Hadley Circulations for Zonally Averaged Heating Centered off the Equator

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ABSTRACT

Consistent with observations, we find that moving peak heating even 2 degrees off the equator leads to profound asymmetries in the Hadley circulation, with the winter cell amplifying greatly and the summer cell becoming negligible. It is found that the annually averaged Hadley circulation is much larger than the circulation forced by the annually averaged heating. Implications for the general circulation are discussed, as are implications for Milankovitch forcing of climate variations and for tropical meteorology and oceanography.

1. Introduction

Understanding the zonally averaged circulation of the atmosphere is, arguably, the oldest scientific problem in dynamic meteorology (Lorenz 1967). The zonally averaged meridional circulation rising from the tropics is known as the Hadley circulation. Beginning with Hadley (1735) and continuing through the 19th Century (Ferrel 1856, Thomson 1857), no serious distinction was made between the zonally averaged circulation and the axially symmetric circulation forced by axially symmetric heating. By the early part of this century (Jeffreys 1926), the idea was being put forth that the zonally averaged circulation might, in large measure, be forced by eddies, and by the post-World War II period, Starr (1948) was going so far as to suggest that the symmetric circulation was inconsequential. These developments are summarized by Lorenz (1967). Surprisingly, despite all arguments, there, in fact, existed no explicit calculation of the symmetric circulation until the late 1970s (Schneider and Lindzen 1977, Schneider 1977). (To be sure, the pro-eddy views may have been so widely held as to suggest that such a calculation was not worthwhile.) Thus, until the late 1970s there really was no objective basis for assessing the shortcomings of the symmetric circulation as an explanation of the zonally averaged zonal wind and temperature.

Calculations of Schneider and Lindzen (1977) and Schneider (1977) showed that purely symmetric circulations could maintain strong subtropical jets and might contribute significantly to the maintenance of surface winds. At least as concerns the subtropical jets, these results strongly suggested that the net effect of eddies would be to reduce these jets—rather than to maintain them. Moreover, Schneider (1977) and Held and Hou (1980, HH hereafter) showed that detailed numerical results could be replicated by the application of some simple balances for angular momentum and thermal energy. The simple theory will be reviewed in section 2 of the present paper.

All the aforementioned calculations of the symmetric circulation used (following Hadley 1735; Thomson 1857; Ferrel 1856) heating distributions symmetric about the equator. Implicit in this was the assumption that the annually averaged symmetric circulation would correspond to the circulation forced by annually averaged heating. In addition, there was a question of what to use for the heating. It was argued by Lindzen (1978), that given the fact that the adjustment time for the atmosphere was much shorter than for the oceanic mixed layer, one should use a forcing which attempted to bring the atmospheric equator to pole temperature difference to that which obtains at the ocean surface (~40°C). That is to say, the atmosphere is, to a first order approximation, forced by the ocean surface. Held and Hou noted, however, that the resultant Hadley circulation was much weaker than the observed annual mean circulation. As we shall see, such a forcing (when centered at the equator) produces a Hadley circulation much weaker than the observed annually averaged circulation. Held and Hou used a forcing which attempted to bring the atmosphere's pole to equator temperature difference toward radiative equilibrium (~100°C); even then, the circulation was somewhat weak.

As it turns out, the observations provide a clue to the problem. Figure 1 from Oort and Rasmussen

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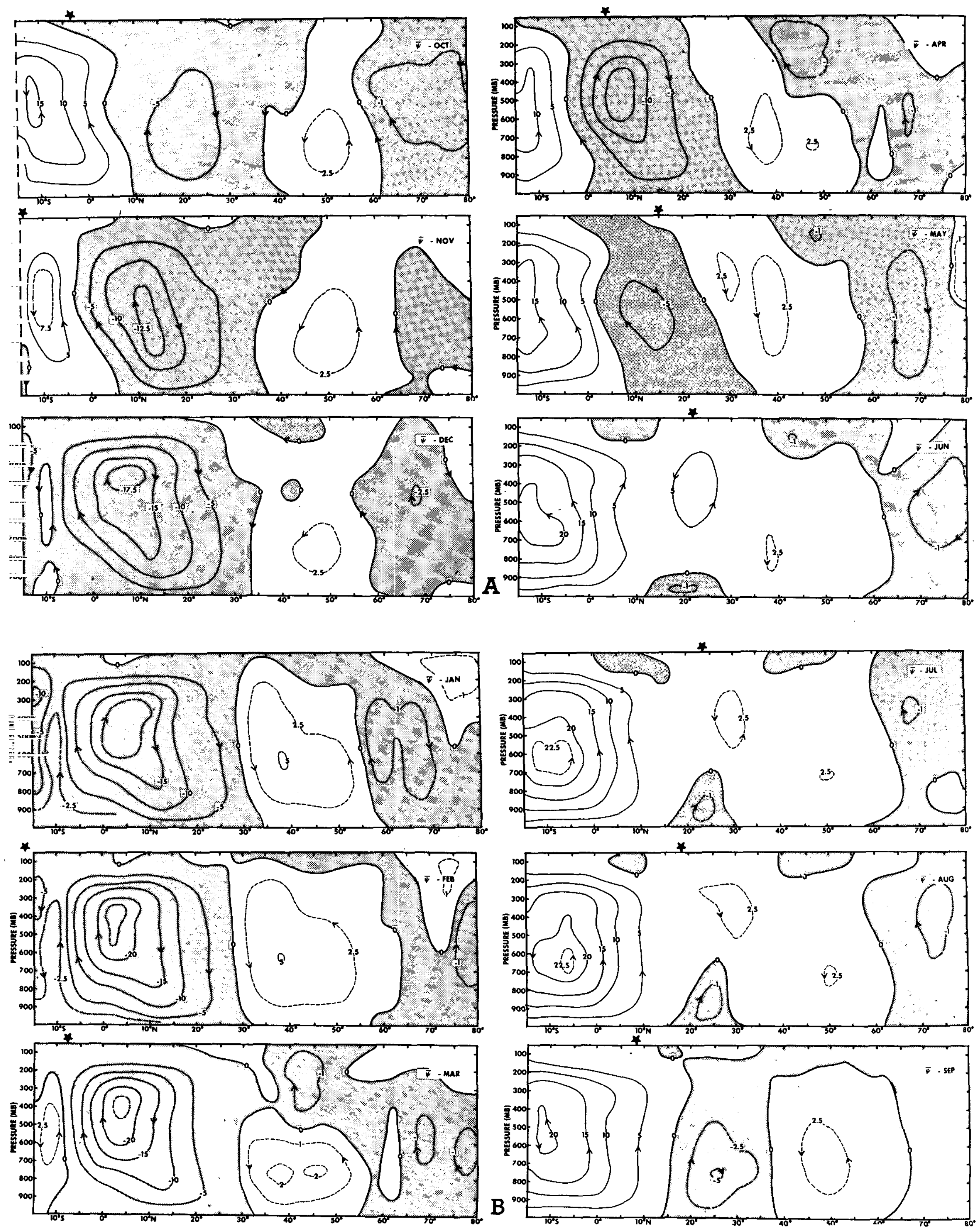

FIG. 1. Streamlines of the mean meridional circulation for each month. The isolines give the total transport of mass northward below the level considered. Units, 10^{13} gm s^{-1}. (Taken from Oort and Rasmussen 1970.)

(1970) shows monthly mean meridional circulations. While the figures only extend to 15°S, it is clear that except for April, the circulation always consists essentially in a single cell extending from deep within the hemisphere, where the sun is, to deep within the opposite hemisphere. Such a circulation is characteristic of solstices (viz. Fig. 2). What the data shows is that the meridional circulation is almost always in a solstitial pattern; the idealized equinoctial pattern is almost never realized.

As we shall show, the observed results are readily accounted for theoretically. In section 2 we will apply HH's simplified theory to the case where zonally averaged heating is centered at a latitude *off* the equator. The simplified calculations show that as the heating center moves off the equator, the latitude separating the winter and summer cells moves much further into the summer hemisphere while the summer cell becomes negligible. Moreover, the winter cell becomes much stronger—the annually averaged theoretical circulation is also much stronger than the equinoctial circulation. Accompanying these realistic features, however, were some unrealistic features such as excessive upper level easterlies at the equator and surface westerlies at the equator.

In section 3 we recalculate the cases of section 2 using a high resolution numerical model. The nonuniform response to equatorial asymmetry remains, but unrealistic features in the wind are eliminated. The results suggest that a smooth sinusoidal seasonal motion of the sea surface temperature maximum back and forth across the equator is accompanied by far more abrupt changes in the Hadley circulation. The latter is accompanied by sharp changes in surface wind—a sharp easterly jet appearing on the winter side of the equator.

In section 4 we will discuss the above results, their limitations, and their implications for the general circulation, climate dynamics and tropical meteorology and oceanography.

2. Simple balance approach to the Hadley circulation

We will assume diabatic heating of the form

$$\frac{\theta - \theta_e}{\tau}, \tag{1a}$$

where

$$\frac{\theta_e}{\theta_0} \approx 1 + \frac{\Delta_H}{3}([1 - 3(\sin\phi - \sin\phi_0)^2]) + \Delta_v\left(\frac{z}{H} - \frac{1}{2}\right) \tag{1b}$$

and

- θ_e radiative–convective equilibrium temperature at $z = H/2$
- θ_0 a reference temperature
- ϕ latitude
- ϕ_0 latitude maximum $\bar{\theta}_e = \theta_0(1 + \Delta_H/3)$
- Δ_H fractional change of potential temperature (from equator to pole when $\phi_0 = 0$)

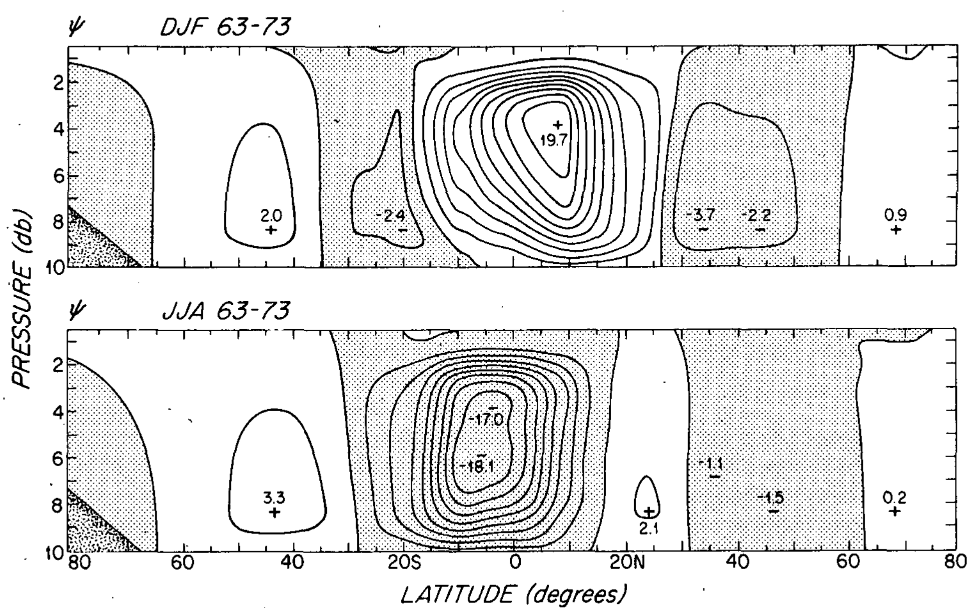

FIG. 2. Time average meridional-height cross sections of the streamfunction for the mean meridional circulation. Units, 10^{13} g s^{-1}; contour intervals, 0.2×10^{13} g s^{-1}. December–February 1963–1973 (upper panel) and June–August 1963–1973 (lower panel). (From Oort 1983.)

Δ_v fractional potential temperature drop from H to ground
τ radiative-convective relaxation time.

Schneider (1977) and HH both found that for $\phi_0 = 0$, the response to (1) approximately satisfied a few simple conditions—which could be used to calculate quickly Hadley circulations (a schematic of the geometry is shown in Fig. 3).

1) Flow in the upper branch of the Hadley circulation conserves angular momentum. Angular momentum per unit mass is given by

$$M = \Omega a^2 \cos^2\phi + ua\cos\phi, \qquad (2)$$

where Ω is earth's rotation rate, a is earth's radius. Assuming that the uppermost streamline originates at the division between summer and winter cells, ϕ_1, and is at relative rest at ϕ_1,

$$M = \Omega a^2 \cos^2\phi_1 \qquad (2a)$$

$$u = \frac{\Omega a(\cos^2\phi_1 - \cos^2\phi)}{\cos\phi}. \qquad (2b)$$

For reasons we will soon explain, $\phi_1 \neq \phi_0$ except when $\phi_0 = 0$.

2) Zonal flow is assumed to be in cyclostrophic balance:

$$fu + \frac{u^2 \tan\phi}{a} = -\frac{1}{a}\frac{\partial\Phi}{\partial\phi} \qquad (3)$$

where $\Phi = p/\rho_0$; p is pressure and ρ_0 is a reference density used in the Boussinesq approximation. We next evaluate (3) at the assumed height of the topmost streamline, H, and at the ground and subtract the two relations:

$$f[u(H) - u(0)] + \frac{\tan^2\phi}{a}[u^2(H) - u(0)] = -\frac{1}{a}\frac{\partial}{\partial\phi}[\Phi(H) - \Phi(0)] \qquad (3a)$$

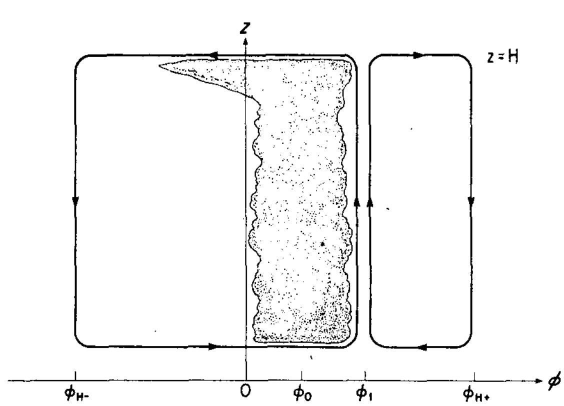

FIG. 3. Schematic illustration of the Hadley circulation.

Assume $|u(0)| \ll |u(H)|$ so that

$$fu(H) + \frac{\tan^2\phi}{a}(u^2(H)) = -\frac{1}{a}\frac{\partial}{\partial\phi}[\Phi(H) - \Phi(0)] \qquad (3b)$$

3) Assume hydrostaticity:

$$\frac{\partial\Phi}{\partial z} \approx \frac{g}{\theta_0}\theta \qquad (4)$$

Integrating (4) from 0 to H and substituting in (3b) yields

$$fu(H) + \frac{\tan^2\phi}{a}u^2(H) = -\frac{gH}{a\theta_0}\frac{\partial\bar{\theta}}{\partial\phi} \qquad (5)$$

where

$$\bar{\theta} = \frac{1}{H}\int_0^H \theta\, dz.$$

Finally, substituting (2b) into (5), we get

$$\frac{1}{\theta_0}\frac{\partial\bar{\theta}}{\partial\phi} = \frac{\Omega^2 a^2}{gH}\left(\frac{\tan\phi}{\cos^2\phi}\cos^4\phi_1 - \sin\phi\cos\phi\right). \qquad (6)$$

Integrating (6) gives us the distribution of $\bar{\theta}$ with ϕ:

$$\frac{\theta(\phi) - \theta(\phi_1)}{\theta_0} = -\frac{\Omega^2 a^2}{2gH}\frac{(\sin^2\phi - \sin^2\phi_1)^2}{\cos^2\phi}. \qquad (7)$$

Equations (2b) and (7) give us the distributions of zonal wind and mean temperature associated with the Hadley circulation. Note that according to Eqs. (2b) and (7), both u and θ are symmetric about the equator even when $\phi_0 \neq 0$. Asymmetry arises because the extents of the summer and winter cells will be different. To complete the solution we need to obtain ϕ_1, as well as ϕ_{H^+} (the outer boundary of the summer cell) and ϕ_{H^-} (the outer boundary of the winter cell). Note that when $\phi_0 = 0$, $\phi_1 = 0$ and $\phi_{H^+} = \phi_{H^-}$ by symmetry; however, when $\phi_0 \neq 0$ these symmetries no longer obtain. The conditions needed follow.

4) Conservation of energy: For the linear heating law (1a) and a constant τ this amounts to

$$\int_{\phi_1}^{\phi_{H^+}} (\bar{\theta} - \bar{\theta}_e)\cos\phi\, d\phi = 0 \qquad (8)$$

$$\int_{\phi_1}^{\phi_{H^-}} (\bar{\theta} - \bar{\theta}_e)\cos\phi\, d\phi = 0. \qquad (9)$$

5) Continuity of temperature: This requires that

$$\bar{\theta}(\phi_{H^+}) = \bar{\theta}_e(\phi_{H^+}) \qquad (10)$$

$$\bar{\theta}(\phi_{H^-}) = \bar{\theta}_e(\phi_{H^-}) \qquad (11)$$

and continuity at ϕ_1.

The requirement of continuity at ϕ_1 was unnecessary when $\phi_0 = 0$. However, when the winter and summer cells are different this condition must be imposed. *The only way it can be satisfied is to leave ϕ_1 as a variable to be determined.* Note that Fig. 3 shows the distinction between ϕ_1 and ϕ_0. Figure 3 schematically suggests that the maximum cumulus activity and hence the maximum vertical flow remain close to ϕ_0. As we shall show later, this is, in fact, likely. In contrast to the symmetric situation where $\phi_1 = \phi_0 = 0$, maximum vertical velocities no longer occur at ϕ_1. Here ϕ_1 is simply the boundary between the "winter" cell, and the (weak, as it will turn out) "summer" cell.

Figure 4 shows ϕ_1, ϕ_{H^+} and ϕ_{H^-} as functions of ϕ_0 for both $\Delta = 1/3$ and $\Delta = 1/6$ (corresponding to θ_e(equator) $- \theta_e$(pole) $\approx 97°$, and $48°$ respectively). In these calculations we have taken $H = 15$ km; note that HH used $H = 8$ km. Notice that ϕ_1 is much larger than ϕ_0 once $\phi_0 \neq 0$. Figure 5 shows both $\bar{\theta}_e$ and $\bar{\theta}_v$, ϕ for $\Delta = 1/6$ and $\phi_0 = 0$ and $\phi_0 = 6°$. The intensity of the Hadley circulation is essentially proportional to the difference between $\bar{\theta}$ and $\bar{\theta}_e$. Note that for $\phi_0 = 0°$ this difference is very small as it is for the small summer cell. However, for the winter cell, it is much larger. In general, ϕ_0 varies between $\pm 6°$–$8°$. *It is easily seen that the average circulation obtained for $\phi_0 = +6°$ and $\phi_0 = -6°$ is much larger than the circulation for $\phi_0 = 0°$.*

Figure 6 shows $u(H, \phi)$ for $\phi_0 = 0°$ and $6°$. For $\phi_0 = 0$ we have $u(H, 0) = 0$ m s^{-1} and two strong jets at $\phi = \pm\phi_H$. For $\phi_0 = 6°$, we have $u(H, 0) = -50$ m s^{-1} (which is very unrealistic), a very strong jet at $\phi = \phi_{H^-}$ and a relatively weaker jet at $\phi = \phi_{H^+}$.

In Fig. 5b we see that the simple equatorial temperature flattening found when $\phi_0 = 0$ is now replaced by a noticeable equatorial minimum. This is indeed characteristic of the real atmosphere in the tropical tropopause region; however, $\bar{\theta}$, here, represents a tropospheric average.

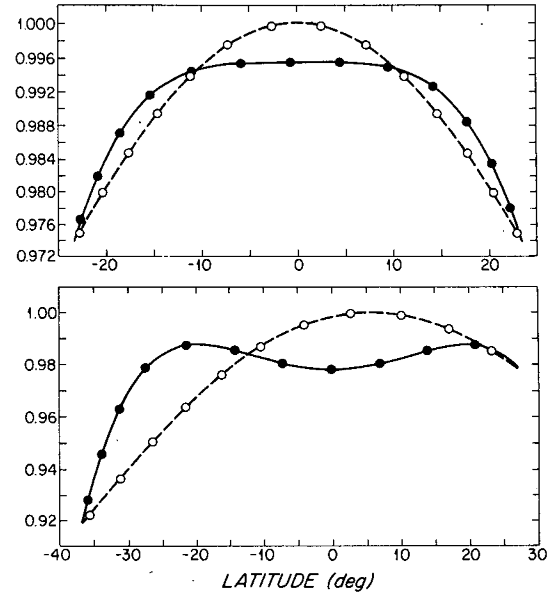

FIG. 5. $\bar{\theta}/\theta_0$ (solid line) and $\bar{\theta}_E/\theta_0$ (dashed line) as functions of ϕ using the simple model. (a) $\phi_0 = 0$. (b) $\phi_0 = 6°$.

Following HH, we may estimate both the intensity of the Hadley circulation and the surface wind distribution. Ignoring vertical heat diffusion, our equation for θ is

$$\nabla \cdot (\mathbf{u}\theta) = -\frac{(\theta - \theta_E)}{\tau} \tag{12}$$

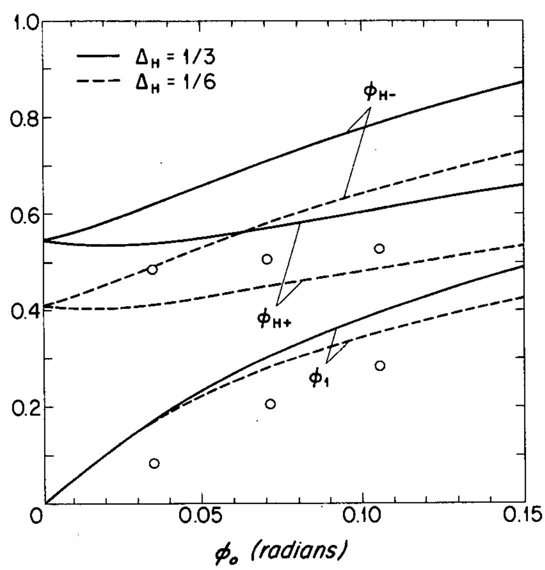

FIG. 4. ϕ_1, ϕ_{H^+} and ϕ_{H^-} as functions of ϕ_0 (see text for definitions). Open circles show results from numerical integration for ϕ_1 and ϕ_{H-1} when $\Delta_H = 1/6$. (Note 1° of latitude $\approx$ 0.0175 radians.)

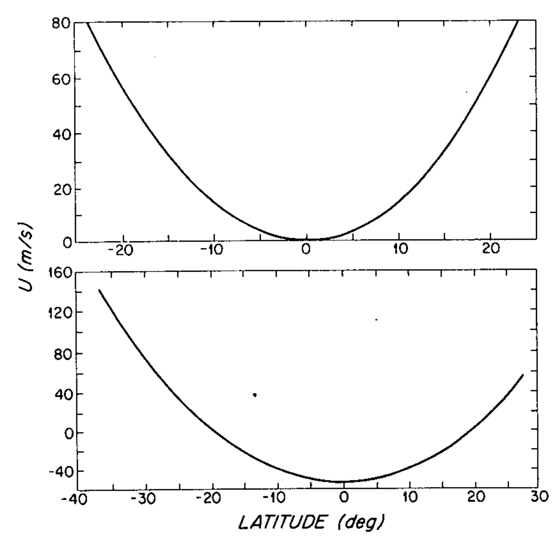

FIG. 6. $u(H, \phi)$ as a function of ϕ using the simple model. (a) $\phi_0 = 0$. (b) $\phi_0 = 6°$.

Integrating (12) between $z = 0$ and H we get

$$\frac{1}{H}\int_0^H \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(v\theta\cos\phi)dz = \frac{\bar{\theta}_E - \bar{\theta}}{\tau} \quad (13)$$

where ($\bar{}$) refers to vertical average. Equation (13) is readily solved for $\int_0^H \theta v dz$. Continuity implies

$$\int_0^H \rho_0 v dz = 0$$

If we assume v is confined to thin layers, then we may solve for the mass flow

$$V\Delta_v = \frac{1}{\theta_0}\int_0^H v\theta dz \quad (14)$$

where V is the mass flux in the upper and lower boundary layers. Our equation for angular momentum is

$$\frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\cos^2\phi\int_0^H uvdz = -cu_0 \quad (15)$$

where c is a linearized surface drag parameter and u_0 a surface zonal wind. Now

$$\int_0^H vudz \approx Vu_M \quad (16)$$

assuming $u_0 \ll u_M$. Using (16) we may solve (15) to obtain $u_0(\phi)$. Figure 7 shows $u_0(\phi)$ for $\Delta = 1/6$ and $\phi_0 = 0°$ and $6°$ where, following HH, we have used $c = 0.005$ m s^{-1}. Several features are immediately evident. First, the magnitudes are significantly larger than HH obtained with $\phi_0 = 0$. Given our linearization of the surface drag formula and the uncertainty in our choice of c, this might not be too worrisome—though the values in Fig. 7 would call into question our assumption that $|u(\phi, 0)| \ll |u(\phi, H)|$. Second, the pattern of equatorial surface easterlies surrounded by surface westerlies has been pushed well into the winter hemisphere. The surface easterlies are now totally in the winter hemisphere; the summer hemisphere and the equator itself have surface westerlies. The last feature is potentially worrisome because it is in violation of Hide's theorem.

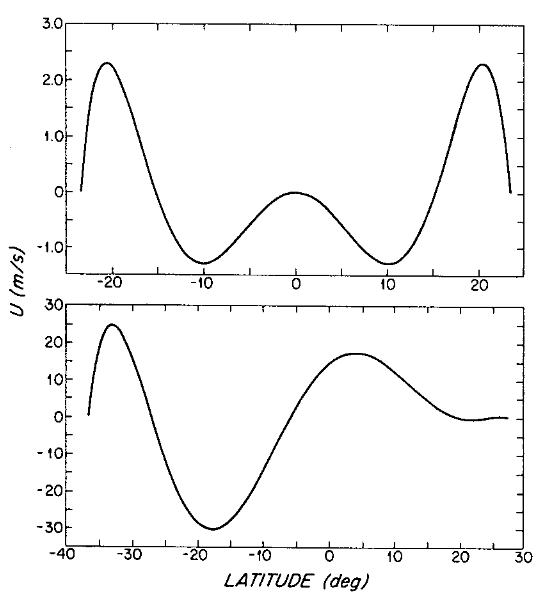

FIG. 7. $u(0, \phi)$ as a function of ϕ using the simple model. (a) $\phi_0 = 0$. (b) $\phi_0 = 6°$.

Hide's theorem is discussed in detail in HH. Briefly, it states that for a symmetric circulation, there can be no extrema of zonal angular momentum within the atmosphere. This, in particular, prohibits equatorial westerlies in the absence of eddies. Maxima of angular momentum can exist only at the surface in regions of surface easterlies; minima can exist only at the surface in regions of surface westerlies. Since Hide's theorem actually depends on down gradient diffusion of momentum which is not specifically accounted for in the Held–Hou simple approach, it is not surprising that the present results do not satisfy Hide's theorem. This will be discussed further, in section 3 where numerical results for a continuous fluid are presented.

It is reasonable at this point to review what we have found so far. It should be stressed that the calculations in this section are based on an "after the fact" study of detailed numerical solutions in which certain balances were found to obtain approximately. These balances led to the present simple calculations. However, the original calculations were for cases where $\phi_0 = 0$ (i.e., equinoctial conditions). Our present results suggest that solutions change markedly when $\phi_0 \neq 0$—and in ways that call into question the validity of the simple model. We will study the more detailed numerical solutions in the following section. It will be seen that a meaningful correspondence to the simple model remains. Here we will cite the major changes the simple model points to when $\phi_0 \neq 0$.

1) For $\phi_0 = 0$, symmetry leads to a certain degeneracy that does not hold when $\phi_0 \neq 0$. In particular, when $\phi_0 = 0$, the latitude separating northern and southern cells, coincides with ϕ_0 (the center of the heating distribution) which is also the latitude of maximum upward vertical flow. This degeneracy is broken when $\phi_0 \neq 0$. Now the latitude separating winter and summer cells, ϕ_1, no longer occurs at ϕ_0. Moreover, it is readily seen that ϕ_1 no longer corresponds to the latitude at which we have maximum upward flow. That latitude remains close to ϕ_0.

2) The ϕ_1 extends much further into the summer hemisphere than does ϕ_0. Moreover, once ϕ_0 is greater than 1°–2°, the circulation in the winter cell is much stronger than either the circulation in the summer cell or the circulation when $\phi_0 = 0$. Indeed, the inferred winter intensity is excessive for $\Delta_H = 1/3$ (as used by HH), $\Delta_H = 1/6$ yields more plausible values. It should

be noted in advance that features (1) and (2) remain in the detailed numerical calculations to follow.

3) The altered meridional circulation described in item (2) implies a sharply altered transport of zonal angular momentum [viz. Eq. (5)]. In the simplified calculations of the present section this leads to the pronounced equatorial easterly maximum at the equator for the outermost streamline [$u(H, 0)$]. It also leads to the intensification and shifting of the surface wind pattern.

The strong upper level equatorial easterlies, the strength of the surface winds and the surface equatorial westerlies are all at variance with observations and the last feature is at variance with basic theory appropriate to continuous fluids. Not surprisingly, all these discrepancies disappear in the detailed numerical integrations.

Some clue to the relevant problem with the simple calculations appears in HH. Held and Hou attempt to carry out their numerical solutions in the limit of vanishing viscosity. They discovered that they could not obtain steady symmetric solutions for viscosity below a certain value. Schneider (1984) subsequently argued that this was a physical rather than a numerical effect; he suggested that a minimum viscosity was necessary for the existence of steady solutions corresponding to the simple results of this section; and that the unsteady solutions found at smaller viscosities might in fact emulate the needed viscosity. These issues were also discussed in HH and Hou (1984). Indeed, at the minimum viscosity, the results of HH closely resemble those resulting from the simple model—except in one significant regard. The simple model takes the angular momentum at $z = H$ as being that pertaining to an atmosphere at rest at $\phi = \phi_1$ (which in HH is also $\phi_0 = 0$). This is mathematically expressed in the present equation (2a). What HH found is shown in Fig. 8. As ν was reduced to the minimum value, M did not tend toward the value given by Eq. (2a) (for $\phi_1 = 0$). Rather, M diminished as one moved off the equator, asymptoting to a value characteristic not of the equator but of a region of about 15° about the equator. Clearly air rising in this region moved poleward along arbitrarily close streamlines across which there was viscous mixing of angular momentum. When $\phi_0 = 0$, the air on these adjacent streamlines, originating from the neighborhood of the equator, has similar values for M and the consequences of mixing are modest. However, when $\phi_0 \neq 0$, the outermost streamlines originates at ϕ_1 but the bulk of the adjacent streamlines originate from the neighborhood of ϕ_0 which is far from ϕ_1. Since the streamlines originating near ϕ_0 have very different values of M from those originating at ϕ_1, the consequences of mixing are far more pronounced.

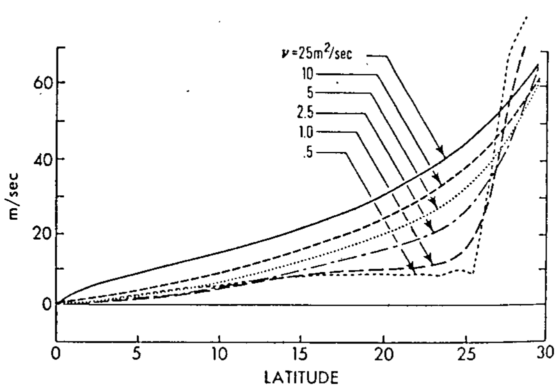

FIG. 8. A measure of M [viz. Eq. (2); namely, $(\Omega a^2 - M)/a$], evaluated at $z = H$ as a function ϕ for diminishing values of viscosity, ν. $\phi_0 = 0$; $H = 8$ km. Results are from the numerical calculations of HH.

4) As in Schneider (1977) and in HH, $u(H, \phi)$ is large and westerly at the edges of the Hadley circulation (viz. Fig. 6b). In the present calculations u is much larger at ϕ_{H^-} than at ϕ_{H^+}; however, both values are larger than is observed. It is now recognized (Wallace 1978; Schneider 1984) that eddies act to reduce the angular momentum in the subtropical jets. This has also been observed in a GCM (Andrews et al. 1983). Thus the angular momentum transported poleward by the Hadley circulation can be viewed as the source of the large scale eddies (baroclinic instability) of midlatitudes, and the resulting total circulation (including eddies) may be expected to have reduced values of zonally averaged u at the jet maxima. This will be discussed further in section 3.

3. Numerical simulations

The use of the simplified model for the Hadley circulation clearly revealed the singular character of the $\phi_0 = 0$ case, as well as the nature of the changes to be expected when $\phi_0 \neq 0$. However, as we saw, the results left one with the impression that one was pushing the simple model to its limits. In this section we redo the calculations using the high resolution numerical model of HH. The equations and the integration scheme are both identical to those used in HH. Since $\phi_0 \neq 0$ we can no longer appeal to symmetry; the equations must be integrated from pole to pole. Our resolution is 1° in latitude and 0.75 km in height. The model assumes a Boussinesq fluid with a lid at $z = H$ and a linear heating law described by Eq. (1a).

This numerical model is hardly an accurate detailed simulation of the atmosphere. The use of a Boussinesq fluid and a rigid lid are unrealistic, and the diabatic forcing is a gross simplification of the actual forcing. It is likely, however, to be adequate for understanding the Hadley circulation and the range of utility of the simple model. In particular, the simple model ought to describe the numerical model of this section, and, in addition, the present numerical model describes a realizable fluid which has enough physics to obey Hide's theorem. It ought, therefore, to be able to show where the simple model went wrong with respect to surface winds. It should also be noted that our simple model is almost identical to the one used by Schneider (1977) to describe the results of a significantly more realistic simulation.

As in HH we carried out numerical integrations for smaller and smaller values of viscosity until solutions failed to converge. However, our search was not as meticulous as that in HH. Our calculations indicated that the minimum value of ν was somewhere around 2.5 m^2 s^{-1}. The need for larger viscosity is not surprising in view of the discussion in section 2. Held and Hou found that for values two to three times the minimum value of ν, solutions were quantitatively similar. In the present case, our calculations have used 5 m^2 s^{-1}. The relative insensitivity of our solutions to the precise choice of ν, suggests that the presence of certain known sources of friction (cumulus friction: Schneider and Lindzen 1976, wave breaking: Lindzen 1981) may not significantly alter the present results. Using ν = 5 m^2 s^{-1}, H = 15 km, τ = 20 days, θ_0 = 300°K, Δ_h = $1/6$ and Δ_v = $1/8$, integrations were carried out for ϕ_0 = 0°, 2°, 4°, 6° and 8°.

As is inevitable in calculations of this sort, it is impractical to present all our results. In Fig. 4 we indicate results from the numerical model for ϕ_1 and ϕ_{H^-}. The numerical results for ϕ_1 are similar but somewhat smaller than those obtained from the simple model. The numerical results for ϕ_{H^-} are smaller than those obtained from the simple model. However, as we shall soon note the determination of the edges of the Hadley circulation in the numerical model is ambiguous and almost meaningless for ϕ_{H^+} (we do not show numerical model results for ϕ_{H^+} in Fig. 4).

Far more illuminating are our results for the meridional streamfunctions defined by

$$v = -\frac{1}{a\cos\phi}\frac{\partial}{\partial z}\left(\frac{\psi}{\rho_0}\right)$$

$$w = \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\frac{\psi}{\rho_0}\right).$$

Distributions of ψ for ϕ_0 = 0, 2° and 6° are shown in Fig. 9. Figure 9a for ϕ_0 = 0° simply duplicates (except for H = 15 km instead of 8 km and Δh = $1/6$ instead of $1/3$) results in HH. Air rises in the neighborhood of the equator with maximum vertical velocity at the equator. Meridional flow tends to be concentrated near the bottom and top boundaries, and the maximum value of ψ (a measure of the intensity of the Hadley circulation) is O(5 × 10^9 kg s^{-1}). Figure 9b for ϕ_0 = 2° already displays profound differences. The winter cell is over 50% stronger than the symmetric cells, while the summer cell is half the strength of the symmetric cells. Even for ϕ_0 = 2°, there is an almost 3 to 1 asymmetry between the winter and summer cells. The separation between the cells is no longer where we have maximum vertical velocity; this remains near ϕ_0. This

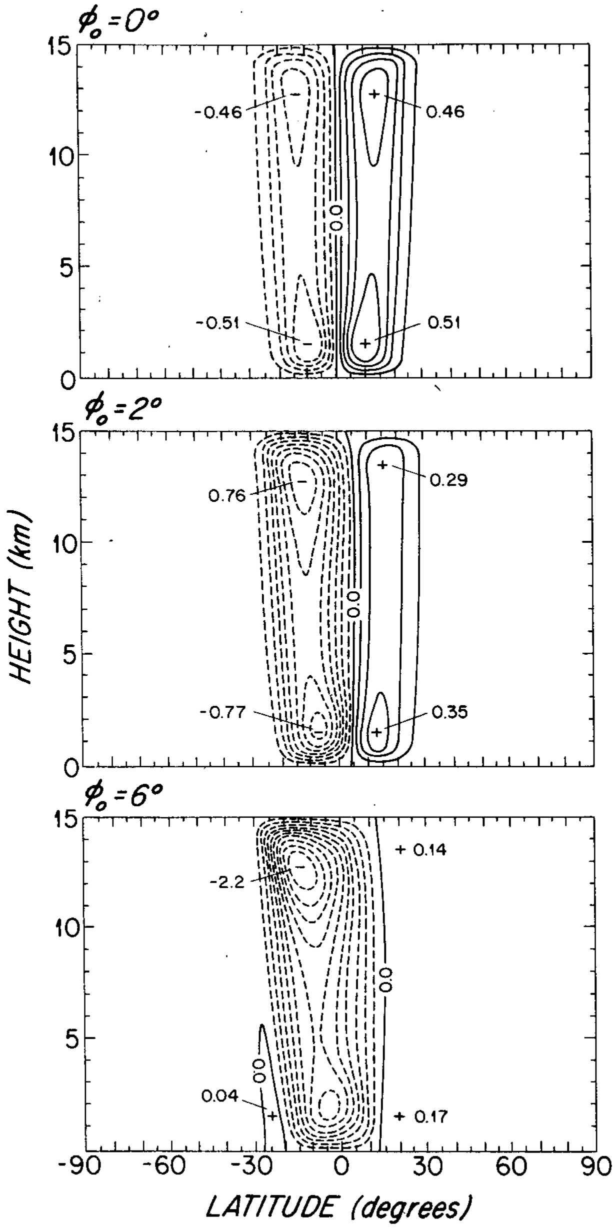

FIG. 9. Numerical model results for ψ for (a) ϕ_0 = 0°, (b) ϕ_0 = 2°, and (c) ϕ_0 = 6°. Units are in 10^{10} kg s^{-1} and the contour interval is 0.1 × 10^{10} kg s^{-1} for (a) and (b); twice this value for (c).

feature is clearer in Fig. 9c. In Fig. 9c for ϕ_0 = 6° we see the complete dominance of the winter cell, which is more than four times as intense as the symmetric cells associated with ϕ_0 = 0. The summer cell by contrast is only about one-fourth the intensity of symmetric cells. This approximately 16 to 1 asymmetry renders the summer cell inconsequential. As already noted in section 2, the annually averaged Hadley circulation will be largely a product of solstitial circulations and much larger than the equinoctial circulation.

Figure 10 shows contours of zonal wind for ϕ_0 = 6°.

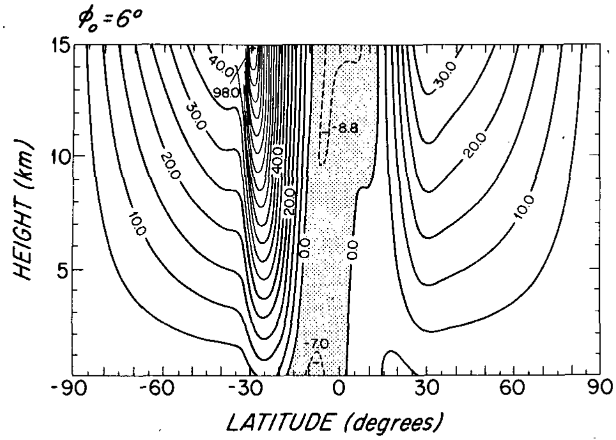

FIG. 10. The calculated zonal wind for $\phi_0 = 6°$ in m s^{-1}. Contour interval is 5 m s^{-1}.

In contrast to Fig. 6, there are no longer strong upper level equatorial easterlies. This is readily understood by reference to Fig. 9c. The poleward moving air at the top of the winter cell originates primarily between the equator and ϕ_0, not at ϕ_1. The angular momentum corresponding to near equatorial latitudes is not associated with strong easterlies at the equator [viz. Eq. (2b)]. Note also that the weak summer cell transports angular momentum so slowly as to be uncompetitive with viscosity. Thus, in the summer hemisphere the zonal winds are approximately what one gets from thermal wind balance with θ_e. The summer maximum in u is located poleward of the main Hadley circulation. In magnitude (30 m s^{-1}) and position (36°) the summer jet is not very different from what is observed. The strong winter cell, by contrast, is a strong transporter of angular momentum and produces a very strong (98 m s^{-1}) jet at the poleward side of the Hadley circulation (29°). Transports by the winter cell almost certainly sustain the large scale winter transient eddies which reduce and broaden the jet. It should be noted that the winter jet is centered closer to the equator than the summer jet—as is the case in the atmosphere.

Figure 11 shows the distribution of surface zonal wind for $\phi_0 = 0°, 2°, 6°$ and 8°. Figure 11a for $\phi_0 = 0°$, basically replicates the results in HH except that magnitudes are smaller. Surface winds are weaker because the values for Δ_h and H are different from those of HH. This also shows where the simple theory fails since according to the scaling of surface winds given in HH, the solution in the two cases should be the same (half the Δ_h value and twice the height). It appears that our streamfunction has a somewhat different vertical structure from HH's for $\phi_0 = 0$ and $\nu = 5$ m^2/5 (their maximum ψ was about 9.9×10^9 kg s^{-1}). There is no zonal wind at the equator but on both sides we have easterlies bounded on each side by westerlies. The results in Fig. 11b for $\phi_0 = 2°$ are already very noticeably different. The winter westerlies and easterlies have been strengthened and moved equatorward. Winds at the equator are now slightly easterly. Winds at ϕ_0 are slightly westerly. Poleward of ϕ_0 we get summer easterlies and westerlies which are weaker and somewhat poleward of their symmetric counterparts. The surface winds for $\phi_0 = 6°$ are shown in Fig. 11c. The winter westerlies and easterlies are much stronger. So too are the westerlies near ϕ_0. The summer easterlies and subtropical westerlies are almost negligible. Overall, the surface winds are such as to satisfy Hide's theorem and their magnitudes are reasonable; nevertheless, their relation to the results of the simple model are clearly discernible. Results in Fig. 11d for $\phi_0 = 8°$ are similar to those for $\phi_0 = 6°$, but the effects noted there are still more pronounced.

Viewed at a particular, near-equatorial latitude [n.b. $\sin(6°) \approx 0.1$], the smooth seasonal variation of ϕ_0 would be accompanied by fairly abrupt changes in surface wind, summer being characterized by surface westerlies and winter by strong easterlies. Moreover, the easterly jet on the winter side of the equator is potentially barotropically unstable. We will briefly discuss the possible role of such an instability in the generation of easterly waves in section 4. Finally, the concentration of tropical easterlies on the winter side of the equator appears in the atmosphere as well (Oort 1983). Nevertheless, one should be cautious about relating the present results for surface wind to the observed surface winds. As discussed in detail by Lindzen and Nigam (1987), the lower troposphere below the trade inversion (~700 mb) forms a relatively mixed boundary layer whose momentum budget involves, among other factors, both direct forcing due to pressure gradients resulting from surface temperature gradients and forcing due to stresses at the top of the layer. Only the latter are involved in the present calculations.

The last feature of the numerical simulations which we will discuss in some detail is the distribution of θ. Figure 12 shows contours of θ for $\phi_0 = 6°$. Not surprisingly, we no longer see the pronounced equatorial temperature minimum present in the results of the simple model. Zonal wind and temperature are related by the thermal wind relation. The temperature minimum in the simple model was associated with the easterly wind maximum at the equator; in the present calculations, the easterly wind maximum is almost eliminated. While there is no hint of an equatorial temperature minimum below 10 km, above 10 km, temperature does increase as one moves into the summer hemisphere from the equator. Presumably, the observed equatorial temperature minimum in the lower stratosphere involves processes other than those included in the present calculation. Indeed, processes do exist which might produce such a minimum. Deposition of easterly momentum by the breaking of gravity waves launched by westward moving squall lines is one possibility.

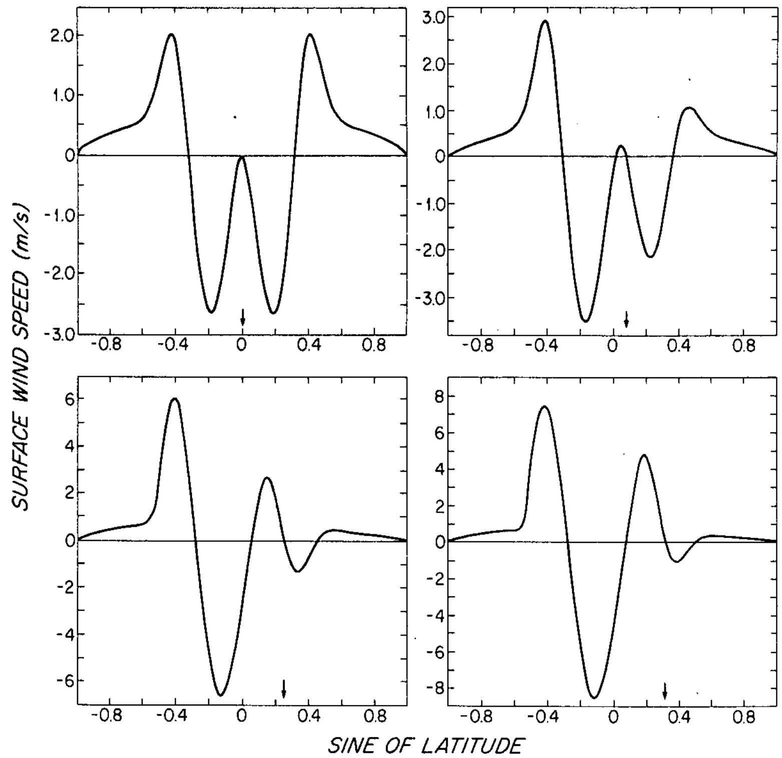

FIG. 11. Distributions of surface winds in m s^{-1} for (a) $\phi_0 = 0°$, (b) $\phi_0 = 2°$, (c) $\phi_0 = 6°$, and (d) $\phi_0 = 8°$. (n.b. $\sin\phi = 0.2$ corresponds to $\phi \approx 11.5°$.)

As already noted at the end of section 2, the numerical calculations substantiate the results of the simple model as concerns the behavior of ϕ_1 and the asymmetries in the strengths of the winter and summer Hadley cells. Unrealistic equatorial easterlies are eliminated in the numerical integrations, as are surface wind distributions which violate Hide's theorem. The surface wind distribution, however, still changes markedly as ϕ_0 departs from zero.

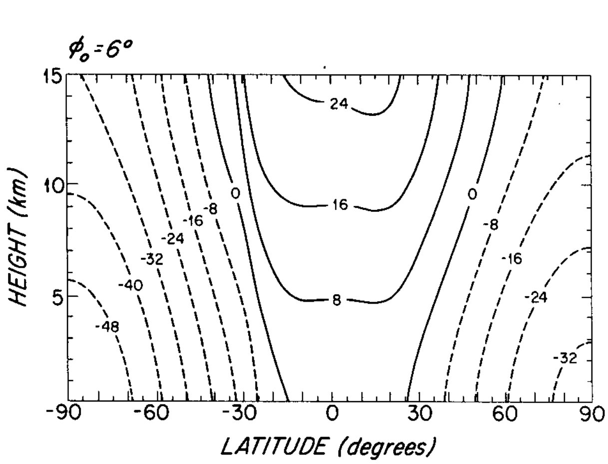

FIG. 12. The calculated temperature field for $\phi_0 = 6°$. The values correspond to departures from θ = 300 K. The contour interval is 8 K.

As noted in Schneider and Lindzen (1977) and in HH, the presence of viscosity will inevitably lead to a meridional circulation. Thus, in contrast to the simple model, Hadley circulations do not cease abruptly. Indeed at those latitudes where the simple model predicts an end to the Hadley circulation, the directly driven Hadley circulation may be weak and the viscously produced solution may dominate. This is what we meant earlier when we noted that ϕ_{H^+} and ϕ_{H^-} are not unambiguously determined in the numerical model. The difficulty is particularly evident for ϕ_{H^+} (the edge of the weak summer cell).

4. Concluding remarks

The present paper's main point has been that axially symmetric circulations depend profoundly on whether thermal forcing is displaced from the equator. An earlier study (dealing with the generation of global cross equatorial circulations on Mars forced by dust storm heating in one hemisphere) by Schneider (1983) failed to display this sensitivity simply because it assumed heating centered far from the equator. In fact, as we have shown, profound changes occur even for small displacements. The results of the present study appear to have substantial implications for our understanding of the general circulation, climate, and tropical meteorology and oceanography. We review a few of these.

1) The annually averaged Hadley circulation would appear to be largely the average of solstitial patterns consisting primarily in a single winter cell which extends from ϕ_1 (well beyond the center of heating, ϕ_0, which can be identified with the ITCZ, into the summer hemisphere) deep into the winter hemisphere. Although the annual average looks like the traditional Hadley picture (2 cells, each rising at the equator), the circulations are much stronger than what is obtained with equinoctial forcing.

2) Hadley transports (especially of angular momentum) are almost entirely into the winter hemisphere where they would appear to excite eddy instabilities. It is traditional in relating the atmosphere to annulus regimes to refer to the summer as a Hadley regime and the winter as a Rossby regime. Ironically it would appear that the summer "Hadley" regime exists because Hadley transports are unimportant for the summer hemisphere, while the winter "Rossby" regime results from the fact that Hadley transports do reach the winter hemisphere.

3) Item 2 supports a picture wherein poleward heat transport is significantly a winter phenomenon. Thus, the annually averaged climate of mid-high latitudes may be expected to be significantly dependent on the summer–winter asymmetry in heating even though this asymmetry may not contribute to the annually averaged heating. This is potentially important for the Milankovitch hypothesis for long period oscillations in glaciation. This hypothesis (Milankovitch 1930) holds that such oscillations are forced by oscillations in the earth's orbital parameters. Such orbital changes tend to be associated with seasonal asymmetries in heating rather than annually averaged heating (Suarez and Held 1976). This has traditionally led to an emphasis on the local radiative budget and on the explicit role of summer in the ice budget. However, such approaches ignore the fact that *temperature is not in general determined by the local radiative forcing.*

4) As has already been noted, the sharp easterly jet structure of the low level winds should contribute significantly to the barotropic instability of easterly waves. The fact that the jet is on the winter side of the equator does not necessarily imply that the unstable mode will be similarly distributed (viz. the Burger–Green modes of the Charney problem, Burger 1962; Green 1960). Also, as noted by Lindzen and Nigam (1987), in order for barotropic instability to modulate significantly cumulus convection it ought to occur at low levels. It must be added that the resulting wave in cumulus convection will thermally force an additional circulation throughout the troposphere. These matters are currently being investigated.

5) If one assumes that ϕ_0 undergoes a sinusoidal annual variation, the Hadley circulation and associated fields (like surface zonal wind) will undergo an annual variation more nearly square wave in shape. Such relatively abrupt changes in wind have been invoked by oceanographers as causes of equatorial ocean waves (Cane 1979; Cane and Sarachik 1979). They have also been recently associated by Salby and Garcia (1987) with the generation of 30–50 day waves in the atmosphere.

All the above remarks still retain a significant degree of speculation, but they are plausible consequences of the largely solstitial character of the Hadley circulation theoretically described in this paper. There remains, however, a particularly disturbing discrepancy with current observational analyses. Oort (1983) suggests that the zonally averaged surface temperature maximizes at the equator or north of it in all seasons. Lindzen and Nigam (1987) find that such a maximum is likely to determine the latitude of maximum cumulus convection and relatedly, diabatic heating. With heating always centered north of the equator, it would be difficult, with the present model, to account for the northern winter Hadley circulation. Given the lack of actual data south of the equator, it is perhaps premature to worry unduly about the discrepancy. Moreover, much of the latent heating in the tropics occurs over land so that sea surface temperatures alone may not be relevant. Nevertheless, there can be no question that the distribution of surface temperature needs more careful and intensive measurement.

Related to the aforementioned is an additional feature worth discussing: namely, the circulation intensities shown in Fig. 9c are substantially weaker than those observed (viz. Figs. 1 and 2). Comparison of Schneider (1977) with HH shows that this is almost certainly due to the fact that the low level convergence, by concentrating cumulus convection and latent heating, acts to redistribute the broad heating described by Eq. (16) in a narrower region about ϕ_0. The simple model of section 2 shows that such redistribution of heating should not affect the calculated behavior of ϕ_1, ϕ_{H^+} and ϕ_{H^-}, but will affect V, the meridional mass flow. The ramifications of this are currently being investigated. Similarly, although Schneider (1977) found that the effects of cumulus friction were not important in nonlinear models, we are reexamining this issue in the present context.

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