap instability until Section 4. Fig. 3 also has curves $Q(x_s)$ for values of D/B larger than that needed to simulate present conditions; ignoring these for the moment, we notice that $Q(x_s)$ for either pure diffusion or for linear heat exchange with small diffusion are qualitatively and quantitatively similar. In both cases a reduction of Q of about 2% causes the snow-ice line to advance to a point beyond which it will continue to advance unstably. The only difference between the two models is the latitude from which unstable advance of the ice line proceeds is 40.5° ($x=0.65$) for the diffusive model and 46.9° ($x=0.73$) for the linear heat exchange model. As we shall soon see, this small difference is of little significance.

An understanding of the unstable behavior is easily obtained by observing $Q(x_s)$ in the diffusive model for values of D/B greater than that needed to simulate present conditions (we could just have well used the Budyko model with increased C/B). The results of such calculations are shown in Fig. 3. D/B is a measure of the effectiveness with which heat transport serves to eliminate horizontal variations in temperature across the entire globe. The maximum heat flux possible in our system would be that which, in fact, renders the system isothermal. This flux is very nearly realized for $D/B=10\times0.3566$. For $\bar{B}=\bar{B}_0=0.3566$, the flux is about three-fourths the maximum value. Thus, for $D\gtrsim10D_0$, T is very nearly constant over the globe. Thus, for ice to set on at the pole, T must approximately equal T_s everywhere, and the value of Q for ice to exist at the pole is much less than required when $D=D_0$. In this sense one may speak of increasing D to $10D_0$ as having stabilized the pole to the onset of ice. However, once ice is established, it must spread unstably over the globe since $T\approx T_s$ everywhere and advancing ice is accompanied by increasing albedo and hence reduced energy input. As D is reduced below $10D_0$, the degree of "stabilization" at the poles is decreased and eventually the "stabilization" at the pole is spread over a finite band of latitudes.[3] It should be noted that for D slightly greater than D_0, it is possible to obtain a $Q(x_s)$ which is almost identical to that obtained with a Budyko model—hence our contention that the difference in $Q(x_s)$ between the two models is inconsequential.

The upshot of the above considerations is that the most important aspect of a heat flux is that it is acting to eliminate temperature variations with latitude; the specific details of the relation between heating and temperature are of much less importance except for the following (which to some extent anticipate results developed later in this paper):

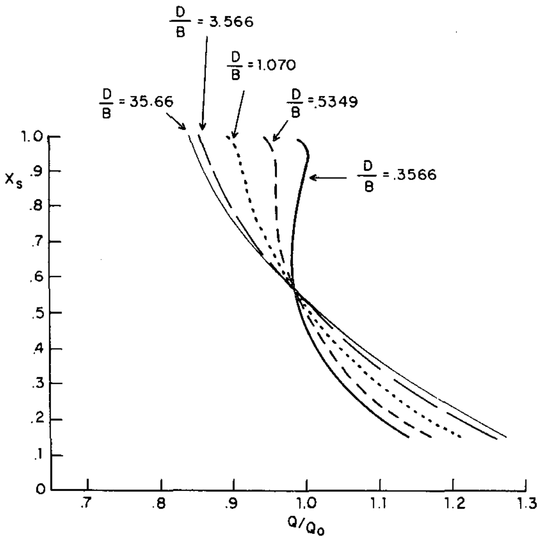

FIG. 3. Equilibrium ice line position x_s as a function of normalized solar constant Q/Q_0 for diffusive heat transport. Curves for values of diffusivity larger than that which best simulates present conditions are also shown.

1) The band of latitudes over which the dynamic heat fluxes are acting to smooth temperatures (in the two models considered in this section, this band ranges from the equator to the pole).

2) The effectiveness of the smoothing (as given, for example by C/B or D/B); this is in effect determined by attempting to simulate present conditions.

Not surprisingly, among the various models which North (1975) showed to have similar $Q(x_s)$ distributions all involved temperature smoothing over the entire globe; hence, they were alike as far as the above two conditions are concerned go. More important, we must conclude that the curves $Q(x_s)$ shown in Figs. 2 and 3 must be essentially correct *unless* the actual mechanisms for heat transport are not all acting to smooth temperatures from the equator to the pole. In the remainder of this paper we will present evidence that the latter is the case, and show what happens in this circumstance.

3. Tropical temperatures and heat fluxes

It has been shown by Schneider (1977) that conservation of angular momentum can only be consistent with the thermal wind if tropical temperatures change very little with latitude over a region extending from the equator to about 30° latitude and that meridional heat transports act to maintain the smallness of the temperature variations. In Schneider's calculations only symmetric atmospheric circulations were considered, and the required heat transports were produced by a

[3] The quantity $a(D/B)^{\frac{1}{2}}$ is a radiative diffusive length scale (where a is the earth's radius). The spreading of the zone of stabilization begins when $(D/B)^{\frac{1}{2}}\lesssim\pi/2$.

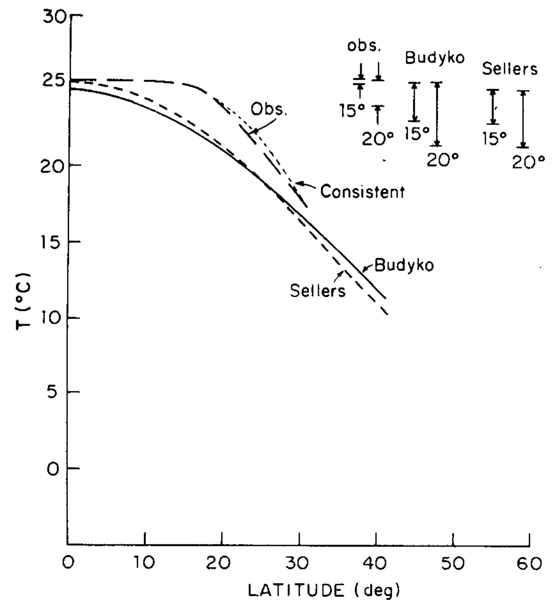

FIG. 4. Temperature as a function of latitude showing the observed distribution (based on Sellers, 1965) as well as calculated distributions obtained from Budyko- and Sellers-type models. Also shown is a distribution resulting from a Hadley circulation which is consistent with both conservation of angular momentum and geostrophy.

Hadley cell; however, what is required is the smoothing—regardless of the specific mechanism. Moreover, although Schneider's proof dealt with an inviscid budget, the same result appears to hold for frictional momentum budgets (Schneider and Lindzen, 1977). In Fig. 4 we see the observed distribution of surface temperature with latitude (based on Sellers, 1965) as well as the distribution theoretically expected under the constraint of angular momentum conservation; to the accuracy of the data they are in agreement. Also shown are the temperature distributions for current climatic conditions obtained from the Budyko-type and Sellers-type models described in Section 2. Clearly, temperatures from these models vary too rapidly in the tropics. Although the differences between these models and observations have been ignored in such papers as North (1975), presumably because they are "small" in some sense, they would lead to extremely unrealistic winds. From this point of view such "small" differences must be reckoned as large for the tropics. Such variations of temperature with latitude are not found on the earth because of a heat transport mechanism which acts to smooth temperatures not over the entire globe, but only within the tropics (out to a latitude of about 30° beyond which observed temperatures and those from the Budyko and Sellers-type models begin to agree). From the discussion of Section 2 we would expect such an effective heat transport mechanism to produce a region of enhanced stability at the poleward edge of the tropics and instability equatorward of this region. Explicit calculations will confirm this.

For the purposes of the present paper there is little point in being particularly sophisticated or detailed in modeling this tropical transport (purely for convenience we will henceforth refer to this transport as a Hadley transport). It will be adequate to model this transport by a kind of convective adjustment wherein a heat flux is assumed to exist which goes to zero for latitudes greater than some $\theta=\theta_h$ and which for $\theta<\theta_h$ is such as to eliminate all temperature variation [the resulting heat flux is very nearly what Newton (1972) obtains for the observed Hadley flux, suggesting that our label for this transport is not entirely inappropriate]. For Budyko-type models the effect of such a heat flux is simply to replace $Q_s(x)\mathcal{A}(x,x_s)$ in (1a) with its average over the region $0\leqslant\theta\leqslant\theta_h$ in that region. The only remaining question is what to use for θ_h. From Fig. 4 we see that Hadley smoothing is important out to 30° but the approximation of the temperature variation by a constant is reasonable only to about 20°. A choice of θ_h somewhere between these values should yield a reasonable estimate. In Fig. 5 we show $Q(x_s)$ for $\theta_h=30°$, 25° and 20°. A marked stabilization at θ_h is seen in each case, though the degree of stabilization diminishes as θ_h decreases. For $\theta_h=30°$, a reduction of 9% in Q is needed to produce an ice-covered earth, while for $\theta_h=20°$ the Hadley stabilization is of no consequence. For most of what follows, we will take $\theta_h=25°$. It should be noted (see Held, 1976) that Held and Suarez, using a truncated spectral, two-level general circulation model, obtained a stability ledge very similar to what we obtain for $\theta_h=30°$. However, as we shall note in Section 5, part of their stability may be attributed to processes other than Hadley adjustment.

4. Some remarks on diffusive models

Clearly, Budyko-type models are easier to use than diffusive models although the use of diffusive models with an efficient numerical algorithm is not at all difficult. Nevertheless, there is no compelling reason to assume diffusive models are indeed better. From our discussion at the end of Section 2 it should be apparent, moreover, that little actually depends on specific model choices. In addition there are certain drawbacks to the use of diffusive models which ought to be noted.

The application of the Hadley "adjustment" described in Section 3 is more complicated. One might be tempted to use the same procedure as was used for a Budyko-type model: i.e., replace $QS(x)\mathcal{A}(x,x_s)$ in the region $0\leqslant\theta\leqslant\theta_h$ by its average in that region. The heat flux implied by this adjustment is indeed close to the observed tropical heat flux. However, the distribution $Q(x_s)$ obtained with this approach differs negligibly from that shown in Fig. 3. The reason for this initially surprising result becomes clear when one looks at the

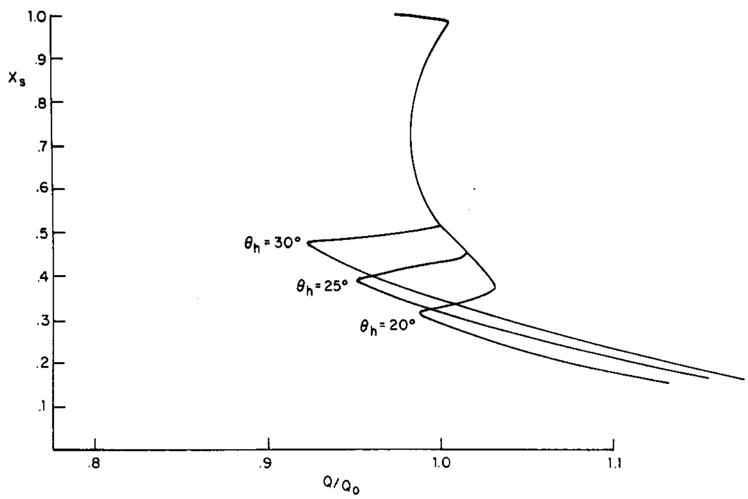

FIG. 5. Equilibrium ice line position x_s as a function of Q/Q_0 for Budyko model (with small diffusivity) with Hadley adjustments extending to latitude θ_h, where $\theta_h=30°$, 25° and 20°.

distribution of temperature with latitude: it is virtually indistinguishable from that shown for the Sellers-type model in Fig. 4. Stated differently, the insertion of a nearly correct Hadley flux did *not* effect a correct Hadley adjustment. For a conductivity given by $D/B=0.3566$ (see Section 2), the diffusive heat flux acts to cancel the Hadley flux to the extent needed to leave an essentially diffusive result. This is similar to what happened in many of the cases described by North (1975) where it was found that the function $Q(x_s)$ was that given by the simple diffusive model despite various physical perturbations to such a model. This behavior was taken by North to indicate a genuine insensitivity. However, as we see here, this is not necessarily the case. Of course, it is perfectly possible to make a correct Hadley adjustment in a diffusive model,[4] and we have found (in calculations which we have not presented) that $Q(x_s)$ then behaves in the same manner found in Section 3.

Apart from the modest additional difficulty of properly including the Hadley adjustment, diffusive models present other difficulties. As noted in Section 2, such models generally display a polar cap instability. Most papers dealing with such models ignore this instability. As noted by Held and Suarez (1974), this instability does not appear when nonlinear diffusion is used; moreover, one sees that no such instability is found in the general circulation model described in Held (1976). There are, in fact, a number of modifications to both the diffusion coefficient and the ice albedo which can eliminate this instability. Thus, one may reasonably conclude that the polar cap instability is a spurious feature of simple diffusive models. The basic reason for this instability is easily understood by considering the limits of very small D/B (see Fig. 2); in this case the instability is confined to the immediate vicinity of the pole. The abrupt decrease of $Q(x_s)$ as $x_s \rightarrow 1$ is due to the fact that the Q necessary to maintain the pole at $T=T_s$ when there is no ice at the pole is less than the value needed when there is ice at the pole. As D/B increases, this transition is spread over a wider region corresponding to a boundary layer thickness determined by D/B. When D/B becomes sufficiently large the polar cap instability merges into the global instability (Fig. 3). While we feel that the polar cap instability is probably not of practical relevance, it does present something of a nuisance in interpreting $Q(x_s)$.

The point of the above discussion is not to rule out the use of simple diffusive climate models. Indeed, there are ways of circumventing the above difficulties. However, for purposes of developing a basic understanding of simple climate models, Budyko-type models are more straightforwardly interpretable.

5. An implication from CLIMAP

a. *Insulation of the tropics*

CLIMAP (1976) has recently presented their reconstruction of the earth's climate in the midst of an ice age 18 000 years ago. During this period, the mean ice line had advanced to about 60° from the present 72°. In addition, according to the reconstruction, mean tropical temperatures 18 000 years ago were no more than about 2°C less than they are at present (despite an apparently larger local change in the eastern Pacific). Now in all existing simple climate models, the advance of the ice line to 60° is always accompanied by a significantly larger temperature decrease in the tropics. Part

[4] For example one could set $D/B=\infty$ for $0\leqslant\theta\leqslant\theta_h$, and tune D/B for $\theta>\theta_h$ as in Section 2. For such a variation in D, the two-mode truncation of North (1975) would be inadequate.

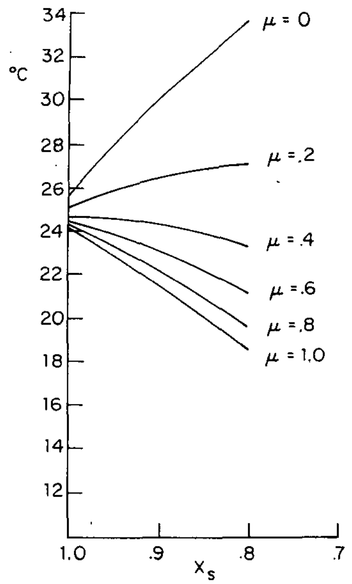

FIG. 6. T_{equator} vs x_s for Q/Q_0 held fixed and equal to 1, for model in which the Budyko heating coefficient equatorward of the ice line $[\gamma(x<x_s)]$ is greater than the coefficient poleward of the ice line $[\gamma(x>x_s)=\mu\gamma(x<x_s)]$. Calculations are shown for various choices of u.

of the reason for the tropical temperature decrease is that the tropics must share heat with the advancing ice-covered region which is receiving less energy due to its high albedo. A second reason is that in a stable configuration, the advance of the ice is associated with a decrease of available insolation.[5] We may isolate the first factor by means of the following somewhat inconsistent calculation (using a Budyko-type model): holding Q *constant* at its present value, we vary x_s [hence, altering $\alpha(x,x_s)$] and calculate the change in T at the equator. The results of such a calculation are shown in Fig. 6. Note that a decrease in x_s from 0.95 to 0.85 (corresponding to θ_s going from 72° to 60°) causes a reduction of T_{equator} of more than 3°C, due entirely to the need for the tropics to share its heat. Of course, as x_s decreases, so must Q (in order for equilibrium to hold) and this in turn further reduces T_{equator}. Thus, the changes in T_{equator} shown in Fig. 6 must be regarded as lower bounds. However, even without the effect of decreasing Q, it is clear that heat sharing implicit in existing climate models implies excessive sensitivity of T_{equator} to x_s. Thus, consistency with CLIMAP results requires some reduction of this sharing. Two ways to achieve this, in the context of Budyko-type models (when we refer to Budyko-type models in this section, we are always assuming the inclusion of Hadley adjustment in addition to the Budyko-type transport laws) are (a) reduce $\gamma=C/B$ for $x>x_s$ and (b) reduce $\gamma=C/B$ for $0\leqslant x<x_a<x_h$, where x_a remains to be specified. Possible physical interpretations for these choices are described in Section 5b. Other approaches are possible, though most are simple variants of the above two models.

[5] We parameterize this, for convenience, by letting Q decrease. However, it is unnecessary to identify this with a decreasing solar constant. For example, in Suarez and Held (1976), such a reduction in the *annual average* of $Qs(x)\alpha$ is due to seasonal variations in s and α and not to changes in Q.

We consider the first possibility, model (a). We shall set $\gamma(x>x_s)=\mu\gamma(x<x_s)$; $\mu<1$. The dynamic heating term is now $-\gamma(T-\tilde{T})$, where

$$\tilde{T}=\frac{\int_0^1 \gamma T dx}{\int_0^1 \gamma dx}.$$

In such a model we will determine x_s by requiring that $T=-6$°C on the equatorward side of x_s instead of requiring $T=-10$°C at x_s. This leads to a unique $Q(x_s)$ without recourse to small diffusion [which now would lead to a more complicated mean than that given by Eq. (13)]; it also implicitly recognizes that our modeling is relatively poor for $x>x_s$. Our choice of $T=-6$°C at $x=x_s-\epsilon$ is based on the calculations described in Section 2 for a Budyko model with small diffusion. We found that when $T(x_s)=-10$°C, $T(x_s-\epsilon)$ was typically about -6°C. We tune γ (for μ given) by requiring $T=-6$°C at $x=x_s-\epsilon=0.95-\epsilon$ for the current value of Q. Having so chosen γ, we can repeat the calculation of the change for T_{equator} while holding Q fixed and varying x_s. The results for various choices of μ are shown in Fig. 6. We see that as $\mu\rightarrow 0$, T_{equator} actually begins to increase as x_s decreases (at least as long as $x_s>x_h$); in other words, we have succeeded in reducing the extent to which the tropics shares its heat. However, in Fig. 7 we show $Q(x_s)$ for various choices

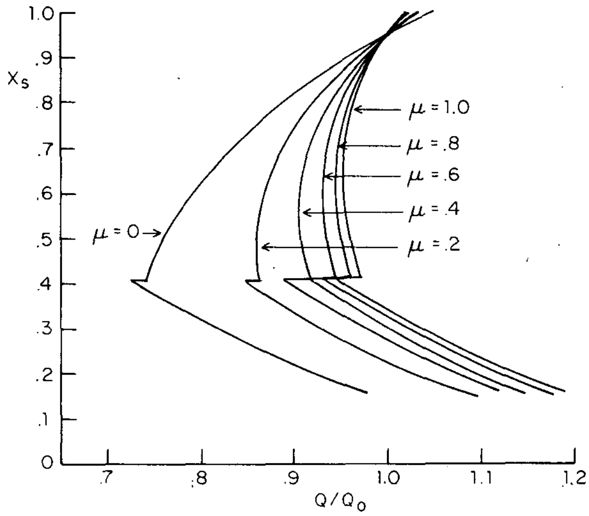

FIG. 7. x_s vs Q/Q_0 for the cases described in Fig. 6.

of μ; we see that as μ decreases below 1, the sensitivity decreases markedly (i.e., the decrease of x_s calls for a larger decrease of Q). When the effects of the decrease of Q are included in the calculation of T_{equator} we obtain the results shown in Fig. 8. Note that as x_s decreases from 0.95 to 0.85, the *smallest* reduction of T_{equator} is associated with $\mu=1$; this is clearly counter to what we wish! The effect of increased insulation of the tropics has, in this case, been overwhelmed by increased stability produced by this insulation.

Before completely rejecting this model, we should recall from Section 2 that the local sensitivity at high latitudes (as measured by dQ/dx_s) can be increased without reducing the insulation of the tropics by simply adding an additional heat transfer mechanism which serves to smooth temperature variations outside the tropics. Suppose that in *addition* to the heat transfer term we have just been considering (with $\mu=0$, for convenience) we include a term $\gamma_a(T-\tilde{\tilde{T}})$, where

$$\gamma_a=\begin{cases}0, & x<x_h\\ \alpha\gamma(x<x_s), & x>x_h\end{cases}$$

$$\tilde{\tilde{T}}=\frac{\int_0^1 \gamma_a T dx}{\int_0^1 \gamma_a dx}.$$

We will henceforth refer to $\gamma(x<x_s)$ as γ_0 when $\mu=0$. Once more, we tune γ_0, for each choice of α, by requiring that $T=-6°\text{C}$ at $x=0.95-\epsilon$ for the current value of Q.

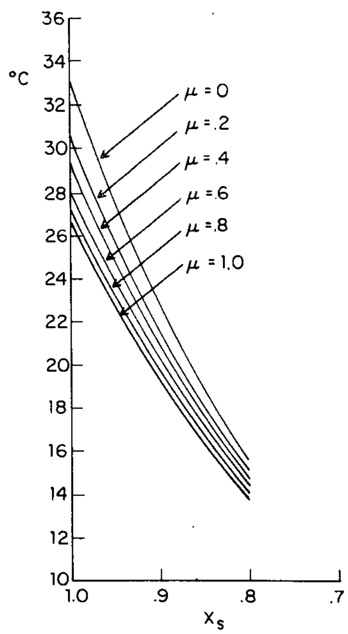

Fig. 8. T_{equator} vs x_s for the cases described in Fig. 6. In this diagram, however, Q/Q_0 is allowed to vary with x_s as indicated in Fig. 7.

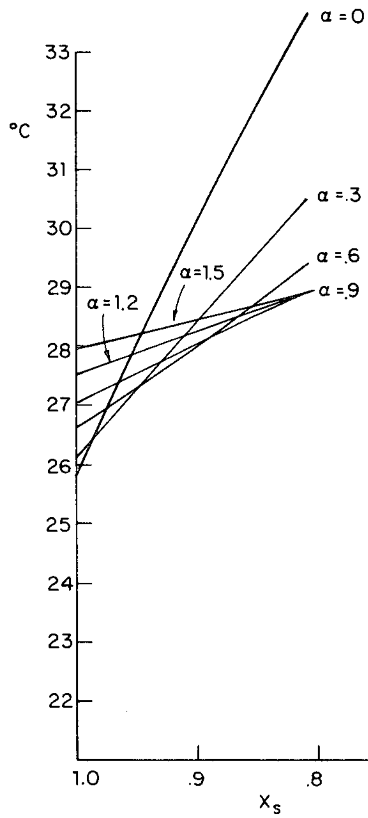

Fig. 9. T_{equator} vs x_s for Q/Q_0 held fixed and equal to 1, for models with both Budyko transport between the equator and $x_s(\gamma_0)$ and between the edge of the Hadley zone and the pole (γ_a). We set $\gamma_a=\alpha\gamma_0$ and show results for several choices of α.

As α increases, relatively smaller values of γ_0 are needed to maintain the temperature at $x_s-\epsilon$. In Fig. 9 we show the variation of T_{equator} for several choices of α

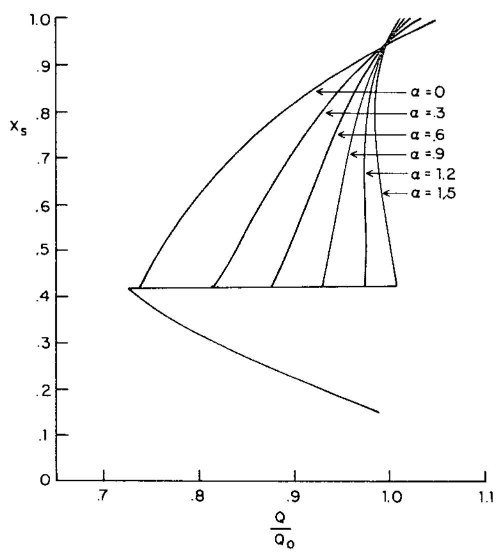

Fig. 10. x_s vs Q/Q_0 for the cases described in Fig. 9.

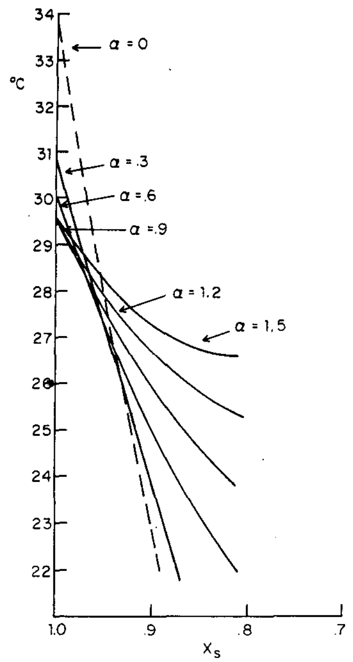

FIG. 11. T_{equator} vs x_s for the cases described in Fig. 9. In this diagram, however, Q/Q_0 is allowed to vary with x_s as indicated in Fig. 10.

as x_s changes and Q is held fixed. We see that the increase in T_{equator} as x_s decreases, associated with $\mu=0$, disappears as $\alpha\rightarrow\infty$. At the same time T_{equator} is much larger for large values of α. The point is that for large α, the bulk of the heat maintaining the temperature at $x_s-\epsilon$ comes from the region $x\geqslant x_h$ and the tropics are truly isolated from higher latitudes; for small values of α, transport associated with γ_0 is important and the tropics are not completely isolated from higher latitudes, but the isolation increases as x_s decreases. In Fig. 10, which shows $Q(x_s)$, we see that as α increases, the sensitivity outside the tropics is markedly increased—much as we expected. Thus, it should come as no surprise to see in Fig. 11, which shows $T_{\text{equator}}(x_s)$ with the effects of varying Q included, that as α increases the variation of T_{equator} can be made arbitrarily small. Indeed, as $\alpha\rightarrow\infty$, the extratropical region becomes unstable and T_{equator} increases as x_s decreases—but this limit is patently unrealistic. For $\alpha=1.2$ we find, in fact, that T_{equator} decreases by less than 2°C as x_s goes from 0.95 to 0.85.

This is perhaps as good a time as any to introduce the concept of *global stability*. We *define* this to be the *percent decrease of Q from its present value needed to provoke an instability which will lead to an ice-covered earth.* Thus for the simple Budyko model (Fig. 2) the global stability is 1.8%, for the Budyko model with a Hadley adjustment where $\theta_h=25°$ the global stability is 5% (Fig. 5), and in the present case (Fig. 10) the global stability is 28% regardless of α. Note that the reduction of stability at high latitudes, associated with increasing α, does not affect global stability. As we will see in this section, insulation of the tropics appears to be generally associated with a large increase in global stability. Before considering whether the present case is in some sense realistic, we will briefly examine model (b).

For simplicity we will restrict ourselves to

$$\gamma=\begin{cases}0, & x<x_a<x_h\\ \gamma_a, & x>x_a.\end{cases}$$

As in the preceding cases we take $\theta_h=25°$, and we consider x_a's associated with $\theta_a=0°$ (simple Budyko model), 10°, 20° and 25°. (The last case corresponds to $\alpha=\infty$ in the preceding calculation.) For each choice of θ_a, we "tune" γ_a so that $T=-6$°C at $x=0.95-\epsilon$ for the current value of Q. In Fig. 12a we examine the variation of T_{equator} with x_s for the "artificial" situation wherein Q is held fixed. In contrast to model (a), the apparent insulation effected by increasing θ_a is rather small until $\theta_a=\theta_h$ at which point the tropics and extratropics are totally decoupled. For example as x_s advances from 0.95 to 0.85, T_{equator} changes 3°C for $\theta_a=0°$ and 2°C for $\theta_a=20°$. The rather weak effect of changing θ_a results from the interaction of our Hadley adjustment with the "tuning" of γ_a. In order for sufficient heat to be transported to $x=0.95-\epsilon$ to maintain it at $T=-6$°C, γ_a must be increased as θ_a increases (the values obtained for γ_a are shown in Fig. 12a). In effect more heat is extracted from a smaller region. The Hadley adjustment serves to distribute the heat loss from the region $\theta_a\leqslant\theta\leqslant\theta_h$ over the entire tropics $(0\leqslant\theta\leqslant\theta_h)$. Thus, as long as $\theta_a<\theta_h$, the insulation of the equator is less than one might expect. Indeed one might wonder why there is any insulation of the tropics in this case. The answer is that as θ_a approaches θ_h, an increasingly large portion of the heat which maintains T at $x_s-\epsilon$ is drawn from latitudes greater than θ_h. This situation reaches an extreme when $\theta_a=\theta_h$. Note from Fig. 12a that γ_a jumps almost to ∞. Indeed, if we had required that T be greater than -6°C at $x=0.95-\epsilon$ for the current value of Q, then for $\theta_a=\theta_h$, no value of γ_a would have been sufficient! We will have more to say later about this need for γ_a to increase as θ_a approaches θ_h. Obviously, if there are limits to the efficiency of heat transfer, there will be a need for γ_a to overlap the tropics to at least some extent. Disregarding this, we may expect that the increase in γ_a (as θ_a approaches θ_h) to lead to destabilization of the extratropics (as explained in Section 2). This is confirmed in Fig. 13a where we show $Q(x_s)$ for various choices of θ_a. The situation in the extratropics when $\theta_a=\theta_h$ is similar to what the Hadley adjustment produces for $\theta<\theta_h$. It is also important to note from Fig. 13a that as θ_a increases (i.e., as the tropics become more insulated), the global stability does increase consistent with our earlier

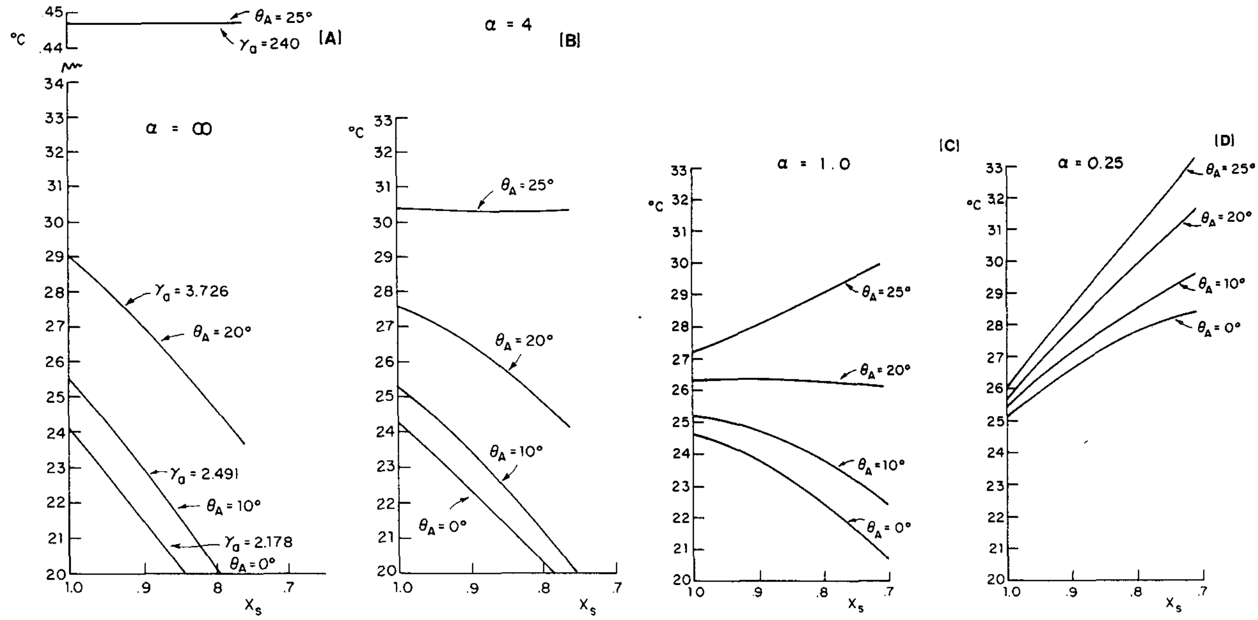

FIG. 12. T_{equator} vs x_s for Q/Q_0 held fixed and equal to 1. The models considered consist in two Budyko transports: one acting between the equator and $x_s(\gamma_0)$ and the other acting between some latitude θ_a within the Hadley zone ($\theta_a \leqslant \theta_h$) and the pole ($\gamma_a$) We set $\gamma_a = \alpha\gamma_0$. Results for $\alpha = \infty$ are shown in (a), for $\alpha = 4$ in (b), for $\alpha = 1$ in (c) and for $\alpha = 0.25$ in (d). Results are various choices of θ_a for shown in each of the diagrams.

remarks. In Fig. 14a we show $T_{\text{equator}}(x_s)$ for different choices of θ_a, when Q varies as indicated in Fig. 13a. For this case increased insulation of the tropics is accompanied not only with increased global stability but also by increased sensitivity within high latitudes. Thus, as θ_a gets larger, T_{equator} decreases less as x_s ad-

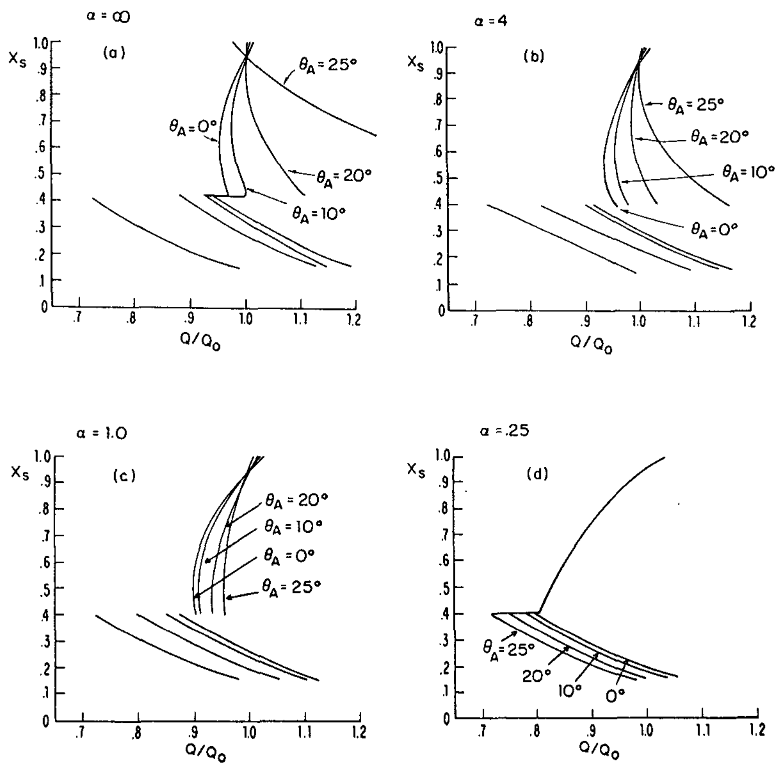

FIG. 13. x_s vs Q/Q_0 for the cases described in Fig. 12.

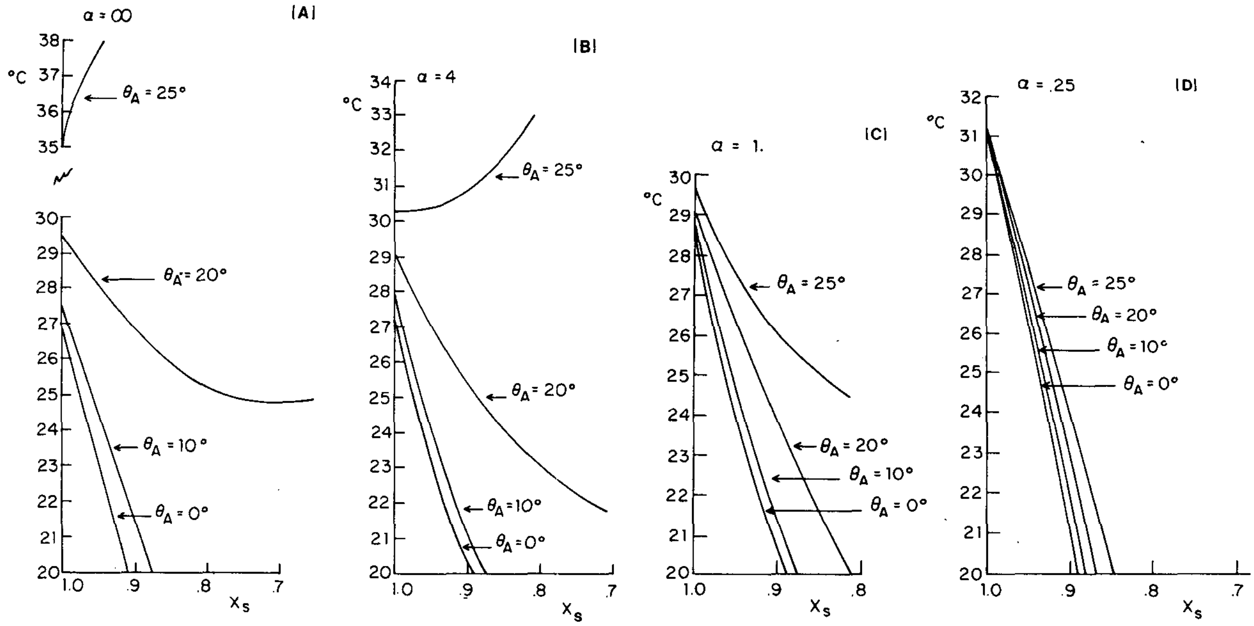

FIG. 14. T_{equator} vs x_s for the cases described in Fig. 12. In this figure, however, Q/Q_0 is allowed to vary with x_s as indicated in Fig. 12.

vances from 0.95 to 0.85. For $\theta_a = 20°$ the change is only 2°C. The behavior for $\theta_a = \theta_h$ is meaningless since it is associated with an unstable climate under current conditions (i.e., an advance of the ice line requires an increase in Q!).

The above calculations show, formally, how one may use Budyko-type heat transports to insulate the tropics in a manner consistent with CLIMAP observations. Moreover, we have shown that such insulation leads, rather generally, to an increase in global stability. However, significant details of $Q(x_s)$ do depend very markedly on whether we use model (a), model (b), or some combination of the two. We shall, in fact, investigate various combinations in detail. We hope that some combination (in conjunction with the Hadley adjustment) will yield a reasonable approach to the actual situation.

b. Implications of different approaches to insulating the tropics

In order to assess the physical plausibility of various combinations of the previously described mechanisms it would be helpful if we could identify our models with specific heat transport mechanisms in the atmosphere and oceans. Current estimates of atmospheric eddy heat transports and oceanic heat transports are suggestive in this regard. Thus, atmospheric eddy heat transport appears to act between some latitude within the tropics and the pole (Newton, 1972; Palmén and Newton, 1969; Lorenz, 1967). As argued in Section 2, the most important characteristics of a heat transfer mechanism (from the point of view of climate stability) are the latitudes over which it is acting to smooth temperatures and the effectiveness with which it does so. It therefore seems reasonable to identify γ_a with atmospheric eddy heat transfer. Similarly, available estimates of oceanic heat transport (Vonder Haar and Oort, 1973; Newton, 1972; Palmén and Newton, 1969) suggest that it extends from the equator to the ice line may, therefore, be identified with γ_0. In fact, we shall, for convenience, refer to the heat transports associated with γ_a and γ_0 as atmospheric eddy and oceanic heat transports, respectively. However, there is ample reason for uncertainty with these designations. For example, if one naively views sea ice as an insulator, then oceanic heat transport would indeed be inconsequential for the surface heat budget beyond the ice line. However, sea ice undergoes significant seasonal excursions, and the annual mean oceanic flux would not cease at the annual mean position of the sea ice. Another potential complication is noted in Held's (1976) description of the Held and Suarez general circulation model. In their model there is no ocean heat transport; in addition, atmospheric eddy transports extend from the equator (we will suggest a reason for this shortly) to the pole. Their model, nevertheless, offers a measure of insulation for the tropics because ice/snow covered regions, in their model, are characterized by relatively large static stabilities which inhibit heat transfer by baroclinic eddies, suggesting a possible contribution of atmospheric eddies to γ_0.[6] A final note of caution is in order concerning the use of Budyko-type heating laws. We saw in

[6] Held and Suarez' stability properties are very similar to those shown in Fig. 7 with $\mu \approx 0.6$.

Section 2 that gross features of $Q(x_s)$ are reasonably independent of whether dynamic heating is represented by a linear law or by a diffusion model. Held and Suarez (1974) and North (1975) have also shown that other heating laws (like nonlinear diffusion) also yield very similar results. Nevertheless, the specific choice of heating law *does* affect certain details of $Q(x_s)$—most notably its behavior at high latitudes. Thus, the extent to which $T_{\text{equator}}(x_s)$ (shown in Figs. 11 and 14a) varies due to variation of $Q(x_s)$ for $x_s>0.8$, is certainly less reliable (and hence less significant) than the degree to which the equator is shielded from ice advances, as shown in Figs. 9 and 12a.

With all the above caveats in mind we proceed to examine the results of models wherein both $\gamma_a \neq 0$ and $\gamma_0 \neq 0$. Our procedure is as follows: we choose a ratio $\alpha=\gamma_a/\gamma_0$, and a θ_a (latitude below which $\gamma_a=0$), and tune γ_0 so that $T=-6°\text{C}$ at $x=0.95-\epsilon$ for the present value Q. We then go on to evaluate T_{equator} with Q held fixed at $Q(x_s)$, and $T_{\text{equator}}(x_s)$ with Q varying as required by $Q(x_s)$. Model (a), discussed earlier (with $\mu=0$) corresponds to $\alpha=0$, and model (b) corresponds to $\alpha=\infty$. Further results have been obtained for $\alpha=0.25$, 1 and 4, and for $\theta_a=\theta_h=10°$, 20° and 25°. The results for the variation T_{equator} (actually T_{tropical} would be more appropriate) as x_s is changed while Q is held fixed are shown in Fig. 12. The results are essentially self-explanatory and entirely consistent with the results obtained from models (a) and (b). Clearly the maximum insulation of the tropics is achieved when $\alpha=\infty$ and $\theta_a=\theta_h=25°$. However, more germane to agreement with CLIMAP results is the degree to which decreases in T_{equator} as x_s decreases from 0.95 to 0.85 are *inhibited*. In general this inhibition is aided by decreasing α (i.e., increasing the relative importance of oceanic heat transport) and by increasing θ_a (i.e., decreasing the overlap between regions of Hadley and atmospheric eddy transport). Assuming climatic stability for x_s between 0.95 and 0.85, a *minimum* requirement for consistency with CLIMAP results is that the decreases in T_{equator} shown in Fig. 12 as x_s goes from 0.95 to 0.85 be less than 2°C. We see that this condition is met for all choices of θ_a as long as α is less than 4. For $\alpha>4$, progressively larger choices of θ_a are necessary; for $\alpha=\infty$, θ_a must be greater than 20°. As already noted, large values of α and θ_a call for large values of γ_a. Some question may arise as to whether arbitrary values of γ_a may ever be achieved. Let us assume eddy transport arises from baroclinic instability; two-level calculations imply that a minimum north-south temperature gradient (not very different from existing values in mid-latitudes) is necessary for the onset of this instability (Thompson, 1961). If this is relevant to the atmosphere, it implies that eddies cannot be arbitrarily effective in smoothing out temperature differences. Thus, if a two-level model, lacking ocean transports, is to simulate the current climate, we would expect its eddy heat transport to begin very near the equator [in fact this appears to be the case in a model described in Held (1976)].

The stability of the above models (with both atmospheric and oceanic transport) as indicated by $Q(x_s)$ is shown in Fig. 13. We see that for $\theta_a=\theta_h=25°$, global stability is 28%, independent of α! We have, in fact, already discussed this case, but it is worth noting that for $\alpha=\infty$, global stability is concentrated at the edge of the Hadley transport. Indeed, for $\alpha=\infty$, middle and high latitudes can be locally unstable, although for decreases in Q less than the global stability the ice would advance no further than θ_h. As α decreases, the global stability becomes more uniformly distributed over the extratropical region. For $\alpha\sim O(1)$, extratropical latitudes are everywhere stable, and for $\alpha=0$, most of the global stability is distributed throughout extratropical latitudes. For $\theta_a<\theta_h$, decreases in α lead to increases in global stability, as well as to a more uniform distribution of global stability throughout the extratropics. (For those cases previously mentioned as being consistent with CLIMAP, global stability ranges from $7\frac{1}{2}$% to 28% in contrast to 5% for $\alpha=\infty$ and $\theta_a=0$; essentially the Budyko model with a Hadley adjustment.)[7] In the absence of oceanic heat transport ($\alpha=\infty$), there is always a significant portion of the extratropics (between θ_h and some $\theta>\theta_h$) which is locally unstable. On the other hand, for comparable oceanic and atmospheric contributions to heat transport [$\alpha\sim O(1)$], climate is stabilized for all $\theta_s>\theta_h$. If our identification of γ_0 with oceanic heat transport should prove correct then this stabilization is likely to be a major contribution of the oceans to the climate. For $\alpha\sim O(1)$ and a reasonable degree of overlap between Hadley and eddy fluxes ($10°<\theta_a<20°$), global stability is between 15 and 20% with about half the global stability distributed through the extratropics.

Turning finally to Fig. 14, wherein the variations of T_{equator} with x_s (when the variations of Q with x_s are included) are shown, we see that only those models with $\alpha>1$ and $\theta_a>20°$ are immediately consistent with the CLIMAP results. This disturbing feature is not in agreement with the known behavior of the current atmosphere (θ_a should be less than 20°). However, it is a feature which depends critically on the detailed behavior of $Q(x_s)$ for $x_s>0.8$ and as we have already noted, the behavior $Q(x_s)$ in this region is dependent upon the specific model of heat transport used. Hence, the results in Fig. 14 are much less reliable than the general results shown in the preceding two figures. Indeed there is reason to believe that Q in reality varies

[7] Differences in sensitivity and global stability between these results and those in Section 2 arise from the fact that different criteria for the position of the ice line are employed. In Section 2 we used $T(x_s)=-10°\text{C}$, while in this section we use $T(x_s-\epsilon)=-6°\text{C}$.

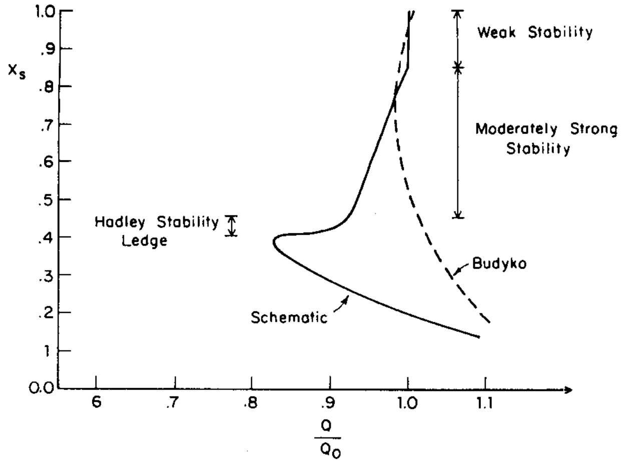

FIG. 15. A schematic illustration of how we expect x_s to vary with Q/Q_0. For comparison purposes, we also show the variation of x_s with Q/Q_0 for a simple Budyko-type model without a Hadley adjustment but with a small amount of diffusion.

very slightly with x_s for $x_s>0.8-0.85$.[8] This belief arises from the recent claim on the basis of paleoclimatological time series that the Milankovitch (1938) hypothesis, wherein major changes in climate are related to variations in the earth's orbit, is correct (Broecker, 1966). Independently, Suarez and Held using a relatively simple seasonally varying climate model, in which the effect of orbital variations on insolation was included, managed to simulate climatic variations on the 20 000–100 000 year time scale (Suarez, 1976; Suarez and Held, 1976). However, as Suarez and Held note, changes in insolation associated with orbital variations are extremely small and in order for a model to simulate the ice ages, the position of the ice line within the model must, in effect, be extremely sensitive to Q. If the Milankovitch hypothesis is indeed correct, it would appear to require only slight stability at high latitudes—in which case the results in Fig. 12 are more appropriate than those in Fig. 14 for determining consistency with CLIMAP results for 18 000 BP (before present). From Fig. 12 we see that a variety of cases are compatible with CLIMAP results including some with $\alpha\sim O(1)$ and a modest overlap between eddy and Hadley transports.

6. Concluding remarks

In the light of the preceding, somewhat rambling, discussion we suggest that the schematic distribution of $Q(x_s)$ shown in Fig. 15 is a reasonable description of the real state of affairs. For comparison we also show $Q(x_s)$ for a simple Budyko model with small diffusion (see Section 2). The reasons for the differences have been described at length in this paper. A brief summary may, however, prove useful at this point:

1) The stability ledge at 25° and the subsequent instability equatorward of 25° arise because of the need for temperatures in this region to be almost constant with latitude. We identify the heat transport required to achieve this with the Hadley cell, but such an identification is unnecessary.

2) The large global stability arises from the insulation of the tropics from higher latitudes required by the CLIMAP reconstruction of the climate of 18 000 BP.

3) The smooth distribution of about half the global stability in the extratropics (for $25°<\theta_s<60°$) arises from the presence of a heat flux which ceases at $\theta=\theta_s$. We tentatively identify this as the oceanic heat flux.

4) The region of weak stability for $\theta_s\gtrsim 60°$ appears to be demanded by the observed correlation between orbital parameters and major climate cycles wherein the *mean* ice line moves between 75° (present position) and 60° (its position during the major glacial period 18 000 BP).

The above picture suggests a climate wherein ~15% of the earth (in the neighborhood of the poles) is sensitive to small perturbations in Q, but where the climate of the remainder of the earth is remarkably stable.

As we have noted in Section 2, simple climate models have a number of obvious shortcomings. Among these, the fact that infrared cooling is not strictly relatable to surface temperature seems serious (Held and Suarez, 1974). Relatedly, dynamic heat fluxes inferred from satellite data ought not be strictly describable in terms of surface temperature, since a portion of the flux is not involved in the surface heat budget. Nevertheless, the fact that attempts to tune transport coefficients by (i) matching observed dynamic heat fluxes (ii) matching pole-to-equator temperature differences or (iii) matching the observed mean position of the ice line all lead to similar results suggests that this type of difficulty may not be crucial for the questions we have been addressing.

It must be emphasized, in connection with approach (i), that matching the observed total heat flux in no way indicates that one has correctly modeled the dependence of the heat flux on surface temperature. The discussion of the Hadley transport in Section 3 shows an example of an important component of the earth's heat transport which was not properly included in simple models—models which nevertheless plausibly reproduced the observed total heat flux. Our discussion in Section 2 was primarily meant to show that the gross features of climate stability depend primarily on two aspects of each of the heat transport mechanisms:

1) The range of latitudes over which the mechanism is acting to smooth temperatures.

2) The effectiveness with which the smoothing is occurring.

[8] We might equivalently say that climate is almost neutrally stable in this region, or that x_s in this region is very sensitive to variations in Q.

Since individual mechanisms (Hadley transport, atmospheric eddy transport, ocean transport) differ with respect to these aspects, they must be included separately. Thus, North (1975) restricted himself to a single, imaginary mechanism which acted to smooth temperatures over the entire globe. Recognizing this limitation of North's work, it would still seem reasonable to conclude from his work that once these two aspects of the heat transport mechanisms are determined, other details of the modeling are relatively unimportant—at least for the calculation of $Q(x_s)$.

The above supports some measure of confidence in Fig. 15. However, some obvious uncertainties remain in the model discussed in this paper. Most notably, we have assumed that the regions over which atmospheric transport mechanisms operate are not affected by x_s. Some evidence exists in Schneider (1977) to suggest that θ_h will depend only slightly on such quantities as x_s; but, the general question certainly warrants further study.

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